The Strategically Ignorant Principal

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April 17, 2013

Abstract

A principal-agent model is considered in which the principal decides how much private information to acquire before making an offer to the agent (e.g., an insurer decides whether to gather information about the risk status of a potential client through medical testing before offering the client a policy). I prove in a general environment that there is a nontrivial set of parameters for which it is strictly suboptimal for the principal to be completely informed, regardless of the continuation equilibrium following any information acquisition choice. This result is robust to the notion that an informed principal could select any desired equilibrium via persuasion over the agent's beliefs. The intuition is that to convince the agent that she is contracting honestly given her private information, the principal may need to severely distort the allocation. This distortion can be very costly ex ante. Choosing to be partially ignorant frees the principal from these incentive constraints and partially mitigates the damage to her ex ante payoff. I also determine in a quasilinear, three state case the optimal information acquisition choice for the principal as a function of the parameters of the model. In particular, I characterize when it is optimal to be fully ignorant of the state, when partial ignorance is preferred and when the principal wants to know the state precisely. Although a small literature has looked at the principal's information acquisition problem in specific environments or for restricted mechanisms, this paper is the first to take a mechanism design approach to the problem in a general environment. This generality is important since it allows the principal to make full strategic use of any information she acquires. Outside of this literature, the principal is assumed to have acquired information; this paper demonstrates that this assumption may be undesirable.

1   Introduction

The problem of a privately informed principal contracting with an agent is known to be relevant to many real world situations, as noted by Akerlof [1], Myerson [15], Maskin and Tirole [12, 13] and Segal and Whinston [20]. For example, an insurer may know more than the client about the risks she faces, or a franchiser may have private access to data about demand in the territory of a franchisee. As observed in this literature, a privately informed principal's payoff can be constrained by her need to convince the agent that she is contracting honestly, which can require inefficient contracts (c.f. Akerlof [1] and Maskin and Tirole [13]).

This paper studies the advantages to the principal of bypassing these constraints by making the *strategic* choice to be ignorant. We consider a standard principal-agent model and extend it by allowing the principal to costlessly learn about the state before making an offer to the agent.[1] Importantly, we allow the principal to offer a menu of contracts from which she chooses one to implement *after* the agent has accepted (à la Segal and Whinston [20]).[2] This approach favours the acquisition of information; by contrast, the simpler alternative of the point-contact leaves the principal no discretion once a contract is accepted and thus subjects her to the agent's arbitrary off-path posterior beliefs which can deter her from exploiting her private information. For example, very inefficient contracts can be supported in equilibrium by punishing deviations from said contracts with agent's beliefs that put probability 1 on the worst possible state. Despite giving the principal full strategic flexibility to exploit her information, we prove that there is a nontrivial set of parameters such that it is strictly suboptimal for the principal to acquire full information. This holds even if an informed principal can choose the continuation equilibrium she most desires.

In our framework, both the principal and the agent care about the state of the world and all choices, including the information acquisition choice, are observable.[3] For example, one could have in mind the market for *term-life* insurance, where an insurance policy can either be guaranteed renewable or renewable subject to requalification. In the latter, the period before the renewal and the requalification process may allow the insurer to accumulate information about the policy holder that the policy holder cannot access.[4] After the insurance policy is

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[1]We use the terminology of the literature by naming the actor that makes offers the *principal* while the actor who responds the *agent*. The principal is labelled as such because she controls mechanism to be played and the opposing party must accept this choice passively. A more informative, though less standard, label for the agent may be the *subordinate* as used in Myerson [15].

[2]Menu contracts are fully general trading mechanisms due to the revelation principle.

[3]Formally, we study an adverse selection model with common values as in Maskin and Tirole [13]. We relax the observability of the information choice in Section 6.

[4]The insurer employs a team of actuaries and underwriters that can make accurate predictions of mortality (Black and Skipper, [4]); even if the policy holder has access to the same information, she lacks the expertise to assess the net benefits of holding a policy.

renewed, the discretion that an insurer typically has in adjusting premiums and approving or denying claims (Macedo, [11]) indicates that the contract has features of a menu-contract. Thus, contracts subject to requalification could correspond to the choice of the principal to be privately informed. By contrast, a guaranteed renewable contract would take away the insurer's option to exploit any private information that it accumulates over the first term of the contract. Hence it resembles, to some extent, the choice of the principal to be strategically ignorant in our model.

Our first result, Theorem 1, proves under general conditions that there are always preferences such that the distortions required to make the menu offer incentive compatible are so severe that for nontrivial priors the principal finds it strictly suboptimal to be fully informed, regardless of continuation equilibria following any information acquisition choice.[5] This results holds despite the fact that information is free in our model and would thus hold *a fortiori* under the more realistic assumption that it is costly to acquire information.

While the proof of this strategic ignorance result is technically complicated, the intuition is straightforward. We choose preferences for the principal such that the difference in payoff functions between two adjacent states is small. This creates an incentive for the principal to lie in one of these states, requiring distortion in the menu-contract to maintain the principal's incentive compatibility. The principal prefers to be uninformed in order to avoid this distortion ex ante. For tractability, this theorem is based on a set of priors under which the equilibrium payoff of the fully informed principal's continuation game is uniquely the lower bound equilibrium payoff of the game. Its formal proof and those of subsequent results are presented in the Appendix.

We next ask the question of whether ignorance can be an optimal strategy when there are other equilibria in the fully informed principal's continuation game that deliver payoffs greater than the lower bound. In the general case, only these lower bound payoffs can be computed; to establish the entire set of expected equilibrium payoffs we specialize to a quasilinear, binary state environment in Section 4. We go beyond Theorem 1 to prove not only that the answer to this next question is yes, but that ignorance is optimal for nontrivial set of parameters of the model even when the principal expects to attain her highest ex ante payoff conditional on becoming informed. Thus, ignorance can be optimal even when the principal has a nontrivial opportunity to choose which equilibrium is played, à la Myerson [15]. Moreover, we prove that the restrictions on preferences needed for Theorem 1 to hold are compatible with quasilinearity, and provide more precise restrictions on the

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[5]Our strategic ignorance result is not to be confused with Myerson's [15] *inscrutability principle*. Myerson notes that the principal can never be worse off by not revealing private information when offering her menu of contracts (thus remaining inscrutable to the agents at this stage), whereas our result claims that foregoing the acquisition of private information can strictly improve payoffs.

preferences and priors for which the ignorance result holds.

In Section 5 we consider the three state case to examine the subtleties of the model when the principal is no longer restricted to being either fully informed or completely uninformed. We prove that complete ignorance is optimal for the principal in a nonempty open set of priors for nontrivial preferences when there are three states of the world.[6] More generally, we characterize the optimal information acquisition choice depending on preferences and priors.

Finally, we show that when the information choice of the principal is not observed by the agent, there is still a nontrivial set of parameters of the model under which ignorance is chosen with positive probability in equilibrium.

1.1   Related Literature

The seminal work on the informed principal problem asks whether and how the principal can exploit her informational asymmetry (Myerson [15]; Maskin and Tirole [12, 13]). These papers endow the principal with information and do not consider her decision to acquire it.

Since, a handful of papers have looked at the principal's information acquisition problem. Nosal [17] and Crémer [6] study finite horizon principal-agent problems in which a principal can acquire information before offering a contract. In both papers, the information acquired by the principal becomes public before the contract is implemented; the principal therefore does not face the same distortionary incentive compatibility constraints that drive our results. Finkle [8] also studies the information acquisition decision of a principal. His principal covertly acquires private information for a cost *after* a contract has been signed but before the contract is implemented. Finkle considers only contracts that induce full information acquisition. Our focus is different since we are concerned about how distortionary contracts can be improved upon by acquiring less than perfect information.

A number of recent papers study the informed principal problem in other environments. With multiple agents with stochastically dependant (privately known) types, Severinov [21] provides a construction that allows a privately informed principal to extract all social surplus. Thus, in this environment, the principal always wishes to obtain as much information as possible. Mylovanov and Tröger [16] focus on a linear, independent private values environment. In contrast to our common values environment, the principal can never lose by having private information but Mylovanov and Tröger determine when the principal is not strictly better off than when her information is public.

Particularly related to the current paper, Silvers [22], Kaya [10], Chade and Silvers [5],

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[6]This result is nontrivial since there is always a nonempty open set of priors such that complete knowledge is optimal for any preferences and there are always preferences such that complete knowledge is optimal for all priors.

and Beaudry [2] study the value of the principal's private information in games with moral hazard and identify parameters when the principal prefers to be ignorant. While we focus on pure adverse selection and do not consider moral hazard, we provide a deeper consideration of the incentives for a principal to acquire information. In each case, these authors restrict the principal to offer only point-contracts to the agent, which leaves her no discretion once a contract is accepted. In contrast we allow the principal to offer menu-contracts, which are fully general trading mechanisms in our framework. By Myerson's inscrutability principle, menu-contracts allow the principal to reveal no information until the agent has accepted the contract then reveal the state afterwards. This is more than a matter of technical generality. Menu-contracts preserve the strategic nature of the informed principal problem captured in the seminal work by Myerson [15] and Maskin and Tirole [12, 13], giving the principal the best opportunity to capitalize on her informational advantage. Moreover, a restriction to point-contracts can be used to exploit mistrust in the trading relationship by using pessimistic posteriors to support very inefficient equilibria, thus increasing the relative value of ignorance. Allowing the principal to offer menu-contracts eliminates these mistrustful equilibria from the game.[7]

2　An Example

The following example illustrates the main results of this paper as well as demonstrates the importance of considering fully general *menu-contracts* instead of simpler point-contracts.

Consider a car manufacturer (the principal) who is negotiating the sale of cars produced via a new production process to a dealership (the agent) who then resells the cars to consumers. Suppose there is some uncertainty in the new production process about how effectively paint can be applied to the cars: in state 1, the standard paint does not adhere properly and requires an additive that is only effective with black paint; in state 2, the standard paint can be applied successfully in the manufacturing process, allowing cars to be painted in any colour. Using the additive raises the cost of painting each car and the lack of variety reduces the demand for the car. Formally, in state i the cost to the manufacturer of producing y units of the good is $C^i(y) := c_i y$ with $0 < c_2 < c_1 < 8$; in state 1, the downstream inverse demand for the car is $P^1(y) := 8 - y$ while in state 2 it is $P^2(y) := 9 - y$.

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[7]The importance of allowing more general mechanisms here is analogous to the work of Segal and Whinston [20]. By generalizing offers in a family of bilateral contracting games to allow for menu contracts, these authors are able to make robust predictions about the game in the sense that they must be satisfied by all equilibria in all such games. Whether restricting the principal to point-contracts has bite depends on the specific environment. In a separate note (Bedard [3]) we characterize moral hazard environments where the principal can get strictly higher ex ante payoffs when allowed to use more general mechanisms.

Thus, given contract (y, t), the payoff to the manufacturer in state i is $V^i(y, t) = t - c_i y$ while the payoff to the dealership is $W^i(y, t) = P^i(y)y - t$. Let π be the common prior belief that the state of the world is 1.

Consider the case where the manufacturer is informed of the state of the world and suppose the manufacturer can only offer a point-contract: a single pair (y, t). Further, suppose that the dealership is highly mistrustful of the manufacturer and rejects any offer that would give her negative payoff in at least one state of the world. Formally, she believes the state is 1 with probability 1 for any offer (y, t) such that $P^1(y)y - t < 0$ and maintains her prior belief π otherwise. The optimal equilibrium point-contract for the manufacturer given these beliefs is $\left(y^{PC}, t^{PC}\right) = \left(\frac{8-c_1}{2}, \left(8 - \frac{8-c_1}{2}\right)\frac{8-c_1}{2}\right)$ regardless of the state. Note that $\left(y^{PC}, t^{PC}\right)$ gives the dealership zero payoff in state 1 and strictly positive payoff in state 2.

The game where the manufacturer can offer point-contracts has other equilibria, some of which are better for the her than the one described above. For example, there is an equilibrium where the manufacturer offers $\left(y_1^{LCS}, t_1^{LCS}\right) := \left(\frac{8-c_1}{2}, \left(8 - \frac{8-c_1}{2}\right)\frac{8-c_1}{2}\right)$ in state 1 and

$$\left(y_2^{LCS}, t_2^{LCS}\right) := \operatorname*{argmax}_{(y,t)} \left\{ t - c_2 y : V^1(y_1^{LCS}, t_1^{LCS}) \geq V^1(y, t), P^2(y)y = t \right\} \quad (1)$$

in state 2. This is the least-cost separating equilibrium and is the best equilibrium for the manufacturer when she can only offer point-contracts.

Now consider the case where the manufacturer can offer menu-contracts. A menu-contract is a list of point-contracts offered to the retailer that gives the manufacturer the discretion to choose which contract to implement after the retailer has accepted. Menu-contracts guarantee that the manufacturer's payoff is at least as high as in the least-cost separating equilibrium; in particular, menu-contracts allow us to get rid of equilibrium contracts based on mistrust like (y^{PC}, t^{PC}). To see this, suppose the manufacturer offers the menu $\left\{ \left(y_1^{LCS}, t_1^{LCS}\right), \left(y_2^{LCS}, t_2^{LCS}\right) \right\}$ in both states the world. This menu is acceptable to the retailer *regardless of her belief*: it gives her nonzero payoff in each state of the world, assuming the manufacturer chooses optimally from the menu. Since we have imposed an incentive compatibility constraint for the manufacturer, this assumption is valid.[8] Thus, the manufacturer can always offer this menu-contract and obtain its payoff. It therefore provides a lower bound on the payoff the manufacturer expects to earn when she is able to offer menu-contracts. This menu-contract is called the RSW menu-contract. It is introduced by Maskin and Tirole [13, p11] and it plays a important role in our analysis below. We present its technical definition and discuss its significance in Section 3.

We will now determine when the manufacturer prefers to learn the state of her production

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[8]See problem (1).

process and when she would rather be uninformed. Let $c_1 = 4$ and $c_2 = 2.9$. The uninformed principal solves the problem

$$\max_{(y,t)}\{t - (8\pi + 9(1 - \pi))y - 4\pi - 2.9(1 - \pi) \mid (8\pi + 9(1 - \pi) - y)y - t \geq 0\}.$$

The value of this problem is

$$\left(\frac{4\pi + 6.9(1 - \pi)}{2}\right)^2. \tag{2}$$

The informed manufacturer's problem potentially has multiple equilibria depending on priors which can give her higher payoffs than the RSW menu. Nevertheless, we will start with the RSW lower bound menu. The RSW menu is given by $\{(2, 12), (3.05, 18.15)\}$ and gives expected payoff

$$4\pi + 9.3(1 - \pi). \tag{3}$$

Notice that y_2^{LCS} is inefficient: marginal cost is greater than marginal revenue; because the manufacturer's incentive constraint is violated at the efficient state 2 production level, production in this state must be increased so that the constraint just binds. State 1 production is always efficient because the manufacturer will never want to pretend to be in state 1 when it is state 2 (i.e. the downward incentive constraint for the manufacturer will never bind).

Expression (2) is strictly greater than (3) if and only if $\pi < 0.82$. So ignorance is preferred when the manufacturer expects the RSW menu to be played in equilibrium as long as the prior is below a cut-off value. This is because the inefficiency in the RSW menu occurs only in state 2; the manufacturer has to expect that state 2 is sufficiency likely to occur to prefer ignorance.

Depending on priors, other menu-contract equilibria can exist that give higher payoffs to the informed manufacturer ex ante. In particular, in Section 4 we characterize the highest payoff the informed manufacturer can expect. Although the details are beyond the scope of this section, one can show that being ignorant of the state delivers strictly higher payoffs for the manufacturer ex ante than any equilibrium menu-contract if and only if $\pi \in (0.62, 0.82)$.

We have discussed why this interval has an upper cut-off. To understand the lower bound on this interval consider that for low π the manufacturer can mitigate the inefficiency in state 2. To see this, first notice that we can assume without loss of generality that the manufacturer offers the same menu in both states of the world.[9] This implies that dealership evaluates the menu offer using her prior belief: i.e. she accepts the offer if and only if her

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[9]This is due to Myerson's [15] inscrutability principle.

participation constraint is satisfied on average:

$$\pi[(8 - y_1)y_1 - t_1] + (1 - \pi)[(9 - y_2)y_2 - t_2] \geq 0. \tag{4}$$

Now suppose we set y_2 to be efficient and at the same time increase t_1 and decrease t_2 until the manufacturer's incentive constraint is just satisfied. When we do this, the first term of (4) becomes negative but the second term becomes positive. For small enough π, (4) will be satisfied and the dealership will accept the menu. Meanwhile, the manufacturer earns the full expected trade surplus at this prior and therefore chooses to become informed. As π increases, eventually full efficiency will not be attainable. In this example, when $\pi = 0.62$, it is just low enough that the closest the manufacturer can get to the efficient y_2 generates ex ante payoffs that are equal to the uninformed manufacturer's payoff.

Finally, in this example the highest payoff the informed manufacturer can achieve under point-contracts is the RSW payoff, by definition the least cost separating equilibrium.[10] For $\pi < 0.82$, even this payoff is less than the uninformed equilibrium payoff (2). In contrast, we can show that if $c_2 < 2.73$ (with $c_1 = 4$), there exists at least one menu-contract for any prior such that it is better to be informed. Thus, if we were only to look at point-contracts in this case (with $c_2 < 2.73$ and low enough π) we would conclude that the principal has a negative value of information whereas this value can be positive when menu-contracts are allowed.

3   The Model and Suboptimality of Full Information

The state space is $N = \{1, \dots, n\}$ for $n < \infty$. The game proceeds in four stages. First, the principal makes an *information acquisition choice*: a partition of the state space. This choice is observable and verifiable and the principal privately observes the partition cell to which the state belongs. There is no cost associated with the information choice. Second, she offers a menu of contracts. Third, the agent accepts or rejects the offer. Rejection leaves all parties with zero payoff. Acceptance leads to the final stage where the principal chooses a contract from the menu and said contract is implemented. The principal and agent can commit to the menu-contract which the agent accepted.

A contract specifies an action-transfer pair $(y, t) \in \mathbb{R}^2$. In state $i \in N$, when contract (y, t) is implemented, the principal earns payoff $V^i(y, t)$ and the agent earns payoff $W^i(y, t)$. We follow the notational convention of Maskin and Tirole [13] by having superscripts on payoff

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[10]It can be shown that no pooling equilibrium can ever be sustained: the state 1 manufacturer will always wish to deviate.

functions indicate the state. Both functions V^i and W^i are continuously differentiable and concave in (y, t). Function V^i is increasing in t and decreasing in y while W^i is increasing in y and decreasing in t. In addition, W^i is increasing in state i for almost all (y, t). We make no explicit assumptions about the principal's preferences over states although item (iii) in Assumption 1 below puts some structure over how the principal's marginal rates of substitution varies by state. Both parties are expected utility maximizers.

We adopt the following standard sorting assumption on preferences from Maskin and Tirole [13]. Subscripts on payoff functions denote partial derivatives: $V^i_y(y, t) = \partial V^i(y, t)/\partial y$, $V^i_t(y, t) = \partial V^i(y, t)/\partial t$ with agent's marginal payoffs defined analogously.

Assumption 1 (Sorting) *(i)* $W^i_y(y, t) \geq 0$ *for all* $(y, t) \in \mathbb{R}^2$ *and there is an* $\epsilon > 0$ *such that* $V^i_y(y, t) < -\epsilon$, $V^i_t(y, t) > \epsilon$, $W^i_t(y, t) < -\epsilon$ *for all* $i \in N$ *and all* $(y, t) \in \mathbb{R}^2$;

(ii) $-W^i_y(y, t)/W^i_t(y, t) \to 0$ *as* $y \to \infty$ *for all* $t \in \mathbb{R}$; *and* $-W^i_y(y, t)/W^i_t(y, t) \to \infty$ *as* $y \to -\infty$ *for all* $t \in \mathbb{R}$;

(iii) $-V^i_y(y, t)/V^i_t(y, t) > -V^j_y(y, t)/V^j_t(y, t)$ *for all* $i < j \in N$ *and all* $(y, t) \in \mathbb{R}^2$.

In this framework, the menu contracts described above are direct revelation mechanisms: a list of n contracts $\{(y_i, t_i)\}_{i=1}^n$ such that the principal offers the menu-contract in stage two of the game and chooses a contract from the menu to implement in stage four of the game. Due to the revelation principle, menu-contracts are fully general trading mechanisms.

An important menu-contracts in the informed principal game is the RSW menu.[11] Introduced by Maskin and Tirole [13, p11], it generates the lower bound payoff for the informed principal and it plays a large role in our analysis below. We now present its technical definition then provide intuition about why it is the principal's lower bound payoff.

Definition 1 *The RSW payoff for the principal in state* j *is the principal's lower bound payoff in that state. It is attained by solving the problem*

$$V^j_r := \max_{\{(y_k, t_k)\}_{k \in N}} V^j(y_j, t_j)$$

$$s.t. \ (RSW\text{-}IC[l,k]) \quad V^l(y_l, t_l) \geq V^l(y_k, t_k) \ for \ all \ l, k \in N; \ and$$

$$(RSW\text{-}IR[k]) \quad W^k(y_k, t_k) \geq 0 \ for \ all \ k \in N.$$

Denote by (y^r_j, t^r_j) *the state* j *principal's contract in her solution to this problem. Let* $\{(y^r_k, t^r_k)\}_{k \in N}$ *denote the menu such that each* (y^r_k, t^r_k) *solves the RSW problem for all* $k \in N$.

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[11]RSW is an acronym for Rothchild-Stiglitz-Wilson, a reference to the similar least cost separating contracts developed in the insurance models of Rothschild and Stiglitz [18] and Wilson [24].

The RSW problem generates lower bound payoffs for the principal in state j since the agent will accept any RSW menu *regardless of her belief about the state of the world*.[12] To see this, note first that the RSW problem for the principal in state j specifies an entire menu: a contract for each state $k \in N$. This menu must be incentive compatible in *every* state $k \in N$, not just state j. Finally, this menu must guarantee the agent her reservation payoff *ex post* in *every* state. Thus, the agent will always accept an RSW menu. The principal in any state $j \in N$ can always deviate to her RSW menu and get payoff V_r^j.[13]

Theorem 1 *Suppose Assumption 1 holds. Then, for any set of payoffs $(W^1, \ldots, W^n)$ for the agent, there are payoffs functions $(V^1, \ldots, V^n)$ for the principal and a nonempty open set of priors such that for any priors in this set, the principal finds it strictly suboptimal to be fully informed regardless of the continuation equilibria (in pure strategies) following information acquisition.*

The formal proof of this theorem and all subsequent results appear in Section A. To prove this theorem, we restrict priors such that within the restricted set the equilibrium payoff of the fully informed principal's continuation game is uniquely the RSW payoff. That is, under the set of priors referred to in the theorem, the principal's payoff when fully informed is unique and is her lower bound payoff for the fully informed continuation game. In the next section we show that the strategic ignorance result holds when there are multiple equilibria with payoffs that are greater than the RSW payoff for the principal in all states.

4 Strategic Ignorance Despite Multiple Equilibria

In this section we specialize to the quasilinear, binary state environment. Here, we are able to characterize the entire set of equilibrium payoffs. We therefore go beyond Theorem 1 to prove that ignorance can be optimal even when there exist equilibrium payoffs higher than the RSW lower bound, and in particular that ignorance is optimal for nontrivial parameters of the model even when the principal expects to attain her highest ex ante payoff conditional on becoming informed. This is shown in Theorem 2. Thus ignorance can be optimal even when principal can choose from among multiple equilibria, conditional on being informed, via persuasion over the agent's beliefs (à la Myerson [15]).

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[12]In terms of Myerson [15], any feasible solution to the RSW problem is *safe*. The RSW menu for the principal in state j is the best safe menu in state j.

[13]For further discussion of RSW menus and a general characterization of equilibrium menus in this framework, see Maskin and Tirole [13]

4.1 Preferences and Supplemental Assumptions

Let $n = 2$. Given contract (y, t), the principal gets payoff $V^i(y, t) = t - C^i(y)$ for $i \in \{1, 2\}$ and the agent gets payoff $W^i(y, t) = U^i(y) - t$. Let $MC^i := dC^i/dy$ and $MU^i := dU^i/dy$ for all $i \in \{1, 2\}$. We will refer to C^i as the principal's *cost* in state i and U^i as the agent's *revenue* in state i.

We assume these payoff functions have the same properties as defined in the Introduction and satisfy Assumption 1. We make the following further assumptions on the principal's cost function.

Assumption 2 *For all states $i \in \{1, 2\}$: (i) C^i is strictly decreasing in i for all $y \neq 0$; and (ii) $dMC^i(\cdot)/dy$ is nondecreasing in i.*

Item (i) says that the principal and the agent agree about which state is the good state.[14] Item (ii) ensures that the RSW contract is unique and deterministic. For example, $C^i(y) = y^2 - iy + 2 - i$ satisfies all our assumptions for $y > 0$.

For $n = 2$ the principal is either fully informed or completely ignorant. If the principal chooses not to learn the state, the offer in stage two is a single contract. Define $\pi := \pi_1$ as the probability that the state is 1. In this case, the contract is the solution to the uninformed principal's problem:

$$V_u(\pi) := \max_{(y,t)} \left\{ t - \pi C^1(y) - (1 - \pi)C^2(y) \mid \pi U^1(y) + (1 - \pi)U^2(y) - t \geq 0 \right\}. \quad (5)$$

An equilibrium consists of an information acquisition choice (either ignorance or knowledge) together with a contract for each known state and a list of accept/reject decisions from the agent corresponding to any information choice and menu offered such that the information strategy, the offer, and list of the agent's decisions constitute a perfect Bayesian Nash equilibrium.

Define

$$\kappa := \sup_y MC^1(y)/MC^2(y) > 1.$$

The parameter κ measures the severity of distortions needed in an informed principal's menu to maintain incentive compatibility as a function of preferences.

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[14]This eases incentive compatibility requirements relative to the case where they disagree. We therefore expect the results to carry over to the latter case.

4.2 Ignorance and the Best Ex Ante Informed Payoff

The following problem delivers the highest equilibrium payoff the principal can expect ex ante conditional on becoming informed. The **ex ante optimal informed principal's problem** is

$$V^*(\pi) := \max_{\{(y_i, t_i)\}_{i \in \{1,2\}}} \sum_{i \in \{1,2\}} \pi_i \left(t_i - C^i(y_i) \right) \tag{6}$$

$$s.t. \ (\text{IC}[i,j]) \quad t_i - C^i(y_i) \geq t_j - C^j(y_j) \text{ for all } i \neq j \in \{1,2\}$$

$$(\text{IR}) \quad \sum_{i \in \{1,2\}} \pi_i \left(U^i(y_i) - t_i \right) \geq 0$$

$$(\text{NB}[i]) \quad t_i - C^i(y_i) \geq V_i^r \text{ for all } i \in \{1,2\}.$$

The constraints NB[i] for $i \in N$ are the *non-blocking* constraints. They state that the informed principal cannot commit to a contract that gives her a payoff lower than her RSW payoff in any state. Maskin and Tirole's [13, p19] Theorem 1 proves that these constraints form sufficient and necessary conditions for a menu-contract to be an equilibrium.

Next, we define an ordering for menus among the principal in different states. One menu is superior to another if it delivers strictly higher payoff to the principal in at least one state and at least as high a payoff in the other.

Definition 2 *A menu* $\{(y_i, t_i)\}_{i \in N}$ *is **superior** to another menu* $\{(y_i', t_i')\}_{i \in \{1,2\}}$ *if* $t_i - C^i(y_i) \geq t_i' - C^i(y_i')$ *for all* $i \in \{1,2\}$ *and there exists* $j \in \{1,2\}$ *such that* $t_j - C^j(y_j) > t_i' - C^j(y_j')$.

Our main result of this section says that there exist preferences such that even when the principal expects to earn V^* *and* that payoff is superior to her RSW payoff, she will still wish to remain ignorant of the state for a nontrivial set of priors.

Theorem 2 *Suppose Assumptions 1 and 2 hold. If κ is sufficiently close to 1, there exists a nonempty, open interval of priors such that, for any priors in this interval, the principal is uninformed regardless of the continuation equilibrium played following information acquisition and there are multiple continuation equilibria following information acquisition that are superior to the informed principal's RSW lower bound.*

In particular, for any preferences and priors π specified in the theorem, choosing to be ignorant delivers strictly higher payoff than becoming informed and earning payoff $V^*(\pi)$.

To discuss the intuition of Theorem 2 we define the first best menu of contracts.

Definition 3 *Let action y_i^E be called efficient in state $i \in \{1,2\}$ if $MC(y_i^E) = MU(y_i^E)$. A menu is **first best** if it is efficient in both states. Define*

$$V^{FB}(\pi) := \pi \left(U^1(y_1^E) - C^1(y_1^E) \right) + (1 - \pi) \left(U^2(y_2^E) - C^2(y_2^E) \right)$$

to be the value of the first best menu to the principal ex ante.

Figure 1 illustrates the following intuition behind Theorem 2. We show in Lemma 11 in Section A.2 that when κ is close to 1, RSW-IC[1,2] binds and as a result $y_2^r > y_2^E$. The RSW menu in the continuation game following full information acquisition is thus distorted away from the first best. For low π (lower than π^{FB} in Figure 1), the menu that solves problem (6) can completely mitigate this inefficiency and the principal can attain the first best payoff ex ante. As π increases, however, this become impossible to do and V^* eventually settles to the RSW lower bound payoff $V_r(\pi) := \pi V_r^1 + (1 - \pi)V_r^2$. We label this point π^r.

In Proposition 1 (to follow), we show that there exists preferences and $\pi^* \leq 1$ such that $V_u(\pi) > V_r(\pi)$ for all priors $\pi \in (0, \pi^*)$: ignorance generates a higher payoff than the expected RSW payoff for the principal. This can be seen in Figure 1. Further, in Proposition 2 we show that that V^* is continuous and that there exists preferences such that $\pi^r < \pi^*$. Thus, $V_u(\pi) - V^*(\pi) < 0$ for $\pi \in \left(0, \pi^{FB}\right]$ and $V_u(\pi) - V^*(\pi) > 0$ for $\pi \in [\pi^r, \pi^*)$. Since both V_u and V^* are continuous, the intermediate value theorem states there must be some $\pi' \in (\pi^{FB}, \pi^r)$ such that $V_u(\pi') = V^*(\pi')$.[15] Thus, for $\pi \in (\pi', \pi^r)$, we have $V_u(\pi) > V^*(\pi) > V_r(\pi)$: the statements of Theorem 2 hold.
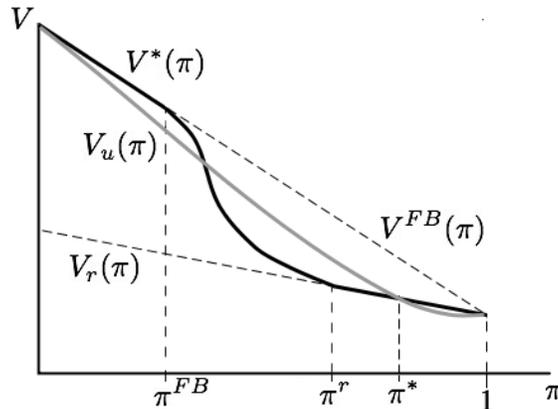

Figure 1: This figure illustrates Theorem 2. Note the nonempty, open set of priors such that $V_u(\pi) > V^*(\pi) > V_r(\pi)$.

The next proposition establishes the value of the ignorant principal's problem (5) relative

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[15]If there are multiple such π', choose the largest.

to the ex ante RSW payoff and characterizes this relative value in terms of preferences and priors.

Proposition 1 *Suppose Assumptions 1 and 2 hold. If κ is sufficiently close to 1 then there exists $\pi^* \in (0,1]$ such that for any priors $\pi \in (0,\pi^*)$, $V_u(\pi) > V_r(\pi)$: the principal strictly prefers her ignorant payoff to her informed ex ante RSW payoff; if $\pi \in (\pi^*,1)$, then $V_u(\pi) < V_r(\pi)$. Moreover, there exists $\underline{\kappa}$ such that $\pi^* = 1$ if $\kappa < \underline{\kappa}$.*

Figure 2 illustrates the following intuition behind Proposition 1. Figure 2(a) plots, in (y,t)-space, the informed RSW solution when the informed principal is constrained by incentive compatibility. It illustrates how the RSW contract entails inefficiently high y in state 2 and efficient y in state 1. To see why the RSW action is efficient in state 1, note that the principal can offer the menu-contract $\left\{ \left(y^E, U(y^E) \right), \left(y^E, U(y^E) \right) \right\}$. It is straight forward to check that this menu is ex post incentive compatible (i.e. satisfies RSW-IC[1,2] and RSW-IC[2,1]) and is individually rational for the agent in both states. Thus, $\left\{ \left(y^E, U(y^E) \right), \left(y^E, U(y^E) \right) \right\}$ is an RSW menu for the principal in state 1. Since $\left(y^E, U(y^E) \right)$ is a tangency point on the agent's indifference curve at her reservation utility, it is the unique state contract that gives the state 1 principal her efficient payoff $U^1(y^E) - C^1(y^E)$ and therefore the unique state 1 contract in the RSW menu. The state 2 contract in the RSW menu is then the least cost separating equilibrium, as plotted in the figure. The Figure 2(b) plots the functions V_u and V_r when κ is sufficiently close to 1 that the state 2 RSW contract is inefficient.
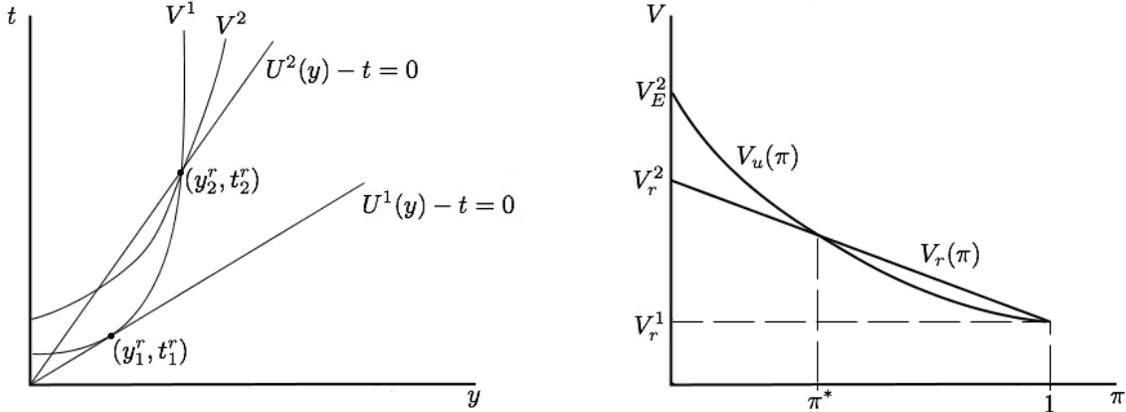
(a) V^i indicates the RSW indifference curve for the principal in state i.

(b) $V_2^E := U^2(y_2^E) - C^2(y_2^E)$ is the value of the efficient contract payoff to the principal in state 2.

Figure 2: Example of informed principal RSW solution and value function and uninfored value function when the principal is constrained by incentive compatibility

Notice that the state i RSW problem is independent of priors; this implies that, even as the probability of state 2 approaches 1, the value of the RSW problem for the state 2 principal

will be less than the value of the first-best menu. Meanwhile, the uninformed principal is unburdened by incentive compatibility constraints and her ex post payoff approaches efficient levels as π approach 0 and 1. Further, the uninformed value function is convex in π. Since V_r is linear in π, these value functions must intersect at most twice as a function of π: once at $\pi = 1$, since the state 1 contract is always efficient when the principal is informed, and once at some $\pi \geq 0$. Denote the first intersection as π increases from 0 to 1 by π^*. As Proposition 1 asserts, $\pi^* > 0$ for κ close enough to 1. For all priors $\pi < \pi^*$, the uninformed principal's payoff will be higher ex ante than the informed principal RSW payoff.

Our next proposition states that there exists preferences and priors such that the optimal ex ante equilibrium payoff is achieved by being ignorant of the state, even when the principal expects to attain V^* upon becoming informed.

Proposition 2 *Suppose Assumptions 1 and 2 hold. If κ is sufficiently close to 1 then there exists $\pi^r < \pi^*$ such that for any priors $\pi \in (\pi^r, \pi^*)$, $V_u(\pi) > V^*(\pi) = V_r(\pi)$: the unique ex ante optimal informed payoff is the RSW payoff and the uninformed principal's payoff is strictly larger.*

Remark 1 While Propositions 1 and 2 may appear to be corollaries of Theorem 1, they are making stronger statements than such a corollary could make. First, our assumptions on preferences (i.e. that κ is sufficiently close to 1) restrict only the second order properties of the payoff functions rather than the entire function as in Theorem 1. Moreover, Theorem 1 could not be specific about which priors admit ignorance as an optimal strategy whereas the results in this section can.

The main task in the proof of Proposition 2 is to characterize the ex ante optimal informed principal problem (6). This allows us to prove the existence of π^r and, importantly, that it is strictly less than 1. Further, we show that V^* is continuous.

The existence of π^r is proved by demonstrating that for high enough π the state 2 RSW contract cannot be altered at all without violating either the state 1 principal's incentive compatibility constraint or the agent's individual rationality constraint. Thus, V^* must equal the ex ante RSW payoff for such priors. To see this, note that to improve on the RSW payoff we must reduce y_2^r closer to its efficient level: since the principal gets all gains from trade in the RSW payoff, the only way to increase her payoff is to increase the gains from trade. Decreasing y_2 requires that we deliver a higher payoff to the state 1 principal to maintain incentive compatibility. Since y_1^r is efficient, however, U^1 is tangent to C^1 at (y_1^r, t_1^r). This implies that the agent's payoff must be less than her reservation value in state 1. We can give the agent a payoff higher than her reservation value in state 2 as we move y_2

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closer to y_2^E to balance out this state 1 deficit ex ante; if π is too large, however, we cannot give the agent a high enough surplus in state 2 to make up for the deficit in state 1 that is required to maintain incentive compatibility. We label π^r as the prior at which this point is just hit as π increases from 0 to 1 and we note that $\pi^r < 1$ since the state 1 indifference curve is everywhere steeper than the state 2 indifference curve. Hence, for $\pi \in [\pi^r, 1)$, we have $V^*(\pi) = V_r(\pi)$. Finally, we can appeal to Proposition 1 and choose κ close enough to 1 such that $\pi^* > \pi_r$. Then for $\pi \in (\pi^r, \pi^*)$, we have $V^*(\pi) = V_r(\pi) < V_u(\pi)$.

The results in this section have so far used the distortionary effects of the incentive constraints conditional on the principal being informed as a sufficient condition for ignorance of the state to be of strategic advantage. The final proposition of this section shows that binding incentive constraints in the menu offered by the informed principal are also necessary.

Proposition 3 *If RSW-IC[1,2] does not bind, then ignorance will never be chosen in equilibrium. Moreover, the informed RSW problem generates the first best menu and the unique equilibrium payoff for all priors.*

5 Optimal Information Structure: Three States

In this section we consider the three state case to examine the subtleties of the model when the principal no longer faces a binary choice of information acquisition. She can now choose *how* informed or *how* ignorant she wishes to be. We show that complete ignorance of the state is optimal for the principal in a nonempty open set of priors for nontrivial preferences. More generally, we characterize optimal information acquisition choice depending on preferences and priors. Further, we find that if the principal is exogenously restricted to choosing between complete knowledge of the state or complete ignorance, there are preferences and a nonempty open set of priors such that complete ignorance is preferred.

5.1 General Information Structures

An information choice by the principal consists of any partition of the set N. Let $\mathscr{P}$ be the set of all partitions of N. We will refer to $p \in \mathscr{P}$ as an information acquisition option; the ith cell of p is denoted p_i and is referred to as an information set. Given information acquisition option p, the state space becomes p in a new informed principal problem with typical state p_i. A choice of information option p generates payoff functions

$$C^{p_i}(y) := \left(\frac{1}{\sum_{j \in p_i} \pi_j} \right) \sum_{j \in p_i} \pi_j C^j(y)$$

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$$U^{p_i}(y) := \left(\frac{1}{\sum_{j \in p_i} \pi_j} \right) \sum_{j \in p_i} \pi_j U^j(y)$$

for each information set $p_i \in p$. Associated with each $p \in \mathscr{P}$ there is an RSW menu which we denote the p-RSW menu.[16]

Our goal is to analyze the optimal information acquisition options in this environment. As in the case of two states, we use the closeness of the relative marginal costs between states to measure the severity of the distortions introduced by the incentive constraints. Since the information acquisition choice is no longer binary, however, we require a second parameter. The second measures the *separateness* of the relative marginal costs between states. Whereas the first provided us with sufficient conditions for ignorance between two states, the second will provide sufficient conditions for the principal to be informed of the two states. Define the following

$$\kappa_i^S := \sup_y \frac{MC^i(y)}{MC^{i+1}(y)}; \text{and} \tag{7}$$

$$\kappa_i^I := \inf_{y > y_i^E} \frac{MC^i(y)}{MC^{i+1}(y)} \tag{8}$$

for all $i \in N \setminus \{n\}$ where y_i^E satisfies $MU^i(y_i^E) = MC^i(y_i^E)$.

5.2 Three states of the world

Our result in this section identifies sufficient conditions for certain information acquisition strategies to be optimal. Figure 3, panels (a) to (c) indicate (the shaded areas) the priors under which Proposition 4 parts (A) to (C) apply respectively in a 3 dimensional simplex.

Proposition 4 *Suppose Assumptions 1 and 2 hold. Let $\Delta_o^3 := \{\pi \in (0,1)^3 | \sum_i \pi_i = 1\}$ be the set of non-degenerate priors and $p^1 = \{\{1,2\}, \{3\}\}, p^2 = \{\{1\}, \{2,3\}\}, p^3 = \{\{1,2,3\}\},$ $p^4 = \{\{1\}, \{2\}, \{3\}\},$ and $p^5 = \{\{1,3\}, \{2\}\}.$*

(A) There exists $\bar{\kappa}_1^S > 1$ and $\underline{\kappa}_2^I$ such that for $\kappa_1^S < \bar{\kappa}_1^S$ and $\kappa_2^I > \underline{\kappa}_2^I$, there exists $\hat{\pi} \in \Delta_o^3$ such that for any

$$\pi \in \left\{ \pi' \in \Delta^3 \ \middle| \ \begin{array}{l} \pi_1 \in (\pi_1', 1), \frac{\pi_1'}{\pi_1' + \pi_2'} \in \left(\frac{\hat{\pi}_1}{\hat{\pi}_1 + \hat{\pi}_2}, 1 \right), \frac{\pi_2'}{\pi_2' + \pi_3'} \in \left(\frac{\hat{\pi}_2}{\hat{\pi}_2 + \hat{\pi}_3}, 1 \right) \\ \pi_1' + \pi_3' \in (\hat{\pi}_1 + \hat{\pi}_3, 1), \frac{\pi_1'}{\pi_1' + \pi_3'} \in \left(\frac{\hat{\pi}_1}{\hat{\pi}_1 + \hat{\pi}_3}, 1 \right) \end{array} \right\}$$

the optimal information acquisition option is p^1;

[16]See Section A.3 for a formal description of the p-RSW menu.

(B) *There exists $\bar{\kappa}_2^S > 1$ and $\underline{\kappa}_1^I$ such that for $\kappa_2^S > \bar{\kappa}_1^S$ and $\kappa_1^I < \underline{\kappa}_2^I$, then there exists $\hat{\pi} \in \Delta_o^3$ such that for any*

$$\pi \in \left\{ \pi' \in \Delta^3 \ \left| \ \begin{array}{c} \pi_2' \in (\hat{\pi}_2, 1), \frac{\pi_2'}{\pi_2' + \pi_3'} \in \left(\frac{\hat{\pi}_2}{\hat{\pi}_2 + \hat{\pi}_3}, 1 \right), \\ \frac{\pi_1'}{\pi_1' + \pi_2'} \in \left(0, \frac{\hat{\pi}_1}{\hat{\pi}_1 + \hat{\pi}_2} \right), \frac{\pi_1'}{\pi_1' + \pi_3'} \in \left(0, \frac{\hat{\pi}_1}{\hat{\pi}_1 + \hat{\pi}_3} \right) \end{array} \right. \right\}$$

the optimal information acquisition option is p^2; and

(C) *There exists $\underline{\kappa}_1^S > 1$ and $\underline{\kappa}_2^S > 1$ such that if $\kappa_1^S < \underline{\kappa}_1^S$ and $\kappa_2^S < \underline{\kappa}_2^S$, then there exists $\hat{\pi} \in \Delta_o^3$ such that for any*

$$\pi \in \left\{ \pi' \in \Delta^3 \ \left| \ \begin{array}{c} \pi_1' + \pi_2' \in (\hat{\pi}_1 + \hat{\pi}_2, 1), \pi_1' \in (\hat{\pi}_1, 1), \\ \frac{\pi_1'}{\pi_1' + \pi_2'} \in \left(\frac{\hat{\pi}_1}{\hat{\pi}_1 + \hat{\pi}_2}, 1 \right), \pi_1' \in (\hat{\pi}_1, 1), \frac{\pi_1'}{\pi_1' + \pi_3'} \in \left(\frac{\hat{\pi}_1}{\hat{\pi}_1 + \hat{\pi}_3}, 1 \right) \end{array} \right. \right\}$$

the optimal information acquisition option is p^3.

(D) *There exists $\bar{\kappa}_1^I > 1$ and $\bar{\kappa}_2^I > 1$ such that if $\kappa_1^I > \bar{\kappa}_1^I$ and $\kappa_2^I > \bar{\kappa}_2^I$ then the optimal information acquisition option is p^4. Moreover, there exists $\hat{\pi} \in \Delta_o^3$ such that if $\pi_3 \in (\hat{\pi}_3, 1)$ then the optimal information acquisition option is p^4.*

Notice that information acquisition options p^1, p^2 and p^5 are two-cell partitions. In the proof of Proposition 4, we treat these as two state informed principal problems to which we can apply Proposition 2 to compare their values to the fully ignorant information acquisition strategy p^3 and characterize the priors and preferences under which they are preferred to p^3 or vice versa. This is straightforward for p^1 and p^2 – they induce preferences that conform to Assumptions 1 and 2 – but to use Proposition 2 on p^5 we must first ensure that the payoff functions it generates conform to Assumptions 1 and 2. For parts (A) and (C) this is done by restricting priors such that π_1 is large relative to π_3 so that event $\{1, 3\}$ is analogous to state 1 in Section 4 and for part (B) we restrict priors such that π_1 is small relative to π_3 so that event $\{1, 3\}$ is analogous to state 2. Comparing the values from these two-cell partitions to the fully informed information acquisition strategy p^4 and characterizing the priors and preferences under which they are preferred to p^4 or vice versa uses techniques similar to those used to prove Theorem 1.

We have no theory to directly compare the value of the two-cell partitions to each other, or to directly compare the fully informed payoff to the fully ignorant payoff. To characterize the priors and preferences under which one is preferred to the other in each case, we use indirect comparisons over which Proposition 2 can be used.

(a) p^1 optimal information acquisition option (κ_1^S close to 1, κ_2^I large).

(b) p^2 optimal information acquisition option (κ_2^S close to 1, κ_1^I large).

(c) p^3 optimal information acquisition option (both κ_1^S and κ_2^S close to 1).

(d) Full ignorance preferred to full knowledge in restricted game (κ_1^S close to 1, κ_2^I large).
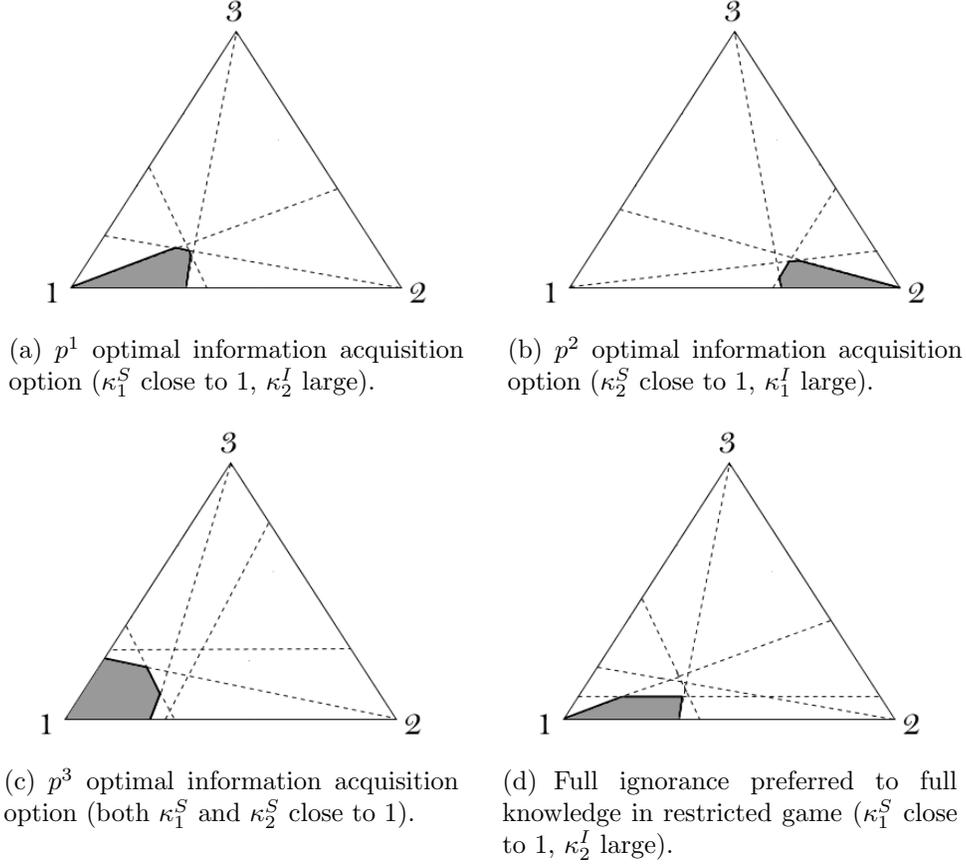
Figure 3: Proposition 4 and Corollary 1 are illustrated in this figure. The labels on the vertices indicate the probability-one state. The dashed lines represent the restrictions on priors stipulated in the propositions. Panels (a) to (c) demonstrate the priors under which Proposition 4 (A), (B), and (C) apply respectively. Panel (d), indicate priors under which Corollary 1 applies if κ_1^S close to 1 and κ_2^I large.

Take for example item (A) of Proposition 4. We first note that in the continuation game following information acquisition option p^1 is a two state informed principal game and the p^1-RSW menu is first best, given the information acquisition option. Therefore, by Proposition 3, the principal must prefer p^1 to the fully ignorant option p^3. Next we characterize priors under which the p^4-RSW payoff is the unique payoff following information acquisition option p^4 *and* the p^4-RSW payoff is strictly lower than any p^1 equilibrium payoff using Proposition 2; this requires that π_1 is sufficiently close to 1 and sufficiently larger than π_2 respectively.

The next two steps compare the value of choosing information acquisition strategy p^1 to p^2 and p^5 *indirectly* by comparing the latter values to information acquisition option p^3. The p^2-RSW payoff is the unique payoff following information acquisition option p^2 *and* the p^2-RSW payoff is strictly lower than any p^3 equilibrium payoff if π_1 is sufficiently close to 1 and π_2 sufficiently larger than π_3. To use Proposition 2 to compare p^5 to p^3 we need to

ensure that p^5 conforms to Assumptions 1 and 2. This is so if π_1 is sufficiently larger than π_3. Then, applying Proposition 2, the p^5-RSW payoff is the unique payoff following information acquisition option p^5 *and* the p^5-RSW payoff is strictly lower than the p^3 payoff if $\pi_1 + \pi_3$ is sufficiently close to 1.

Thus, we have developed a set of restriction on priors such that within this set of priors, ex ante, the principal knows that if she chooses any information acquisition option other than p^1, she will attain her RSW payoff for that information acquisition option and this payoff is necessarily less than the payoff to choosing information acquisition option p^1. We note that this intersection is open and nonempty, since any priors such that π_1 is sufficiently large (but less than 1) and π_2 is sufficiently larger than π_3 is in this intersection.

In the final result in this section, we present a corollary to Proposition 4 where we consider an environment in which it is technologically infeasible for the principal to choose *any* partition of N. In particular, we suppose that she is restricted to choosing either to acquire full information or no information.

Corollary 1 *Suppose the principal was restricted to choose between complete knowledge and complete ignorance. If either κ_1^S or κ_2^S is close to 1, (so some ignorance is desired in the unrestricted game) there is a nonempty set of priors for which the principal prefers complete ignorance.*

Figure 3, panel (d) indicate the priors under which Corollary 1 applies in a 3 dimensional simplex if κ_1^S close to 1 and κ_2^I large.

6 Information Acquisition as Hidden Action

In this section we examine the case where the principal's information acquisition decision is her private information. The problem becomes one of an informed principal with three states in which one of the states is endogenously chosen by the principal: the informed principal in each of the two states and the uninformed state of the principal.

A menu-contract is a list $\{(y_0, t_0), (y_1, t_1), (y_2, t_2)\}$ where state 0 is the uninformed state. Let $\alpha \in [0, 1]$ denote the probability that the principal becomes informed. Thus, α is the principal's information acquisition strategy. Finally, define $C^0(y) := \pi C^1(y) + (1 - \pi)C^2(y)$ to be the expected cost of implementing effort y for the principal and $U^0(y) := \pi U^1(y) + (1 - \pi)U^2(y)$ to be the expected revenue of effort y for the agent.

Our first result shows that there is always an equilibrium where the principal is informed with zero probability.

Lemma 3 *There always exists an equilibrium with $\alpha = 0$.*

On the other hand, we assert in our next proposition that a payoff equivalent equilibrium exists in which the principal is uninformed with strictly positive probability if κ is close enough to 1.

Proposition 5 *Suppose Assumptions 1 and 2 hold. If κ is sufficiently close to 1, then there exists a nonempty open interval of priors such that the principal remains ignorant with positive probability.*

As shown in the proof of Lemma 3, incentive compatibility ensures that the payoff to the uninformed principal will never be larger than that of the informed principal in expectation. To prove Proposition 5, we start with an equilibrium where the principal acquires information with zero probability and construct a payoff equivalent equilibrium where she acquires information with strictly positive probability. As long as κ is sufficiently small, there is an interval of priors such that the contract is inefficient in at least one state. This allows us to increase the agent's payoff while maintaining the principal's payoff, thus creating a surplus for the agent in this state. By choosing a sufficiently low but positive α, we can leverage this surplus to increase the payoff to the uninformed principal sufficiently high to make her indifferent between being informed and being ignorant while maintaining the individual rationality constraint. The formal construction of the payoff equivalent contract is demonstrated in the proof.

7   Conclusion

We have studied a principal-agent problem where the principal can decide how much private information to (costlessly) acquire before offering a contract to an uninformed agent. Importantly, the state is directly payoff relevant to both the principal and the agent. In this setting we have found that the principal will not choose to be completely informed of the state for some priors as long as her payoffs between at least two states of the world are sufficiently close. Indeed, this result holds regardless of the continuation equilibrium played following any information acquisition choice and is robust to the existence of multiple equilibria in the informed principal continuation game. We show further, in a three state, quasilinear environment, that the principal chooses to be completely ignorant of the state for nontrivial parameters of the model. Notably, these results were obtained in a full mechanism design framework: the principal was given full strategic flexibility to make use of whatever information she decides to acquire.

A Proofs

We assume that incentive compatibility constraints are still imposed at degenerate priors.

A.1 The Suboptimality of Full Information

Proof of Theorem 1 Before we prove Theorem 1, we first describe how Assumption 1 simplifies the computation of an RSW menu according to Proposition 2 in Maskin and Tirole [13, p12].

Proposition 2, Maskin and Tirole [13] *Suppose Assumption 1 holds. The RSW allocation (within the class of deterministic solutions) is obtained by successively solving the following programs:*

$$\max_{(y_1,t_1)} V^1(y_1,t_1) \qquad\qquad (\text{RSW}^1)$$

$$s.t. \ (RSW\text{-}IR[1]) \ W^1(y_1,t_1) = 0$$

and for all $k = 2, \ldots, n$, *given* $(y_1,t_1), \ldots, (y_{k-1},t_{k-1})$

$$\max_{(y_k,t_k)} V^k(y_k,t_k) \qquad\qquad (\text{RSW}^k)$$

$$s.t. \ (RSW\text{-}IC[k\text{-}1,k]) \ V^k(y_{k-1},t_{k-1}) \geq V^{k-1}(y_k,t_k); \ and$$

$$(RSW\text{-}IR[k]) \qquad W^k(y_k,t_k) = 0$$

Further, $y_{k-1} < y_k$ *and* $t_{k-1} < t_k$ *for all* $k = 2, \ldots, n$.

Remark 2 Note that, (i) the RSW individual rationality constraints in each state always bind; (ii) of all the incentive compatibility constraints, only those of the form RSW-IC$[j, j+1]$ for all $j \in \{1, \ldots, n-1\}$ can possibly bind; (iii) the constraint RSW-IC$[j-1, j]$ only shows up in the RSW problem of the principal in state j; (iv) the choice variable in each state is now a single contract rather than a full menu; and (v) (y_j^r, t_j^r) is strictly increasing in the state j.

Let $\pi \in \Delta^n := \left\{ \hat{\pi} \in [0,1]^n : \sum_{i \in N} \hat{\pi}_i = 1 \right\}$ be the common prior belief over the state space N. We begin by defining two information acquisition options for the principal (one partially ignorant, one fully informed) and their payoffs. Choose any $i \in N$ and consider:

(a) FI: The *full information option* reveals the precise state before the contract is offered;

(b) PI: The *partial ignorance option* reveals all states precisely unless that state is either i or $i + 1$; if the state is either i or $i + 1$, it is only revealed that the state is in $\{i, i + 1\}$.

We refer to the continuation game following the information acquisition option FI as the *original game* and the continuation game following the information acquisition option PI as the *modified game*. Our goal is to compare the ex ante RSW payoffs for each game.

Consider the principal in the interim stage who knows that the state is in $\{i, i+1\}$; call her the $\{i, i+1\}$-state principal. Let

$$\alpha = \frac{\pi_i}{\pi_i + \pi_{i+1}}.$$

The $\{i, i+1\}$-state principal's interim expected payoff from choosing FI is $V_{FI}^{\{i,i+1\}}(\alpha) := \alpha V_r^i + (1-\alpha)V_r^{i+1}$.

Consider the modified game that treats $\{i, i+1\}$ as a single state: the state space is $\hat{N} = \{1, \ldots, i-1, \{i, i+1\}, i+2, \ldots, n\}$, the principal has payoff $V^j(y,t)$ and the agent has payoff $W^j(y,t)$ in all states $j = 1, \ldots, i-1, i+2, \ldots, n$ and payoffs $V^{\{i,i+1\}}(y,t) := \alpha V^i(y,t) + (1-\alpha)V^{i+1}(y,t)$ and $W^{\{i,i+1\}}(y,t) := \alpha W^i(y,t) + (1-\alpha)W^{i+1}(y,t)$ respectively in state $\{i, i+1\}$, given contract (y,t).

The following lemma establishes the state $\{i, i+1\}$ RSW problem for the principal who chooses PI.

Lemma 4 *The interim expected payoff for the principal from playing PI is represented by the problem*

$$V_{PI}^{i,i+1}(\alpha) := \max_{(y,t)} \ \alpha V^i(y,t) + (1-\alpha)V^{i+1}(y,t) \tag{9}$$
$$s.t \ \ \alpha W^i(y,t) + (1-\alpha)W^{i+1}(y,t) = 0$$
$$V^{i-1}(y_{i-1}^r, t_{i-1}^r) \geq V^{i-1}(y,t).$$

Proof The result follows from Proposition 2 of Maskin and Tirole [13] if we can show that the modified game with state space $\hat{N} = \{1, \ldots, \{i, i+1\}, \ldots, n\}$ satisfies the associated Sorting Assumption 1. In the modified game, we treat the combined states $\{i, i+1\}$ as a single state.

By inspection, items (i) and (ii) of Sorting Assumption 1 are satisfied in the modified game. For item (iii) we need to show that $-\frac{V_y^{i+2}}{V_t^{i+2}} < -\frac{\alpha V_y^i + (1-\alpha)V_y^{i+1}}{\alpha V_t^i + (1-\alpha)V_t^{i+1}} < -\frac{V_y^{i-1}}{V_t^{i-1}}$.[17] Recall that $V_t > 0$ and $V_y < 0$. Assumption 1 for the original game has

$$-\frac{V_y^i}{V_t^i} > -\frac{V_y^{i+1}}{V_t^{i+1}} \Leftrightarrow -V_y^i V_t^{i+1} > -V_y^{i+1} V_t^i. \tag{10}$$

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[17]Recall that the subscripts on the payoff functions indicate partial derivatives.

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Then

$$-\frac{\alpha V_y^i + (1-\alpha)V_y^{i+1}}{\alpha V_t^i + (1-\alpha)V_t^{i+1}} = -\frac{\alpha V_y^i + (1-\alpha)V_y^{i+1}}{\alpha V_t^i + (1-\alpha)V_t^{i+1}} \cdot \frac{V_t^{i+1}}{V_t^{i+1}}$$

$$> \frac{-\alpha V_y^{i+1}V_t^i - (1-\alpha)V_y^{i+1}V_t^{i+1}}{\alpha V_t^i + (1-\alpha)V_t^{i+1}} \cdot \frac{1}{V_t^{i+1}}$$

$$= -\frac{V_y^{i+1}}{V_t^{i+1}} > -\frac{V_y^{i+2}}{V_t^{i+2}} \tag{11}$$

where the first inequality follows from inequality (10) and the second results from the Sorting Assumption 1. And, by a symmetric argument $-\frac{\alpha V_y^i + (1-\alpha)V_y^{i+1}}{\alpha V_t^i + (1-\alpha)V_t^{i+1}} < -\frac{V_y^{i-1}}{V_t^{i-1}}$ as needed. ∎

Denote by $(y(\alpha), t(\alpha))$ the solution to this problem. The following four lemmas characterize $V_{PI}^{\{i,i+1\}}$ and bound it from below.

Lemma 5 $V_{PI}^{\{i,i+1\}}(1) = V_r^i$.

Proof By Proposition 2 of Maskin and Tirole [13],

$$V_r^i = \max_{(y_i,t_i)} \left\{ V^i(y_i, t_i) : V^{i-1}(y_{i-1}^r, t_{i-1}^r) \geq V^{i-1}(y_i, t_i) \text{ and } W^i(y_i, t_i) = 0 \right\}.$$

Problem (9) at $\alpha = 1$ is

$$V_{PI}^{\{i,i+1\}}(1) = \max_{(y,t)} \left\{ V^i(y, t) : V^{i-1}(y_{i-1}^r, t_{i-1}^r) \geq V^i(y, t) \text{ and } W^i(y, t) = 0 \right\}$$

due to the previous lemma. These problems are equivalent. ∎

Lemma 6 *The payoff to the information acquisition option PI can be expressed as*

$$V_{PI}^{i,i+1}(\alpha) = V_r^i - \int_\alpha^1 \left(V^i(y(a), t(a)) - V^{i+1}(y(a), t(a)) \right) da$$

$$- \int_\alpha^1 \lambda(a) \left(W^i(y(a), t(a)) - W^{i+1}(y(a), t(a)) \right) da.$$

Proof Consider the optimization problem (9). By the integral form of the envelope theorem (Milgrom and Segal, Corollary 5, [14]), its value is

$$V_{PI}^{\{i,i+1\}}(\alpha) = V^{i+1}(y(0), t(0)) + \int_0^\alpha \left(V^i(y(a), t(a)) - V^{i+1}(y(a), t(a)) \right) da \tag{12}$$

$$+ \int_0^\alpha \lambda(a) \left(W^i(y(a), t(a)) - W^{i+1}(y(a), t(a)) \right) da$$

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where λ is the multiplier on the first constraint. Simple algebra on equation (12) shows that

$$V_{PI}^{\{i,i+1\}}(\alpha) = V^{i+1}(y(0), t(0)) + \int_0^\alpha \left(V^i(y(a), t(a)) - V^{i+1}(y(a), t(a))\right) da \qquad (13)$$

$$+ \int_0^1 \lambda(a) \left(W^i(y(a), t(a)) - W^{i+1}(y(a), t(a))\right) da$$

$$- \int_\alpha^1 \lambda(a) \left(W^i(y(a), t(a)) - W^{i+1}(y(a), t(a))\right) da.$$

By Lemma 5 we can plug V_r^i in for $V_{PI}^{\{i,i+1\}}(1)$ in equation (13) evaluated at $\alpha = 1$ and rearrange to get

$$\int_0^1 \lambda(a) \left(W^i(y(a), t(a)) - W^{i+1}(y(a), t(a))\right) da = \qquad (14)$$

$$V_r^i - V^{i+1}(y(0), t(0)) - \int_0^1 \left(V^i(y(a), t(a)) - V^{i+1}(y(a), t(a))\right) da$$

Now plug (14) into (13) to get

$$V_{PI}^{\{i,i+1\}}(\alpha) = V_r^i - \int_\alpha^1 \left(V^i(y(a), t(a)) - V^{i+1}(y(a), t(a))\right) da$$

$$- \int_\alpha^1 \lambda(a) \left(W^i(y(a), t(a)) - W^{i+1}(y(a), t(a))\right) da.$$

as needed. ∎

Lemma 7 *Let $\mathscr{V}$ denote the set of payoff functions for the principal that satisfy all our assumptions with typical element $V = (V^1, \ldots, V^n)$. For any $V \in \mathscr{V}$, define*

$$M(\alpha; V) := \frac{-V_t^{\{i,i+1\}}(y(\alpha), t(\alpha)) \left[\frac{V_y^i(y(\alpha), t(\alpha))}{V_t^i(y(\alpha), t(\alpha))} - \frac{V_y^{i-1}(y(\alpha), t(\alpha))}{V_t^{i-1}(y(\alpha), t(\alpha))}\right]}{-\frac{V_y^{i-1}(y(\alpha), t(\alpha))}{V_t^{i-1}(y(\alpha), t(\alpha))} W_t^{\{i,i+1\}}(y(\alpha), t(\alpha)) + W_y^{\{i,i+1\}}(y(\alpha), t(\alpha))},$$

$$M(V) := \min_{\alpha \in [0,1]} M(\alpha; V).$$

Choose small $\delta > 0$ such that $\bar{\mathscr{V}} := \{V \in \mathscr{V} : M(V) > \delta\} \neq \varnothing$. Then for all $V \in \bar{\mathscr{V}}$, $\lambda(\alpha) > \delta$ for any $\alpha \in [0, 1]$.

Proof We claim that $M(\alpha; V)$ is well defined and strictly positive for all α and V. To see this, note first that by the Sorting Assumption 1 the numerator in $M(\alpha; V)$ is strictly

negative for all $\alpha \in [0,1]$ and $V \in \mathcal{V}$. Define

$$Z(\alpha; V) = -\frac{V_y^{i-1}(y(\alpha), t(\alpha))}{V_t^{i-1}(y(\alpha), t(\alpha))} W_t^{\{i,i+1\}}(y(\alpha), t(\alpha)) + W_y^{\{i,i+1\}}(y(\alpha), t(\alpha))$$

To see that the $Z(\alpha; V) < 0$ for all $\alpha \in [0,1]$ and all $V \in \mathcal{V}$ suppose by contradiction that there is some $\alpha \in [0,1]$ and some $V \in \mathcal{V}$ such that $Z(\alpha; V) \geq 0$. Let μ be the Lagrange multiplier on the second constraint in problem (9). To demonstrate a contradiction, consider the Lagrangian for problem (9) evaluated at the maximum

$$\mathcal{L} = V^{\{i,i+1\}}(y(\alpha), t(\alpha)) + \lambda(\alpha) W^{\{i,i+1\}}(y(\alpha), t(\alpha)) + \mu(\alpha) \left(V^{i-1}(y_{i-1}^r, t_{i-1}^r) - V^{i-1}(y(\alpha), t(\alpha)) \right)$$

and the following deviation from the optimal contract $(y(\alpha), t(\alpha))$: $(\hat{y}, \hat{t}) := (y(\alpha) + \delta_y, t(\alpha) + \delta_t)$ for small $\delta_y, \delta_t > 0$ such that $V_y^{i-1}(y(\alpha), t(\alpha)) \delta_y + V_t^{i-1}(y(\alpha), t(\alpha)) \delta_t = 0$.

By the Sorting Assumption 1, part (iii)

$$-\frac{V_y^{i+1}(y(\alpha), t(\alpha))}{V_t^{i+1}(y(\alpha), t(\alpha))} < -\frac{V_y^i(y(\alpha), t(\alpha))}{V_t^i(y(\alpha), t(\alpha))} < -\frac{V_y^{i-1}(y(\alpha), t(\alpha))}{V_t^{i-1}(y(\alpha), t(\alpha)} = \frac{\delta_t}{\delta_y} \tag{15}$$

Cross multiplying and rearranging the (15) gives $\delta_y V_y^i(y(\alpha), t(\alpha)) + \delta_t V_t^i(y(\alpha), t(\alpha)) > 0$ and $\delta_y V_y^{i+1}(y(\alpha), t(\alpha)) + \delta_t V_t^{i+1}(y(\alpha), t(\alpha)) > 0$. Taking a convex combination of these expressions gives (weighting by α and $1 - \alpha$)

$$dV^{\{i,i+1\}}(y(\alpha), t(\alpha)) := \delta_y V_y^{\{i,i+1\}}(y(\alpha), t(\alpha)) + \delta_t V_t^{\{i,i+1\}}(y(\alpha), t(\alpha)) > 0$$

Let $\hat{\mathcal{L}}$ denote the value of the Lagrangian at the deviation $(\hat{y}, \hat{t})$. The net gain from the deviation is

$$\begin{aligned} \hat{\mathcal{L}} - \mathcal{L} &= dV^{\{i,i+1\}}(y(\alpha), t(\alpha)) + \lambda(\alpha) \left[\delta_y W_y^{\{i,i+1\}}(y(\alpha), t(\alpha)) + \delta_t W_t^{\{i,i+1\}}(y(\alpha), t(\alpha)) \right] \\ &= dV^{\{i,i+1\}}(y(\alpha), t(\alpha)) + \delta_y \lambda(\alpha) \left[W_y^{\{i,i+1\}}(y(\alpha), t(\alpha)) + \frac{\delta_t}{\delta_y} \cdot W_t^{\{i,i+1\}}(y(\alpha), t(\alpha)) \right] \\ &= dV^{\{i,i+1\}}(y(\alpha), t(\alpha)) + \delta_y \lambda(\alpha) Z(\alpha; V) > 0 \end{aligned}$$

where the third equality follows from the equality in (15) and the definition of Z and the inequality follows since we have assumed $Z(\alpha; V) \geq 0$. If δ_y, δ_t are sufficiently small, the deviation contract $(\hat{y}, \hat{t})$ strictly increases the Lagrangian which contradicts the supposition that $(y(\alpha), t(\alpha))$ is an optimum. Thus, $Z(\alpha; V) < 0$ so $M(\alpha; V) > 0$ for all $\alpha \in [0,1]$ and $V \in \mathcal{V}$ and so $M(V) > 0$ for all $V \in \mathcal{V}$. Thus, $\bar{\mathcal{V}}$ is a nonempty for sufficiently small $\delta > 0$.

To see that $\lambda(\alpha) > \delta$ for any $V \in \bar{\mathcal{V}}$, suppose there exists $V \in \bar{\mathcal{V}}$ such that $\lambda(\alpha) \leq \delta$ and consider the same deviation proposed above. Choose any $\alpha \in [0,1]$. From equality in (15)

$$\delta_y V_y^i(y(\alpha), t(\alpha)) + \delta_t V_t^i(y(\alpha), t(\alpha)) = M(\alpha; V) \cdot Z(\alpha; V) \cdot \left[-\frac{V_t^{\{i,i+1\}}(y(\alpha), t(\alpha))}{\delta_y V_t^i(y(\alpha), t(\alpha))} \right]^{-1}$$
$$= -\delta_y V_t^i(y(\alpha), t(\alpha)) \cdot M(\alpha; V) \frac{Z(\alpha; V)}{V_t^{\{i,i+1\}}(y(\alpha), t(\alpha))}$$

and due to the first inequality and the equality in (15)

$$\delta_y V_y^{i+1}(y(\alpha), t(\alpha)) + \delta_t V_t^{i+1}(y(\alpha), t(\alpha)) > M(\alpha; V) \cdot Z(\alpha; V) \cdot \left[-\frac{V_t^{\{i,i+1\}}(y(\alpha), t(\alpha))}{\delta_y V_t^{i+1}(y(\alpha), t(\alpha))} \right]^{-1}$$
$$= -\delta_y V_t^{i+1}(y(\alpha), t(\alpha)) \cdot M(\alpha; V) \frac{Z(\alpha; V)}{V_t^{\{i,i+1\}}(y(\alpha), t(\alpha))}.$$

Summing the last two expressions (weighted by α and $1 - \alpha$) we get

$$dV^{\{i,i+1\}}(y(\alpha), t(\alpha)) > -\delta_y M(\alpha; V) Z(\alpha; V)$$
$$= -M(\alpha; V) \left[\delta_t W_t^{\{i,i+1\}}(y(\alpha), t(\alpha)) + \delta_y W_y^{\{i,i+1\}}(y(\alpha), t(\alpha)) \right]$$
$$> -M(V) \left[\delta_t W_t^{\{i,i+1\}}(y(\alpha), t(\alpha)) + \delta_y W_y^{\{i,i+1\}}(y(\alpha), t(\alpha)) \right]$$
$$> -\delta \left[\delta_t W_t^{\{i,i+1\}}(y(\alpha), t(\alpha)) + \delta_y W_y^{\{i,i+1\}}(y(\alpha), t(\alpha)) \right].$$

The equality follows from the definition of Z and the equality in (15). The last two inequalities follow since the term in the square brackets is negative.[18] Using this last inequality, the gain from deviation is

$$\hat{\mathscr{L}} - \mathscr{L} = dV^{\{i,i+1\}}(y(\alpha), t(\alpha)) + \lambda(\alpha) \left[\delta_y W_y^{\{i,i+1\}}(y(\alpha), t(\alpha)) + \delta_t W_t^{\{i,i+1\}}(y(\alpha), t(\alpha)) \right]$$
$$> -(\delta - \lambda(\alpha)) \left[\delta_y W_y^{\{i,i+1\}}(y(\alpha), t(\alpha)) + \delta_t W_t^{\{i,i+1\}}(y(\alpha), t(\alpha)) \right] \geq 0.$$

where the final inequality is due to our assumption that $\delta \geq \lambda(\alpha)$. If δ_y, δ_t are sufficiently small, the deviation contract $(\hat{y}, \hat{t})$ strictly increases the Lagrangian which contradicts the supposition that $(y(\alpha), t(\alpha))$ is an optimum. Thus, $\lambda(\alpha) > \delta$ for any $V \in \bar{\mathcal{V}}$. Since α was chosen arbitrarily, this holds for all $\alpha \in [0,1]$. ∎

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[18]Otherwise, the deviation is strictly better for the $\{i, i+1\}$ principal, at least a good for the agent and maintains the incentive compatibility constraint, a contradiction that $(y(\alpha), t(\alpha))$ is an optimum.

Lemma 8 *There exists a $\xi > 0$ such that for any specification of preferences $W^i(y(\alpha), t(\alpha)) - W^{i+1}(y(\alpha), t(\alpha)) < -\xi$ for any $\alpha \in [0,1]$.*

Proof Since $0 < y_i^r < y(\alpha) < y_{i+1}^r$ and $0 < t_i^r < t(\alpha) < t_{i+1}^r$ and we assume that W^i is strictly increasing in i[19]

$$W^i(y(\alpha), t(\alpha)) - W^{i+1}(y(\alpha), t(\alpha)) \leq \max_{(y,t) \in [y_i^r, y_{i+1}^r] \times [t_i^r, t_{i+1}^r]} W^i(y, t) - W^{i+1}(y, t) < -\xi$$

as needed. ∎

Lemma 9 *For any $V \in \bar{\mathscr{V}}$ such that $\|V^i - V^{i+1}\|_\infty < \frac{1}{2}\delta\xi$, we have for any $\alpha \in [0,1)$, $V_{PI}^{\{i,i+1\}}(\alpha) > V_{FI}^{\{i,i+1\}}(\alpha)$.*

Proof Using Lemma 6

$$
\begin{aligned}
V_{PI}^{i,i+1}(\alpha) - V_{FI}^{i,i+1}(\alpha) &\geq (1-\alpha)\left(V^i(y_{i+1}^r, t_{i+1}^r) - V^{i+1}(y_{i+1}^r, t_{i+1}^r)\right) \qquad (16) \\
&\quad - \int_\alpha^1 \left(V^i(y(a), t(a)) - V^{i+1}(y(a), t(a))\right) da \\
&\quad - \int_\alpha^1 \lambda(a)\left(W^i(y(a), t(a)) - W^{i+1}(y(a), t(a))\right) da \\
&> -(1-\alpha)\delta\xi + (1-\alpha)\delta\xi = 0
\end{aligned}
$$

where the first inequality due to the RSW-IC[$i,i+1$] constraint, the second holds due to Lemmas 7 and 8 and since $\|V^i - V^{i+1}\| < \frac{1}{2}\delta\xi$. ∎

Lemma 10 *Let $V_r^j(PI)$ denote the RSW payoff of the principal in state $j \neq \{i, i+1\}$ in the continuation game following information acquisition option PI and let $(y_j^r(PI), t_j^r(PI))$ be the associated RSW contract.[20] Then $V_r^j(PI) \geq V_r^j$ for all $j \neq \{i, i+1\}$.*

Proof Take $j \neq \{i, i+1\}$. For $j < i$, $V_r^j(PI) = V_r^j$ due to item (iii) in Remark 2.

We claim that the incentive compatibility constraint in the state $j \geq i+2$ PI-RSW problem is weaker than in the state j FI-RSW problem. The argument is illustrated in Figure 4.

If $j = i+2$, then the incentive compatibility constraint is weaker. To see this, define the indifference curve of any principal in state $l \in N \cup \{i, i+1\}$ at payoff K to be $\bar{t}^l(y; K)$ such that $V^l(y, \bar{t}^l(y; K)) = K$ for any y.[21] Note that the slope of the curve $\bar{t}^l(y; K)$ with respect

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[19]The ordering of RSW actions and transfers is stated in Proposition 2 of Maskin and Tirole [13].

[20]We have suppressed the dependence of these objects on α for clarity.

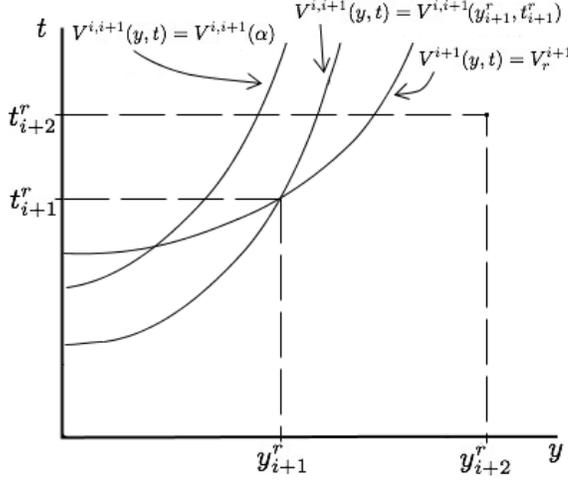[21]The existence of such a $\bar{t}$ is guaranteed by the implicit function theorem.

Figure 4: If the original game is incentive compatible, we can replace states i and $i+1$ with $\{i, i+1\}$ and maintain incentive compatibility.

to y is $-V_y^l(y,t)/V_t^l(y,t)$ and $\bar{t}^l(y;K)$ is strictly increasing in K since V^l is strictly increasing in t for all $l \in N \cup \{i, i+1\}$.

By inequality (11) we have

$$\bar{t}^{\{i,i+1\}}(y; V^{\{i,i+1\}}(y_{i+1}^r, t_{i+1}^r)) \begin{cases} = \bar{t}^{i+1}(y; V_r^{i+1}) \text{ if } y = y_{i+1}^r \\ > \bar{t}^{i+1}(y; V_r^{i+1}) \text{ if } y > y_{i+1}^r \\ < \bar{t}^{i+1}(y; V_r^{i+1}) \text{ if } y < y_{i+1}^r \end{cases} \qquad (17)$$

Since $y_{i+2}^r > y_{i+1}^r$, by the middle line of (17) we have

$$\bar{t}^{\{i,i+1\}}(y_{i+2}^r; V^{\{i,i+1\}}(y_{i+1}^r, t_{i+1}^r)) > \bar{t}^{i+1}(y_{i+2}^r; V_r^{i+1})) \geq t_{i+2}^r \qquad (18)$$

where the last inequality follows since $V^{i+1}(y_{i+1}^r, t_{i+1}^r) \geq V^{i+1}(y_{i+2}^r, t_{i+2}^r)$ by the definition of the RSW menu.

Finally, note that $V^{\{i,i+1\}}(\alpha) \geq V^{\{i,i+1\}}(y_{i+1}^r, t_{i+1}^r)$ since (y_{i+1}^r, t_{i+1}^r) is a feasible solution for problem (9) for all $\alpha \in (0,1)$. Then, by (18) $\bar{t}^{\{i,i+1\}}(y_{i+2}^r; V^{\{i,i+1\}}(\alpha)) > t_{i+2}^r$ so that the principal in state $\{i, i+1\}$ will not misrepresent the state as $i+2$ when the state $i+2$ principal gets her RSW contract (y_{i+2}^r, t_{i+2}^r): $V^{\{i,i+1\}}(\alpha) > \alpha V^i(y_{i+2}^r, t_{i+2}^r) + (1-\alpha)V^{i+1}(y_{i+2}^r, t_{i+2}^r)$.

Moreover, by the Sorting Assumption 1, for $j = i+2, \ldots, n$, if we assign to the state j principal (y_j^r, t_j^r), the state $\{i, i+1\}$ principal will not misrepresent the state as j.

Thus, if $\{(y_k^r, t_k^r)\}_{k \in N}$ is an RSW menu for the continuation game following the full information acquisition option, then $\{(y_1^r, t_1^r), \ldots, (y_{i-1}^r, t_{i-1}^r), (y(\alpha), t(\alpha)), (y_{i+2}^r, t_{i+2}^r), \ldots, (y_n^r, t_n^r)\}$ is a safe menu: it is incentive compatible and the agent will accept it regardless of her be-

liefs.[22] Therefore, the RSW payoff in each state for the modified game is at least as high as that in the original game. ∎

Maskin and Tirole [13] show that there is a nonempty set of priors such that the RSW payoff $\sum_j \pi_j V_r^j$ is the unique equilibrium payoff when the principal is perfectly informed of the state. Choose any prior π' in this set; π' determines some α'. Then, by Lemma 9, for $\|V^i - V^{i+1}\|_\infty$ sufficiently small

$$\sum_j \pi'_j V_r^j - \left[\sum_{j \neq i, i+1} \pi'_j V_r^j(PI) + (\pi'_i + \pi'_{i+1}) V_{PI}^{\{i,i+1\}}(\alpha') \right]$$
$$\leq \sum_{j \neq i, i+1} \pi'_j V_r^j + (\pi'_i + \pi'_{i+1}) V_{FI}^{\{i,i+1\}}(\alpha') - \left[\sum_{j \neq i, i+1} \pi'_j V_r^j + (\pi'_i + \pi'_{i+1}) V_{PI}^{\{i,i+1\}}(\alpha') \right]$$
$$= (\pi'_i + \pi'_{i+1}) \left(V_{PI}^{\{i,i+1\}}(\alpha') - V_{FI}^{\{i,i+1\}}(\alpha') \right) < 0.$$

The first term in both lines is the expected (unique) equilibrium payoff for the fully informed principal. The second term is expected equilibrium payoff if she confounds states i and $i+1$. The first inequality follows from Lemma 10, the second from Lemma 9. ∎

A.2   Strategic Ignorance Despite Multiple Equilibria

Proof of Proposition 1

We begin by showing that if κ is small, the RSW-IC[1,2] binds.

Lemma 11 *If κ is sufficiently close to 1, then $t_1^r - C^1(y_1^r) = t_2^r - C^1(y_2^r)$.*

Proof By way of contradiction, assume that $t_1^r - C^1(y_1^r) > t_2^r - C^1(y_2^r)$. Let $(\hat{y}_1, \hat{t}_1)$ denote the optimal contract for the principal in state 1 when she has convinced the agent that she is in state 2:

$$(\hat{y}_1, \hat{t}_1) = \arg \max_{y_1, t_1} \{t_1 - C^1(y_1) | t_1 = U^2(y_1)\} \tag{19}$$

This solution is uniquely characterized by $MC^1(\hat{y}_1) = MU^2(\hat{y}_1)$ and $\hat{t}_1 = U^2(\hat{y}_1$. Further, the state 1 principal's RSW contract is characterized by $MC^1(y_1^r) = MU^1(y_1^r)$ and $t_1^r = U^1(y_1^r)$ and since RSW-IC[1,2] does not bind, the state 2 principal's RSW contract is characterized by $MC^2(y_2^r) = MU^2(y_2^r)$ and $t_2^r = U^2(y_2^r)$. Now,

$$MC^2(\hat{y}_1) < MC^1(\hat{y}_1) = MU^2(\hat{y}_1) = MC^1(\hat{y}_1) \left(\frac{MC^2(\hat{y}_1)}{MC^2(\hat{y}_1)} \right) \leq \kappa MC^2(\hat{y}_1).$$

---

[22]Recall that the solution to problem (9) requires that the state $i-1$ principal not wish to misrepresent the state as $\{i, i+1\}$.

So, if κ is close enough to 1, since costs are convex, we can bound the difference between the two maximizers for some small $\delta_a > 0$: $y_2^r - \hat{y}_1 < \delta_a$. In a similar way, we can show there exists small $\delta_y^b > 0$ such that $\hat{y}_1 - y_1^r < \delta_b$. Thus, we can choose κ sufficiently close to 1 such that $y_2^r - y_r^1 < \delta_a + \delta_b$ and hence $C^1(y_2^r) - C^1(y_1^r) < \delta_c := \min_y\{U^2(y) - U^1(y)\}$. Then

$$0 > U^2(y_2^r) - C^1(y_2^r) - [U^1(y_1^r) - C^1(y_1^r)] = U^2(y_2^r) - U^1(y_1^r) - [C^1(y_2^r) - C^1(y_1^r)]$$
$$> \delta_c - [C^1(y_2^r) - C^1(y_1^r)] > 0$$

where the first inequality follows from the fact that we have assumed RSW-IC[1,2] does not bind. This is a contradiction so we must have RSW-IC[1,2] bind for κ close to 1. $\blacksquare$

Now we characterize payoffs for the ignorant strategy and the informed strategy. Note that the uninformed principal's problem (5) can be expressed as $V_u(\pi) = \max_y \ \pi[U^1(y) - C^1(y)] + (1 - \pi)[U^2(y) - C^2(y)]$ since IG-IR constraint always binds.

Fix κ such that RSW-IC[1,2] binds. The first statement of Proposition 1 results from the following properties of the payoff functions: (a) $V_r^1 = V_u(1)$ since the maximand and constraints are identical in the RSW and uninformed problems at $\pi = 1$; (b) $V_2^r < V_u(0)$ since the state 2 RSW problem is more constrained (i.e. by RSW-IC[1,2]) than the uninformed principal's problem at $\pi = 0$ by our choice of κ; (c) $V_u(\pi)$ is convex and downward sloping in π since the maximand is linear in π; and (d) $V_r(\pi)$ is linear and downward sloping in π.

Properties (c) and (d) imply that the equation $V_u(\pi) = V_r(\pi)$ has at most two solution. Clearly, one solution is always $\pi = 1$. Due to properties (b) - (d), a second solution $\pi^* > 0$ exists and $V_u(\pi) > V_r(\pi)$ for all $\pi \in (0, \min(1, \pi^*))$.

The following lemma completes the proof of Proposition 1.

Lemma 12 *Fix C^1 and U^i for $i \in \{1, 2\}$. There exists κ^* such that if $\kappa < \kappa^*$, then $V_u(\pi) > V_r(\pi)$ for all $\pi \in (0, 1)$.*

Proof Define $S^i(y) := U^i(y) - C^i(y)$. By the integral form of the envelope theorem (Milgrom and Segal, [14]) $V_u(\pi) = V_u(0) + \int_0^\pi (S^1(y(\pi)) - S^2(y(\pi)))d\pi$. As in Lemma 6 we can write $V_u(\pi) = V_r^1 - \int_\pi^1 (S^1(y(\pi)) - S^2(y(\pi)))d\pi$ and so

$$V_u(\pi) - V_r(\pi) = (1 - \pi)(V_r^1 - V_r^2) - \int_\pi^1 (S^1(y(\pi)) - S^2(y(\pi)))d\pi. \qquad (20)$$

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Note that

$$S^1(y(\pi)) - S^2(y(\pi)) < S^1(y_1^r) - S^2(y_1^r) \tag{21}$$
$$= V_r^1 - (U^2(y_1^r) - C^2(y_1^r))$$
$$= -(U^2(y_1^r) - U^1(y_1^r)) - (C^1(y_1^r) - C^2(y_1^r))$$
$$\leq -(U^2(y_1^r) - U^1(y_1^r)) - \Delta C(0).$$

for all $\pi \in (0,1)$ where $\Delta C(0) = C^1(0) - C^2(0)$ is the difference in fixed costs between states.

Further,

$$V_r^1 - V_r^2 = t_2^r - C^1(y_2^r) - (t_2^r - C^2(y_2^r)) \tag{22}$$
$$= -(C^1(y_2^r) - C^2(y_2^r)))$$
$$= -\int_0^{y_2^r} \left[MC^1(y) - MC^2(y)\right] dy - (C^1(0) - C^2(0))$$
$$\geq -(\kappa - 1)\left[C^2(y_2^r) - C^2(0)\right] - \Delta C(0)$$

The first equality follows from the fact that $t_1^r - C^1(y_1^r) = t_2^r - C^1(y_1^r)$.

Now, applying inequalities (21) and (22) to equation (20) we have

$$V_u(\pi) - V_r(\pi) \geq -(\kappa - 1)\left[C^2(y_2^r) - C^2(0)\right] - \Delta C(0) + U^2(y_1^r) - U^1(y_1^r) + \Delta C(0)$$
$$= \underbrace{-(\kappa - 1)\left[C^2(y_2^r) - C^2(0)\right]}_{A} + U^2(y_1^r) - U^1(y_1^r) \tag{23}$$

The RSW actions y_i^r for all $i = 1, 2$ will be the same for all κ: both are determined solely by the cost function of the state 1 principal. The term A in (23) can be made arbitrarily small by taking κ close to 1 since $C^2(y_2^r) < C^1(y_2^r)$. Moreover, the term $U^2(y_1^r) - U^1(y_1^r) > 0$ and does not change with κ. Therefore 3 for $\kappa > 1$ sufficiently close to 1, we have $V_u(\pi) > V_r(\pi)$ for all $\pi \in (0,1)$. Define $\underline{\kappa} := \sup\{\kappa | V_u(\pi) > V_r(\pi) \text{ for all } \pi \in (0,1)\} > 1$. ∎ ∎

Proof of Proposition 2 We begin by proving three useful lemmas. Then, in Lemma 16, we characterize V^*. The important fact derived in this lemma is that the RSW payoff is the unique equilibrium payoff for all $\pi \in [\pi^r, 1)$ for some $\pi^r < 1$.

Lemma 13 *If RSW-IC[i, i+1] is strictly binding, for any $i = 1, \ldots, n-1$ (i.e. $t_i^r - C^i(y_i^r) = t_{i+1}^r - C^i(y_{i+1}^r)$) then the state i+1 RSW contract is inefficient: $MC^{i+1}(y_{i+1}^r) > MU^{i+1}(y_{i+1}^r)$.*

Proof Suppose $MC^{i+1}(y_{i+1}^r) < MU^{i+1}(y_{i+1}^r)$ and consider the following deviation for type 2 in the RSW problem: $y' = y_{i+1}^r + \varepsilon$; and $t' \in \left(t_{i+1}^r + \varepsilon MC^2(y_{i+1}^r), t_{i+1}^r + \varepsilon \min\left\{MU^2(y_{i+1}^r), MC^i(y_{i+1}^r)\right\}\right)$.

Then for sufficiently small $\varepsilon > 0$, this deviation is profitable and feasible:

$$t' - C^i(y') < t^r_{i+1} - C^i(y^r_{i+1}) = t^r_i - C^i(y^r_i);$$
$$t' - C^{i+1}(y') > t^r_{i+1} - C^{i+1}(y^r_{i+1}); \text{ and}$$
$$U^{i+1}(y') - t' > U^{i+1}(y^r_{i+1}) - t^r_{i+1}.$$

If $MC^{i+1}(y^r_{i+1}) = MU^{i+1}(y^r_{i+1})$, then RSW-IC$[i, i+1]$ is not strictly binding. ∎

Lemma 14 *If $\{(y_i, t_i)\}_{\in N}$ is an equilibrium of the informed principal problem, then $y_2 \leq y^r_2$.*

Proof Suppose $y_2 > y^r_2$. If RSW-IC$[1,2]$ is not binding, then the first best contract is possible and $y_2 = y^r_2$; this is a contradiction.

If RSW-IC$[1,2]$ is binding, first note that since $C^1(y^r_1)$ is tangent to $U^1(y^r_1)$ (so that $t^r_1 = U^1(y^r_1)$), any state 1 contract that satisfies NB^1 must have $t^r_1 \geq U^1(y^r_1)$. By Lemma 13, $MC^2(y^r_2) > MU^2(y^r_2)$. This implies, that since C^2 is convex and increasing and U^2 is concave and increasing, if $t_2 - C^2(y_2) \geq V^1_r = U^2(y^r_2) - C^2(y^r_2)$ then $t_2 > U^2(y_2)$ for $y_2 > y^r_2$. But this violates the individual rationality constraint of the agent, a contradiction that $y_2 > y^r_2$ can occur in equilibrium. ∎

Lemma 15 $V^*(\pi)$ *is continuous.*

Proof Consider the ex ante optimal informed principal's problem (6) and its value function $V^*(\pi)$. Let $\mathbf{y} = (y_1, y_2)$ and $\mathbf{t} = (t_1, t_2)$. We will show that the feasibility correspondence

$$\Gamma(\pi) = \left\{ (\mathbf{y}, \mathbf{t}) \in \mathbb{R}^4 \;\middle|\; \begin{array}{ll} (\text{IC}[i,j]) & t_i - C^i(y_i) \geq t_j - C^j(y_j) \text{ for all } i \neq j \in N \\ (\text{IR}) & \sum_i \pi_i \left(U^i(y_i) - t_i\right) \geq 0 \\ (\text{NB}[i]) & t_i - C^i(y_i) \geq V^r_i \text{ for all } i \in N \end{array} \right\}$$

is both upper and lower hemi-continuous in π.

Due to Lemma 14 and Assumption 2, without lost of generality we can restrict the feasibility correspondence to

$$\Gamma'(\pi) = \left\{ (\mathbf{y}, \mathbf{t}) \in [0, y^r_2]^2 \times [0, T]^2 \;\middle|\; \begin{array}{ll} (\text{IC}[i,j]) & t_i - C^i(y_i) \geq t_j - C^j(y_j) \text{ for all } i \neq j \in N \\ (\text{IR}) & \sum_i \pi_i \left(U^i(y_i) - t_i\right) \geq 0 \\ (\text{NB}[i]) & t_i - C^i(y_i) \geq V^r_i \text{ for all } i \in N \end{array} \right\}$$

for some large finite T. Then the graph of Γ'

$$Gr(\Gamma') = \left\{ (\pi, \{(y_i, t_i)\}^2_{i=1}) \in [0,1] \times [0, y^r_2]^2 \times [0, T]^2 : \{(y_i, t_i)\}^2_{i=1} \in \Gamma'(\pi) \right\}$$

is closed. Moreover, for any closed interval $\Pi \subseteq [0,1]$, $\Gamma'(\Pi)$ is bounded. So by Theorem 3.4 in Stokey and Lucas [23], Γ' is upper hemi-continuous.

As for lower-hemicontinuity, we first note, that of the five possible constraints, at most four will bind. To see this, suppose there is $(\mathbf{y}, \mathbf{t}) \in \Gamma(\pi)$ for some π such that all five constraints bind. Then we have the following series of implications

(a) NB[i] binds for $i = 1, 2$ implies that state i contract is on the state i principal's RSW indifference curve;

(b) IC[1,2] binds implies that (y_2, t_2) is on the state 1 principal's indifference;

(c) IC[21] binds implies that (y_1, t_1) is on the state 1 principal's indifference;

(d) Items (b) and (c) imply that $y_1 = y_2 =: y'$ and $t_1 = t_2 =: t'$ since the indifference curves cross only once due to item (a.iii) of Assumption 2

(e) Items (a), (b) and (d) imply that $y' = y_2^r$, $t' = U^2(y_2^r)$ since y_2^r is defined such that $U^2(y_2^r) - C^1(y_2^r) = V_r^1$;

(f) $U^1(y') - t' < 0$ since $U^1(\cdot)$ is tangent to $C^1(\cdot)$ at y_r^1 and therefore any $y \neq y_1^r$ results in $U^1(y) - t < 0$;

(g) Items (e) and (f) imply IR is violated: $\pi(U^1(y) - t) + (1 - \pi)(U^2(y) - t) = \pi(U^1(y) - t) < 0$.

The final item contradicts the assumption that $(\mathbf{y}, \mathbf{t}) \in \Gamma(\pi)$. Thus, for any π at most four constraints are active.

The following argument is due to Duggan and Kalandrakis [7]. Suppose four constraints bind at $\pi^0 \in (0, 1)$. Take any $(\mathbf{y}^0, \mathbf{t}^0) \in \Gamma(\pi^0)$. Let $f_s(\mathbf{y}, \mathbf{t}, \pi)$ for $s = 1, \ldots, 4$ denote the four binding constraints. Then the Jacobian matrix of $F(\mathbf{y}, \mathbf{t}, \pi) := (f_s(\mathbf{y}, \mathbf{t}, \pi))_{s=1}^4$ is invertible at $(\mathbf{y}^0, \mathbf{t}^0, \pi^0)$. So, by the implicit function theorem there exists a continuous function $h(\pi)$ such that $h(\pi^0) = (\mathbf{y}^0, \mathbf{t}^0)$ and $F(h(\pi), \pi) = \mathbf{0}$ in an open neighbourhood around π^0. Since the remaining constraint is slack at π^0, it is also slack in an open neighbourhood around π^0. Thus, there is an open neighbourhood of π^0 such that $h(\pi) \in \Gamma(\pi)$ for all π in this neighbourhood and we conclude that Γ is lower hemi-continuous at π^0.

If only $d < 4$ constraints bind at π^0, then let $f_s(\mathbf{y}, \mathbf{t}, \pi)$, for $s = 1, \ldots, d$ denote the d binding constraints and define $g_s(\mathbf{y}, \mathbf{t}, \pi)$, for $s = d + 1, \ldots, 4$ as affine linear functions that are constant in π, satisfy $g_s(\mathbf{y}^0, \mathbf{t}^0, \pi^0) = 0$ for all $s = d + 1, \ldots, 4$, and have total derivative $D_{(\mathbf{y}, \mathbf{t})} g_s(\mathbf{y}, \mathbf{t}, \pi) = \mathbf{v}_s$ such that the matrix

$$\left(\left(D_{(\mathbf{y}, \mathbf{t})} f_s(\mathbf{y}^0, \mathbf{t}^0, \pi^0) \right)_{s=1}^d, (\mathbf{v}_s)_{s=d+1}^4 \right)$$

has full rank and is invertible. As above, we can apply the implicit function theorem to conclude that Γ is lower hemi-continuous at π^0.

So by the Theorem of the Maximum (Stokey and Lucas [23, Theorem 3.6]), $V^*(\pi)$ is continuous in π. ∎

Lemma 16 *If RSW-IC[1,2] binds, there are two cutoff points $0 < \pi^{FB} < \pi^r < 1$ such that $V^*(\pi)$ is the first best payoff if $\pi \leq \pi^{FB}$ and the ex ante RSW payoff if $\pi \geq \pi^r$.*

Proof **Claim 1** *If π is close enough to 1, $V^*(\pi) = V_r(\pi)$.* This holds by Lemma 17 below: $i^* = 1$. □

Claim 2 *If π is sufficiently small, then $V^*(\pi) = V_{FB}(\pi)$.* Recall that the superscript E indicates the efficient action. To see that the first best solution is attainable for small π set $y_i^* = y_i^E$, set t_2^* such that $t_2^* - C^2(y_2^E) = V_r^2$ and set t_1^* sufficiently high such that IC[1,2] is satisfied. To see that we can do this last step while satisfying the IR constraint, note that, by Lemma 13, $y_2^E < y_2^r$ which implies that $t_2^* < t_2^r$. Finally, since $U^2(y_2^r) = t_2^r$, Lemma 13 implies that $U^2(y_2^E) - t_2^* > 0$. Thus, we can find small enough π such that $\pi(U^1(y_1^E) - t_1^*) + (1 - \pi)(U^2(y_2^E) - t_2^*) = 0$. □

Define $\pi^r := \inf \left\{\pi \in [0,1] : V^*(\pi) = V_r(\pi)\right\}$. This infimum is attained in $[0,1)$ due to Claim 1 above and Proposition 4 of Maskin and Tirole [13] which says that the set of beliefs relative to which the RSW payoff is the unique equilibrium payoff consists entirely of strictly positive vectors. As a result, $\pi^r < 1$ regardless of κ so $[\pi^r, 1)$ is always well defined and nonempty. By definition, $V^*(\pi) = V_r(\pi)$ if and only if $\pi \in [\pi^r, 1)$.

Further, by assumption, RSW-IC[1,2] binds which implies, by Lemma 13, that the state 2 contract is inefficient. Thus, $V^{FB}(\pi) > V_r(\pi)$ for all $\pi \in (0,1)$. Given Claim 2, we must have $\pi^r > 0$; otherwise, $V^{FB}(\pi)$ and $V_r(\pi)$ must coincide, which is a contradiction.

Define $\pi^{FB} := \sup \left\{\pi \in [0,1] : V^*(\pi) = V^{FB}(\pi)\right\}$. This supremum is attained in $(0,1)$ by Claim 2. By definition $V^*(\pi) = V_{FB}(\pi)$ if and only if $\pi \in \left(0, \pi^{FB}\right]$. This point exists and is strictly greater than 0 by Claim 3. Further, $\pi^{FB} < \pi^r$. To see this, suppose $\pi^{FB} \geq \pi^r$. Then there exists $\tilde{\pi} \in [\pi^r, \pi^{FB}]$. But, by the definitions of π^r, π^{FB} this implies $V^*(\tilde{\pi}) = V_{FB}(\tilde{\pi}) = V_r(\tilde{\pi})$ a contradiction, since by Lemma 13 the state 2 contract is inefficient. Thus, $0 < \pi^{FB} < \pi^r < 1$. Figure 1 plots V^*, V_u, V_r and V^{FB}. ∎

Since $\pi^r < 1$, due to Proposition 1 (item (ii) of the second statement) there is κ close enough to 1 such that $\pi^r < \pi^*$ and for all $\pi \in (\pi^r, \pi^*)$ Proposition 2 holds. ∎

Proof of Theorem 2 By Lemma 11 RSW-IC[1,2] binds since κ is assumed to be sufficiently close to 1. Recall that for such κ, $0 < \pi^{FB} < \pi^r < \pi^* \leq 1$ (see proof of Proposition 2).

Consider the following facts

(a) V^* and V_u are continuous: the former is proved in Lemma 16 (Claim 1), the latter is immediate by inspection of problem (5);

(b) $V^*(\pi) > V_u(\pi)$ for all $\pi \in (0, \pi^{FB}]$: this holds since $V_u(\pi)$ cannot be efficient in both states where as $V^*(\pi)$ is first best by definition in this domain;

(c) $V^*(\pi) < V_u(\pi)$ for all $\pi \in [\pi^r, \pi^*)$: established by Proposition 2;

(d) $V^*(\pi) > V_r(\pi)$ for all $\pi \in (0, \pi^r)$: by definition of π^r in Lemma 16 (Claim 2).

Due to items (a) through (c), the intermediate value theorem guarantees the existence of a $\hat{\pi} \in (\pi^{FB}, \pi^r)$ such that for all $\pi \in (\hat{\pi}, \pi^r)$, $V_u(\pi) > V^*(\pi)$. This confirms the first statement of Theorem 2. Since $\hat{\pi} \in (0, \pi^r)$, by item (d) we also have that $V^*(\pi) > V_r(\pi)$ thus confirming the second statement of Theorem 2. ∎

Proof of Proposition 3 When RSW-IC[1,2] does not bind, the RSW contract in both states is efficient. To see this, recall that the state 1 contract is always efficient and note that, according to Proposition 2 of Maskin and Tirole [13], the problem of the state 2 principal in this case is $\max_{(y_2, t_2)} \{t_2 - C^2(y_2) : U^2(y_2) - t_2 = 0\}$. So $V_u(0) = V_r^2$ and $V_u(1) = V_r^1$. Since V_u is convex and V_r is linear (see the proof of Proposition 1, items (c) and (d)), $V_u(\pi) < V_r(\pi)$ for all $\pi \in (0, 1)$. ∎

A.3 Optimality of Complete Ignorance: Three States

Before proving the results of this section, we define the principal's problems and strategies relative to p. For this, we need some additional notation.

The RSW problem relative to information strategy p for principal in p-state i is to choose $\{(y_i, t_i)\}_{i \in I(p)}$ to solve

$$\begin{aligned}
\max \ \ & t_i - C^{p_i}(y_i) \\
\text{s.t.} \ \ (p\text{-RSW-IC}[i,j]) \ \ & t_j - C^{p_j}(y_j) \geq t_k - C^{p_j}(y_k) \quad \text{for all } j, k \in I(p); \text{ and} \\
(p\text{-RSW-IR}[j]) \ \ & U^{p_j}(y_j) \geq t_j \quad\quad\quad\quad\quad \text{for all } j \in I(p).
\end{aligned}$$

We will refer to this problem as the p-RSW problem for p-state i or the p_i-RSW problem. Let $V_r^{p_i}(\pi; p)$ denote the p_i-RSW given priors π.

Our first lemma in this section characterizes the priors under which the RSW payoff is unique for the fully informed principal problem.

Lemma 17 *Consider the problem of the fully informed principal when there are either two or three states. Let $E \subset N$ denote the set of states for which the RSW contracts are efficient. Define $I := N/E$ to be the set of states with inefficient RSW contracts and let $i^* = \max \{i \in E | i < \min I\}$.*

Then: (i) If $I = \varnothing$, the RSW payoff is the unique payoff for all priors; (ii) if $|I| = 1$, then if π_{i^*} is sufficiently large, the RSW payoff is the unique equilibrium payoff; and (iii) if

$|I| = 2$, *then if π_1 and $\pi_2/(\pi_2 + \pi_3)$ are sufficiently close to 1, the RSW payoff is the unique equilibrium payoff. Moreover, all of these bounds on priors are strictly less than 1.*

Proof First note that $1 \in \{i \in E | i < \min I\}$ since state 1 is always efficient. Therefore, i^* is always well defined.

If $I = \varnothing$, then all states are efficient and the RSW contract is first best (see Proposition 3). The RSW payoff is therefore the unique payoff for all priors.

Now suppose $I \neq \varnothing$. By Theorem 1 in Maskin and Tirole [13] $\{\hat{y}_i, \hat{t}_i\}_{i \in N}$ is an equilibrium menu if and only if it satisfies the following conditions

$$
\begin{cases}
(\text{IC}[i,j]) & t_i - C^i(y_i) \geq t_j - C^j(y_j) \text{ for all } i \neq j \in N \\
(\text{IR}) & \sum_i \pi_i \left(U^i(y_i) - t_i \right) \geq 0 \\
(\text{NB}[i]) & t_i - C^i(y_i) \geq V_i^r \text{ for all } i \in N.
\end{cases}
$$

Suppose there exists a menu $\{\hat{y}_i, \hat{t}_i\}_{i \in N}$ gives payoff strictly higher than the RSW menu in equilibrium. For each $i \in I$, the action in the state i RSW contract is higher than the efficient level (see Lemma 13). For all $i \in I$, define $\delta_i = U^i(\hat{y}_i) - \hat{t}_i$. This is the surplus given to the agent in state i by the proposed menu.

If the proposed menu delivers strictly higher payoff than the RSW menu, there must exist at least one $i \in I$ such that $\delta_i > 0$. To see this, suppose not: for all $i \in I$, $U^i(\hat{y}_i) - \hat{t}_i = 0$. Call this assumption $(\star)$. Note that for all $k \in E$, C^k is tangent to U^k at (y_k^r, t_k^r). This implies that for all (y_k', t_k') such that $t_k' - C^k(y_k') > V_k^r$, $U^k(y_k') - t_k' < 0$. This last implication, along with $(\star)$ and the equilibrium condition IR implies that $(\hat{y}_k, \hat{t}_k) = (y_k^r, t_k^r)$ for all $k \in E$. So, we have that $\{\hat{y}_i, \hat{t}_i\}_{i \in N}$ satisfies IC$[i,j]$ for all $i, j \in N$ and $U^i(\hat{y}_i) - \hat{t}_i = 0$ for all $i \in N$. But the RSW menu is the best of all menus that satisfy these assumptions so that the menu $\{\hat{y}_i, \hat{t}_i\}_{i \in N}$ cannot give a strictly higher payoff than the RSW menu.

Thus, there exist at least one state $i \in I$ such that $\delta_i > 0$. Note that this implies that $\hat{y}_i < y_i^r$ by Lemma 13. For each $i \in I$, define $\bar{\delta}_i := \max_{(y_i, t_i)} \{U^i(y_i) - t_i | t_i - C^i(y_i) \geq V_r^i\}$. So, $\bar{\delta}_i$ is the largest surplus we can assign to the state i agent for $i \in I$. Note that this maximum is achieved at the efficient state i action along the state i RSW indifference curve.

Now consider two cases.

Case 1: $|I| = 1$. Then $i^* + 1 \in I$ by definition and $\delta_{i^*+1} > 0$. We claim that if the principal receives her RSW contract in state i^* she will have strict incentive to lie given state $i^* + 1$ contract $(\hat{y}_{i^*+1}, \hat{t}_{i^*+1})$. To see this first note that, by Proposition 3, RSW-IC$[i^*, i^* + 1]$ must bind: $t_{i^*}^r - C^{i^*}(y_{i^*}^r) = t_{i^*+1}^r - C^{i^*}(y_{i^*}^r)$. Since $MC^{i^*} > MC^{i^*+1}$ the indifference curves of the principal in states i^* and $i^* + 1$ cross only once and the latter crosses the former from below.

Consider the indifference curves that pass through the RSW contracts

$$V_r^{i^*} + C^{i^*}(y) - \left(V_r^{i^*+1} + C^{i^*+1}(y)\right) \begin{cases} < 0 & \text{if } y < y_{i^*+1}^r \\ = 0 & \text{if } y = y_{i^*+1}^r \\ > 0 & \text{if } y > y_{i^*+1}^r \end{cases} \tag{24}$$

Note that

$$V_r^{i^*} + C^{i^*}(\hat{y}_{i^*+1}) - \left(V_r^{i^*+1} + C^{i^*+1}(\hat{y}_{i^*+1})\right) \geq V_r^{i^*} + C^{i^*}(\hat{y}_{i^*+1}) - \left(\hat{t}_{i^*+1} - C^{i^*+1}(\hat{y}_{i^*+1}) + C^{i^*+1}(\hat{y}_{i^*+1})\right)$$
$$= V_r^{i^*} - \left(\hat{t}_{i^*+1} - C^{i^*}(\hat{y}_{i^*+1})\right) \tag{25}$$

where the inequality follows from the NB[$i^* + 1$] condition. Since $\hat{y}_{i^*+1} < y_{i^*+1}^r$, by the first line of expression (24) we have that $0 > V_r^{i^*} + C^{i^*}(\hat{y}_{i^*+1}) - \left(V_r^{i^*+1} + C^{i^*+1}(\hat{y}_{i^*+1})\right)$ which, given (25), implies that $V_r^{i^*} < \hat{t}_{i^*+1} - C^{i^*}(\hat{y}_{i^*+1})$.

Thus, given the state $i^* + 1$ contract $\left(\hat{y}_{i^*+1}, \hat{t}_{i^*+1}\right)$, to satisfy incentive compatibility we must give the state i^* principal payoff that is strictly higher than her RSW payoff.

Since $i^* \in E$, C^{i^*} is tangent to U^{i^*} at the RSW contract; thus, any contract that increases the payoff to the principal in this state necessarily assigns a strictly positive deficit to the agent. Denote this deficit by $\underline{\delta}_{i^*} := \hat{t}_{i^*} - U^{i^*}(\hat{y}_{i^*}) > 0$.

Without loss of generality, set $(\hat{y}_i, \hat{t}_i) = (y_i^r, t_i^r)$ for all $i \in E/\{i^*\}$ and assume the resulting contract is incentive compatible. Then, if π_{i^*} is close enough to 1

$$\sum_i \pi_i \left(U^i(y_i) - t_i\right) = \sum_{i \in E/\{i^*\}} \pi_i \left(U^i(\hat{y}_i) - \hat{t}_i\right) - \pi_{i^*}\underline{\delta}_{i^*} + \pi_{i^*+1}\delta_{i^*+1}$$
$$= -\pi_{i^*}\underline{\delta}_{i^*} + \pi_{i^*+1}\delta_{i^*+1} \leq -\pi_{i^*}\underline{\delta}_{i^*} + \pi_{i^*+1}\bar{\delta}_{i^*+1} < 0$$

where the first equality follows since there is zero surplus for the agent in states $E/\{i^*\}$. This contradicts the assumption that $\{\hat{y}_i, \hat{t}_i\}_{i \in N}$ is an equilibrium.

Case 2: $|I| = 2$. The state 2 and three contracts are inefficient. Then RSW-IC[1,2] and RSW-IC[2,3] bind by Proposition 3. If $\delta_2 > 0$ or $\delta_2 = 0$ and $\delta_3 > 0$ then the argument in Case 1 can be applied in much the same way; if π_1 is sufficiently large, IR cannot hold and $\{\hat{y}_i, \hat{t}_i\}_{i \in N}$ cannot be an equilibrium.

Now suppose that $\delta_3 > 0$ and $\delta_2 < 0$. Without loss of generality, set $(\hat{y}_1, \hat{t}_1) = (y_1^r, t_1^r)$. As above, if the state 2 principal receives her RSW contract she will have a strict incentive to lie given the state three contract $(\hat{y}_3, \hat{t}_3)$. Thus, the menu $\{\hat{y}_i, \hat{t}_i\}_{i \in N}$ must give the state 2 principal strictly higher payoff than her RSW contract. Define $\underline{\delta}_2 := -\delta_2 > 0$. If $\pi_2/(\pi_2 + \pi_3)$

is close enough to 1, then

$$\sum_i \pi_i \left(U^i(y_i) - t_i \right) = \pi_1 \left(U^1(\hat{y}_1) - \hat{t}_1 \right) - \pi_2 \underline{\delta}_2 + \pi_3 \delta_3$$

$$= -\pi_2 \underline{\delta}_2 + \pi_3 \delta_3 \leq (\pi_2 + \pi_3) \left[-\frac{\pi_2}{\pi_2 + \pi_3} \underline{\delta}_2 + \frac{\pi_3}{\pi_2 + \pi_3} \bar{\delta}_3 \right] < 0$$

where the first equality follows since there is zero surplus for the agent in state 1. This contradicts the assumption that $\{\hat{y}_i, \hat{t}_i\}_{i \in N}$ is an equilibrium.

Finally note that Maskin and Tirole [13, Proposition 4] asserts that the set of beliefs relative to which the RSW payoff is unique consists of strictly positive vectors. Thus, the bounds we have placed on priors in this lemma are strictly less than one. ∎

Proof of Proposition 4 Note that α is used below to denote conditional priors. Be aware that α is redefined in subsequent lemmas. Further, any priors (conditional or unconditional) superscripted with r are meant to be analogous to those in Propositions 1 and 2.

This proof proceeds by applying Proposition 2 to the various subgames associated with choosing different information acquisition options. Recall that in Proposition 2, as long as $\kappa < \underline{\kappa}$, $\pi^* = 1$. To ease exposition, when we apply Proposition 2 we sacrifice its generality (i.e. allowing the upper bound on priors to be less than 1) and simply assume all the the starred priors (conditional or unconditional) are 1.

We first characterize priors such that p^5 conforms to the Assumption 2.

Lemma 18 *Let*
$$C^{\{1,3\}}(\cdot) := \frac{\pi_1}{\pi_1 + \pi_3} C^1(\cdot) + \frac{\pi_3}{\pi_1 + \pi_3} C^3(\cdot)$$

and define $U^{\{1,3\}}$ in the same way. There exists priors $\hat{\pi} \in \Delta^3$ such that for all

$$\frac{\pi_1}{\pi_1 + \pi_3} \in \left[\frac{\hat{\pi}_1}{\hat{\pi}_1 + \hat{\pi}_3}, 1 \right)$$

Assumptions 1 and 2 are satisfied for the two state informed principal game with principal payoff functions ordered $(V^{\{1,3\}}, V^2) = (t - C^{\{1,3\}}, t - C^2)$ and agent payoff functions ordered $(W^{\{1,3\}}, W^2) = (U^{\{1,3\}} - t, U^2 - t)$.

Proof We will prove that part (iii) from Assumption 1 holds. Parts (i) and (ii) of Assumption 1 are immediate. Both parts of Assumption 2 are proved in a similar manner.

Let $\alpha := \pi_1/(\pi_1 + \pi_3)$. By Assumption 1, there exists $\delta > 0$ such that $\frac{MC^1(y)}{MC^2(y)}, \frac{MC^2(y)}{MC^3(y)} > \delta + 1$. Define

$$\hat{\alpha} := \frac{\kappa_2^S - 1}{\delta(\delta + 1) + \kappa_2^S - 1} < 1. \tag{26}$$

Then for all $\alpha \in [\hat{\alpha}, 1)$ we have

$$
\begin{aligned}
MC^{\{1,3\}}(y) - MC^2(y) &= \alpha \left(MC^1(y) - MC^2(y) \right) + (1 - \alpha) \left(MC^3(y) - MC^2(y) \right) \\
&> MC^3(y) \left[\alpha \left(\frac{MC^1(y)}{MC^3(y)} - \frac{MC^2(y)}{MC^3(y)} \right) + (1 - \alpha) \left(1 - \kappa_2^S \right) \right] \\
&= MC^3(y) \left[\alpha \left(\frac{MC^1(y)}{MC^2(y)} - 1 \right) \frac{MC^2(y)}{MC^3(y)} + (1 - \alpha) \left(1 - \kappa_2^S \right) \right] \\
&> MC^3(y) \left[\alpha \delta(\delta + 1) + (1 - \alpha)(1 - \kappa_2^S) \right] > 0
\end{aligned}
$$

where the first inequality follows from the definition of κ_2^S in (7) and the second follows from the definition of $\hat{\alpha}$. The lemma is proved. ∎

(A) The proof of the statement is in the form of a series of claims, each describing conditions on priors such that the principal prefers information acquisition strategy p^1 to each of the others. First, we prove that the state three RSW action is efficient given either information acquisition strategy p^1 or p^4, under the assumptions of claim (A).

Lemma 19 *There exists $\underline{\kappa_2^I}$ such that for all $\kappa_2^I > \underline{\kappa_2^I}$, the p^4-RSW and p^1-RSW state 3 actions are efficient.*

Proof We first prove the statement for the p^4-RSW state 3 action. Let $V_r^3(\pi; p)$ denote the state three p-RSW payoff. Note that

$$
V_r^3(\pi; p^4) + C^2(y) = U^3(y) \tag{27}
$$

has two solutions since C^2 is convex, U^i is concave and $V_r^3(\pi; p^4) + C^2(y_2^r) = U^2(y_2^r) < U^3(y_2^r)$. Define $\hat{y}$ as the larger solution to (27). As y increases in a neighbourhood around $\hat{y}$, the left hand side of (27) crosses the right hand side from below. Since $MC^2(\hat{y}) > MU^3(\hat{y})$ we have $V_r^3(\pi; p^4) + C^2(y) > U^3(y)$ for all $y \geq \hat{y}$.

Since $MU^3(y_3^E) = MC^3(y_3^E)$, as we increase κ_2^I, y_3^E increases towards infinity. Thus, there exists a $\underline{\kappa_2^I}$ such that for all $\kappa_2^I > \underline{\kappa_2^I}$, $y_3^E > \hat{y}$. Thus, by the previous paragraph, $V_r^3(\pi; p^4) + C^2(y_3^E) > U^3(y_3^E)$ and therefore RSW-IC[1,2] does not bind and the lemma holds.

To see that this holds for the p^1-RSW state 3 action, define $\alpha := \pi_1/(\pi_1 + \pi_2)$ and replace C^2 and U^2 above with $\alpha C^1(\cdot) + (1 - \alpha)C^2(\cdot)$ and $\alpha U^1(\cdot) + (1 - \alpha)U^2(\cdot)$ respectively. ∎

The next lemma characterizes priors such that information acquisition strategy p^1 is strictly preferred to information acquisition strategy p^4.

Lemma 20 *Define $\alpha = \pi_1/(\pi_1 + \pi_2)$. There exists $\underline{\pi_1} < 1$, $\alpha^r(p^4) < 1$ such that for all $\pi_1 \in (\underline{\pi_1}, 1)$ and $\alpha \in (\alpha^r(p^4), 1)$ the unique payoff following information acquisition strategy p^4 is*

the p^4-RSW payoff and any continuation payoff following information acquisition strategy p^1 is strictly larger.

Proof From Lemma 11, the state 2 p^4-RSW contract is inefficient for sufficiently small κ_1^S and from Lemma 19 we know that the state three p^4-RSW contract is efficient. Thus, from Lemma 17 part (ii), $i^* = 1$ so there exists $\underline{\pi}_1$ such that the RSW payoff is the unique payoff following information strategy p^4 for $\pi \in (\underline{\pi}_1, 1)$.

By Lemma 19 the state 3 p^1-RSW contract is efficient. Thus, $V_r^3(\pi; p^4) = V_r^3(\pi; p^1)$.

Now, consider the RSW problem of the state $\{1, 2\}$ principal

$$V_r^{\{1,2\}}(\pi; p^1) := \max_{(y_{12}, t_{12})} \left\{ t_{12} - \alpha C^1(y_{12}) - (1-\alpha)C^2(y_{12}) : \alpha U^1(y_{12}) + (1-\alpha)U^2(y_{12}) = t_{12} \right\} \quad (28)$$

Since κ_1^S is small, we can apply Proposition 2 to conclude that there exists $\alpha^r(p^4)$ such that $\alpha^r(p^4) < 1$ and for all $\alpha \in (\alpha^r(p^4), 1)$ we have $V_r^{\{1,2\}}(\pi; p^1) > \alpha V_r^1(\pi; p^4) + (1-\alpha)V_r^2(\pi; p^4)$. So $(\pi_1 + \pi_2)V^{\{1,2\}}(\pi; p^1) + \pi_3 V_r^3(\pi; p^1) > \sum_i \pi_i V_r^i(\pi; p^4)$. ∎

The next lemma characterizes priors such that information acquisition strategy p^3 is strictly preferred to information acquisition strategy p^2.

Lemma 21 *Define $\alpha := \pi_2/(\pi_2 + \pi_3)$. There exists $\pi^r(p^2) < 1$ and $\underline{\alpha} < 1$ such that for all $\pi_1 \in (\pi^r(p^2), 1)$ and $\alpha \in (\underline{\alpha}, 1)$ the unique payoff following information acquisition strategy p^2 is the p^2-RSW payoff and the p^3 payoff is strictly larger.*

Proof The continuation game following information strategy p^2 is a two state game with priors $(\pi_1, \pi_2 + \pi_3)$. Define $\kappa(p^2) := \sup_y MC^1(y)/(\alpha MC^2(y) + (1-\alpha)MC^3(y))$. According to Proposition 2, if $\kappa(p^2)$ is sufficiently small, there exists $\pi^r(p^2)$ such that $\pi^r(p^2) < 1$, the unique payoff following information acquisition strategy p^2 is the p^2-RSW payoff for all $\pi_1 \in (\pi^r(p^2), 1)$ and the p^3 payoff is strictly larger.

We now show that $\kappa(p^2)$ can be made sufficiently small given the hypotheses of the proposition $\kappa(p^2) < \kappa_1^S/(\alpha + (1-\alpha)/\kappa_2^S)$. For fixed κ_2^S, if we take α and κ_1^S close enough to 1, $\kappa(p^2)$ can be made sufficiently small to apply Proposition 2. The lemma is proved. ∎

The next lemma characterizes priors such that information acquisition strategy p^3 is strictly preferred to information acquisition strategy p^5.

Lemma 22 *Let $\alpha = \pi_1/(\pi_1 + \pi_3)$. There exists $\underline{\alpha} < 1$ and $\pi^r(p^5) < 1$ such that for all $\alpha \in (\underline{\alpha}, 1)$ and $\pi_1 \in (\pi^r(p^5), 1)$ the unique payoff following information acquisition strategy p^5 is the p^5-RSW payoff and the p^3 payoff is strictly larger.*

Proof By Lemma 18, there exists an $\hat{\alpha}$ such that for $\alpha \in (\hat{\alpha}, 1)$ the problem for the principal who chooses information acquisition strategy $p^5 = \{\{1, 3\}, \{2\}\}$ is a two state informed principal problem with priors $(\pi_1 + \pi_3, \pi_2)$ that satisfies Assumption 2.

Define $\kappa(p^5) := \sup_y MC^{13}(y)/MC^2(y)$. Proposition 2 applies and the claim is proved if $\kappa(p^5)$ is sufficiently close to 1. We now check whether $\kappa(p^5)$ can be sufficiently close to 1. Note that $\kappa(p^5) \le \alpha\kappa_1^S + (1-\alpha)/\kappa_2^S < \alpha\kappa_1^S + (1-\alpha)$ where the first inequality follows from the convexity of the supremum operator. Choosing α less than but close to 1 and small κ_1^S, we can make $\kappa(p^5)$ small and Proposition 2 applies. ∎

Finally, note that for the informed game with state space p^1, the first best payoff has been achieved since each p^1-state principal is producing her efficient output. It follows from Proposition 3 that introducing further ignorance (i.e. an information strategy of p^3) will not improve payoffs. Thus, information acquisition strategy p^1 is strictly preferred to information acquisition strategy p^3 for any priors.

By Lemma 21, the principal prefers p^3 to p^2 for appropriately restricted priors for any equilibrium following the choice of p^2; thus, she prefers p^1 to p^2 on these priors as well. Moreover, by Lemma 22 the principal prefers p^3 to p^5 for appropriately restricted priors for any equilibrium following the choice of p^5; thus, she prefers p^1 to p^5 on these priors as well.

To see that the intersection of the sets characterized in Lemmas 20 to 22 is open and nonempty, note that any priors such that π_1 is sufficiently large (but less than 1) and π_2 is sufficiently larger than π_3 is in this intersection.

(B) Follows same procedure as part **(A)**.

(C) As in part **(A)**, this part is shown in a series of lemmas each characterizing the set of priors such that ignorance is better than each of the other information acquisition options. The first lemma characterizes the set of priors such that information acquisition strategy p^3 is strictly preferred to information acquisition strategy p^1.

Lemma 23 *There exists $\pi^r(p^1)$ such that $\pi^r(p^1) < 1$ and for any $\pi_1 + \pi_2 \in (\pi^r(p^1), 1)$ the unique payoff following information acquisition strategy p^1 is the p^1-RSW payoff and the completely uninformed principal's payoff is strictly larger.*

Proof Define $\alpha := \pi_1/(\pi_1 + \pi_2)$ and $\kappa(p^1) := \sup_y (\alpha MC^1(y) + (1-\alpha)MC^2(y))/MC^3(y)$. The game following information strategy p^1 is a two state informed principal problem with priors $(\pi_1 + \pi_2, \pi_3)$. Since the supremum operator is convex $\kappa(p^1) < \alpha\kappa_1^S\kappa_2^S + (1-\alpha)\kappa_2^S$. Thus, we choose κ_1^S, κ_2^S sufficiently small to apply Proposition 2 and our claim follows. ∎

The next lemma characterizes the set of priors such that information acquisition strategy p^3 is strictly preferred to information acquisition strategy p^2.

Lemma 24 *There exists $\pi^r(p^2)$ such that $\pi^r(p^2) < 1$ and for any $\pi_1 \in (\pi_r(p^2), 1)$ the unique payoff following information acquisition strategy p^2 is the p^2-RSW payoff and the uninformed principals payoff is strictly larger.*

Proof This proof is analogous to that of Lemma 21. Since $\kappa(p^2) < \sup_y \frac{MC^1(y)}{MC^3(y)} \leq \kappa_1^S \kappa_2^S$ we can choose κ_1^S, κ_2^S sufficiently small to apply Proposition 2. $\blacksquare$

The next lemma characterizes the set of priors such that information acquisition strategy p^1 is strictly preferred to information acquisition strategy p^4.

Lemma 25 *Define $\alpha = \pi_1/(\pi_1 + \pi_2)$. There exists $\alpha^r(p^4) < 1$ and $\underline{\pi}_1$ such that for any $\alpha \in (\alpha^r(p^4), 1)$ and $\pi_1 \in (\underline{\pi}_1, 1)$ the unique payoff following information acquisition strategy p^4 is the p^4-RSW payoff and the p^1-RSW payoff is strictly larger.*

Proof From Lemma 11, we know that the state 2 and 3 p^4-RSW contracts are inefficient for sufficiently small κ_1^S and κ_2^S respectively. Thus, from Lemma 17 item (iii), there exists priors $\underline{\pi}$ the p^4-RSW payoff is the unique payoff following information acquisition strategy p^4 for $\pi_1 \in (\underline{\pi}_1, 1)$ and any $\pi_2/(\pi_2 + \pi_3) \in (\underline{\pi}_2/(\underline{\pi}_2 + \underline{\pi}_3), 1)$.

This remainder analogous to Lemma 20 except we appeal to Lemma 10 to ensure that $V_r^3(\alpha; p^1) \geq V_r^3(\alpha; p^4)$ instead of Lemma 11.[23] $\blacksquare$

Our final lemma characterizes the set of priors such that information acquisition strategy p^1 is strictly preferred to information acquisition strategy p^5.

Lemma 26 *Let $\alpha = \pi_1/(\pi_1 + \pi_3)$. There exists, $\underline{\alpha} < 1$ and $\pi^r(p^5)$ such that $\pi^r(p^5) < 1$ and for any $\pi_1 + \pi_3 \in (\pi_r(p^5), 1)$ and $\alpha \in (\underline{\alpha}, 1)$ the unique payoff following information acquisition strategy p^5 is the p^5-RSW payoff and the uninformed principals payoff is strictly larger.*

Proof This follows immediately from Lemma 22. Although Lemma 22 is proved under the assumptions of claim (A), only the hypothesis that k_1^S is sufficiently small was used in the proof. Since claim (C) shares this hypothesis, the lemma applies here as well. $\blacksquare$

To see that the intersection of the sets characterized in Lemmas 23 to 26 is nonempty and open, note any priors with π_1 sufficiently large (but less than 1) is in this intersection. $\blacksquare$

(D) If π_1 is small enough, we can achieve the first best ex ante payoff using the same technique as in Claim 2 of Lemma 16. If κ_1^I and κ_2^I are large enough, we can show that the p^4-RSW menu is efficient and therefore achieves the first best ex ante payoff using the same technique as in Lemma 19.

Proof of Corollary 1 If both κ_1^S and κ_2^S are close to 1, simply apply Proposition 4 (C).

Suppose κ_2^S is close to 1 and κ_2^I is large so that p^1 is optimal on the set of priors described in Proposition 4 (A): $V(\pi; p^1) := (\pi_1 + \pi_2)V_r^{\{1,2\}}(\alpha; p^1) + \pi_3 V_r^3 > \sum \pi_i V_r^i$. Since $V(\pi; p^1)$ is continuous in π, $V(\pi; p^1) \to V(\pi; p^3) := V_r^{\{1,2,3\}}(\pi, p^3)$ as $\pi_3 \to 0$. So for small π_3 there exists $\delta > 0$ such that $V(\pi; p^1) - V(\pi; p^3) = \delta$ and $V(\pi; p^3) - \sum_i \pi_i V_r^i = V(\pi; p^1) - \delta - \sum_i \pi_i V_r^i > 0$.

For κ_1^I large and κ_2^S small close to 1, the proof is similar. $\blacksquare$

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[23] The p^4 payoffs are constant in α so the statement trivially holds for all $\alpha \in [0, 1]$.

A.4 Information Acquisition as Hidden Action

Denote the value of the principal's RSW problem in state $k \in \{0,1,2\}$ by

$$V_k^r := \max_{\{(y_i,t_i)\}_{i \in \{0,1,2\}}} t_k - C^k(y_k)$$

$$s.t. \quad (\text{IC}[i,j]) \quad t_i - C^i(y_i) \geq t_j - C^i(y_j) \quad \text{for all } i,j \in \{0,1,2\} \text{ and}$$

$$(\text{RSW-IR}[i]) \quad U^i(y_i) = t_i \qquad \text{for all } i \in \{0,1,2\}$$

Let (y_i^r, t_i^r) denote the RSW contract for the state $i \in \{0,1,2\}$ principal.[24]

The following lemma gives the necessary and sufficient conditions for equilibrium in this environment.

Lemma 27 *The contract $\{(y_0^*, t_0^*), (y_1^*, t_1^*), (y_2^*, t_2^*)\}$ and the information acquisition strategy α is an equilibrium if and only if*

$$(\text{MIX}) \quad \alpha \in \arg\max \left\{ \alpha \left[t_0^* - C^0(y_0^*) \right] + (1-\alpha) \sum_{i=1,2} \pi_i \left(t_i^* - C^i(y_i^*) \right) \right\}$$

$$(\text{IR}) \quad \alpha \sum_{i=1,2} \pi_i \left(U^i(y_0^*) - t_0^* \right) + (1-\alpha) \sum_{i=1,2} \pi_i \left(U^i(y_i^*) - t_i^* \right) \geq 0$$

$$(\text{IC}) \quad t_i^* - C^i(y_i^*) \geq t_j^* - C^i(y_j^*) \qquad\qquad\qquad \textit{for all } i,j \in \{0,1,2\}$$

$$(\text{NB}) \quad t_i^* - C^i(y_i^*) \geq V_i^r \qquad\qquad\qquad\qquad\quad \textit{for all } i \in \{0,1,2\}$$

Proof of Lemma 27 Sufficiency: Suppose the contract $\{(y_0^*, t_0^*), (y_1^*, t_1^*), (y_2^*, t_2^*)\}$ and the information acquisition strategy α satisfy MIX, IR, IC, and NB. Then, the contract $\{(y_0^*, t_0^*), (y_1^*, t_1^*), (y_2^*, t_2^*)\}$ is an equilibrium contract given α by Theorem 1 in Maskin and Tirole [13]. Moreover, given, the contract $\{(y_0^*, t_0^*), (y_1^*, t_1^*), (y_2^*, t_2^*)\}$, the MIX condition ensures that the principal cannot deviate profitably by choosing a different α.

Necessity: Suppose, IR, IC, or NB is violated. Then by Theorem 1 in Maskin and Tirole [13] $\{(y_0^*, t_0^*), (y_1^*, t_1^*), (y_2^*, t_2^*)\}$ cannot be an equilibrium given α. If MIX is violated, then the principal has a profitable deviation to another α. ∎

Proof of Lemma 3 Due to the IC conditions of the equilibrium $t_2^r - C^2(y_2^r) \geq t_0^r - C^2(y_0^r)$ and $t_1^r - C^1(y_1^r) \geq t_0^r - C^1(y_0^r)$. Weighting each of these by the appropriate prior we have $\sum_{i=1}^{2} \pi_i^r (t_i^r - C^i(y_i^r)) \geq t_0^r - C^0(y_0^r)$. ∎

Proof of Proposition 5 Let $\{(y_0^*, t_0^*), (y_1^*, t_1^*), (y_2^*, t_2^*)\}$ be an equilibrium contract with information strategy α^*. Due to lemma 3, without loss of generality we can set $\alpha^* = 0$. We are therefore considering an equilibrium in a 2 state informed principal problem (while still respecting the extra incentive compatibility constraint of the uninformed principal). Thus, by Proposition 2, since κ is close to 1, we know that there exists an interval of priors such that the action is inefficient in at least one state.

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[24]We have suppressed the dependance of the uninformed principal's RSW strategies and payoffs on priors.

Suppose the inefficient state is state 2. Let (y_2', t_2') be a contract for the state 2 principal that lies on the same indifference curve as the contract (y_2^*, t_2^*) but is closer to the efficient level of y. Then the agent receives a higher payoff at (y_2', t_2') in state 2 than at (y_2^*, t_2^*).[25]

Choose (y_0', t_0') to be the (unique) intersection between the state 1 and state 2 indifference curves passing through the points (y_1^*, t_1^*) and (y_2^*, t_2^*) respectively. Then $t_1^* - C^1(y_1^*) = t_0' - C^1(y_0')$ and $t_2' - C^2(y_1') = t_0' - C^2(y_0')$. Weighting by the appropriate prior and summing these two equations we get

$$t_0' - C^0(y_0') = \pi \left(t_1^* - C^1(y_1^*)\right) + (1 - \pi)\left(t_2' - C^2(y_1')\right) \tag{29}$$

Now we check the agents IR constraint. First note that

$$\pi(U^1(y_1^*) - t_1^*) + (1 - \pi)(U^2(y_2') - t_2') > \pi(U^1(y_1^*) - t_1^*) + (1 - \pi)(U^2(y_2^*) - t_2^*) \geq 0$$

where the first inequality follows by our choice of (y_2', t_2') and the second follows since $\{(y_0^*, t_0^*), (y_1^*, t_1^*), (y_2^*, t_2^*)\}$ is assumed to be an equilibrium and $\alpha^* = 0$. Thus, there exists $\alpha' > 0$ such that $\alpha' \left(U^0(y_0') - t_0'\right) + (1 - \alpha') \left[\pi(U^1(y_1^*) - t_1^*) + (1 - \pi)(U^2(y_2') - t_2')\right] = 0$.

Since (y_2', t_2') is on the same indifference curve as (y_2^*, t_2^*)

$$\pi \left(t_1^* - C^1(y_1^*)\right) + (1 - \pi) \left(t_2^* - C^2(y_1^*)\right) = \pi \left(t_1^* - C^1(y_1^*)\right) + (1 - \pi) \left(t_2' - C^2(y_1')\right)$$

and due to equation (29)

$$\alpha' \left(t_0' - C^0(y_0')\right) + (1 - \alpha') \left[\pi \left(t_1^* - C^1(y_1^*)\right) + (1 - \pi) \left(t_2' - C^2(y_1')\right)\right] = \pi \left(t_1^* - C^1(y_1^*)\right) + (1 - \pi) \left(t_2^* - C^2(y_1^*)\right).$$

Thus, the expected payoff to the principal from offering contract $\{(y_0', t_0'), (y_1^*, t_1^*), (y_2', t_2')\}$ with α' is equal to the expected payoff from offering $\{(y_0^*, t_0^*), (y_1^*, t_1^*), (y_2^*, t_2^*)\}$ with α^*.

If the state 1 contract is inefficient, we can similarly find a payoff equivalent menu with positive probability of being ignorant. ■

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[25] To see this note that, fixing the payoff to the principal, the greatest payoff to the agent is at the efficient level of y: $\max_{(y,t)}\{U^i(y) - t : t - C^i(y) = \bar{V}\} = \max\{U^i(y) - C^i(y) - \bar{V}\} = U^i(y_i^E) - C^i(y_i^E)$ where $\bar{V}$ is a constant.

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