

# Collective Decision with Costly Information: Theory and Experiments

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# Condorcet's Jury Theorem

On trouve de plus, que si la probabilité de la voix de chaque Votant est plus grande que  $\frac{1}{2}$ , c'est-à-dire, s'il est plus probable qu'il jugera conformément à la vérité, plus le nombre des Votants augmentera, plus la probabilité de la vérité de la décision sera grande: la limite de cette probabilité sera la certitude [...]

Une assemblée très-nombreuse ne peut pas être composée d'hommes très-éclairés; il est même vraisemblable que ceux qui la forment joindront sur bien des objets beaucoup d'ignorance et beaucoup de préjugés.

Condorcet (1785)[1986, p. 29]

# Condorcet's idea

elections serve to make good collective choices by aggregating the information dispersed among the voters

- ▶ a jury situation
- ▶ a society making a choice between two policy proposals
- ▶ democratic accountability: deciding whether or not a party in power ought to be reelected

... **epistemic** foundation for majority rule

# Problems for information aggregation

However,

- ▶ **ignorance**: voters may decline acquiring costly information
- ▶ **biased judgement**: voters may not make correct inferences at the voting booth, leading to biased judgement

# This paper

- ▶ model of information aggregation in committees where information is costly
- ▶ solution concept allowing for biased judgements (subjective beliefs)
- ▶ laboratory exploration of Bayesian equilibria and subjective equilibria of the model
- ▶ evidence of rational ignorance
- ▶ evidence of biased judgement, not consistent with cursed behavior

# Related literature, 1

**strategic voting** literature and information aggregation:

- ▶ Austen-Smith and Banks (APSR 1996)
- ▶ Feddersen and Pesendorfer (AER 1996, Ecta 1997)
- ▶ McLennan (APSR 1998)
- ▶ Myerson (GEB 1998)
- ▶ Duggan and Martinelli (GEB 2001), Meirowitz (SCW 2002)

... Condorcet's reasoning remains valid with strategic voters in a variety of situations with a **common interest** component of preferences

## Related literature, 2

### Rational ignorance:

- ▶ committees with endogenous decision to acquire information and common preferences: Mukhopadhaya (2005), Persico (2004), Gerardi and Yariv (2008)
- ▶ large elections with continuous distribution of costs: Martinelli (2006, 2007), Oliveros (2011)

... this literature does not contemplate biased judgements

### Experimental literature:

- ▶ Guarnaschelli, McKelvey and Palfrey (2000)
- ▶ Battaglini, Morton and Palfrey (2010)

... empirical support for the swing voter's curse

# This presentation

1. motivation and preview ✓
2. formal model of collective decision
3. equilibrium under majority rule
4. equilibrium under unanimity rule
5. experiment design
6. experimental results
7. structural estimation
8. conclusions

## The model: basics

- ▶  $n$  committee members must choose between two alternatives,  $A$  and  $B$
- ▶ two equally likely states of the world,  $\omega_A$  and  $\omega_B$
- ▶ **common value**: all voters get 1 if decision matches state, zero otherwise
- ▶ voters do not observe state of the world but can acquire information at a cost  $c$ , drawn independently from continuous distribution with support  $[0, \bar{c})$  and  $F(0) = 0$
- ▶ if voter acquires information, receives a signal in  $\{s_A, s_B\}$  that is independently drawn across voters conditional on the state of the world
- ▶ probability that the signal is correct is  $1/2 + q$

## The model: voting rules

- ▶ committee members can vote for  $A$ , for  $B$ , or abstain

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- ▶ Under **simple majority**,  $V_M$ , the alternative with most votes is chosen, with ties broken by a fair coin toss. That is:

$$V_M(v^A, v^B) = \begin{cases} A & \text{if } v^A > v^B \\ B & \text{if } v^B > v^A \end{cases}$$

with ties broken randomly

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- ▶ Under **unanimity**,  $V_U$ , in our specification,  $A$  is chosen unless every vote that is cast favors  $B$ , with  $A$  being chosen if every member abstains. That is:

$$V_U(v^A, v^B) = \begin{cases} B & \text{if } v^B > 0 = v^A \\ A & \text{otherwise} \end{cases}$$

## The model: preferences

Given a voter's cost of information  $c_i$ , the utility,  $U_i$ , of voter  $i$  net of information acquisition costs is given by:

$$U_i = \begin{cases} b - c_i & \text{if } d = A \text{ and the state is } \omega_A \\ b - c_i & \text{if } d = B \text{ and the state is } \omega_B \\ -c_i & \text{otherwise} \end{cases}$$

if the voter acquires information. If voter  $i$  does not acquire information, then

$$U_i = \begin{cases} b & \text{if } d = A \text{ and the state is } \omega_A \\ b & \text{if } d = B \text{ and the state is } \omega_B \\ 0 & \text{otherwise} \end{cases} .$$

## The model: subjective beliefs

- ▶ private belief that the state of the world is  $\omega_A$  is  $1/2 + \epsilon$
- ▶  $\epsilon$  is iid across voters according to a symmetric distribution function  $M$  with support  $[-\beta, \beta]$  for some  $\beta \in [0, 1/2]$
- ▶ for every  $\kappa > 0$ ,  $M(\kappa) - M(-\kappa) > 0$ , prior beliefs that are arbitrarily close to the correct priors have positive probability
- ▶  $\epsilon = 0$ : unbiased voter
- ▶  $\epsilon \neq 0$ : biased voter

# The model: types, actions and strategies

- ▶ a voter's **type** is a triple  $(\epsilon, c, s)$  specifying prior beliefs, cost of information acquisition, and private signal
- ▶ An **action** is a pair  $a = (i, v)$ ,  $i \in \{1, 0\}$ ,  $v \in \{A, B, 0\}$ , indicating whether the voter acquires or not information and whether the voter votes for  $A$ ,  $B$ , or abstains
- ▶ A **strategy function** is a mapping  $\sigma$  assigning to each type a probability distribution over the set of actions
- ▶ notation:  $\sigma(a|t)$  is the probability that a voter chooses action  $a$  given type  $t$
- ▶ constraint:  $\sigma((0, v)|(\epsilon, c, s_A)) = \sigma((0, v)|(\epsilon, c, s_B))$

# The model: equilibrium

- ▶ a **subjective equilibrium** is a strategy profile such that for each voter  $j$ ,  $\sigma_j$  is a subjective best response; that is,  $\sigma_j$  maximizes the subjective expected utility of voter  $j$  given the strategies of other voters and given voter  $j$  prior beliefs about the states
- ▶ an equilibrium is **symmetric** if every voter uses the same strategy
- ▶ if  $\beta = 0$ , all voters have correct prior beliefs with probability one, and the subjective equilibrium is a **Bayesian equilibrium**

## Simple majority: neutral strategies

- ▶ a strategy  $\sigma$  is **neutral** if

$$\sigma((0, A)|(\epsilon, c, s_d)) = \sigma((0, B)|(-\epsilon, c', s_{d'}))$$

for all  $d, d'$  and almost all  $\epsilon, c, c'$ , and

$$\sigma((1, A)|(\epsilon, c, s_A)) = \sigma((1, B)|(-\epsilon, c', s_B))$$

and

$$\sigma((1, A)|(\epsilon, c, s_B)) = \sigma((1, B)|(-\epsilon, c', s_A)) = 0$$

for almost all  $\epsilon, c, c'$

- ▶ a neutral strategy does not discriminate between the alternatives except on the basis of the private signal and prior beliefs

# Simple majority: Bayesian equilibria

## Theorem

*Under majority rule,*

1. *For any solution  $c^*$  to*

$$c^* = bq \sum_{i=0}^{\lfloor (n-1)/2 \rfloor} \binom{n-1}{2i} \binom{2i}{i} F(c^*)^{2i} (1 - F(c^*))^{n-1-2i} \left(\frac{1}{4} - q^2\right)^i$$

*there is some  $\beta^* \in (0, q)$  such that if  $0 \leq \beta \leq \beta^*$ , a strategy profile is a symmetric, neutral, informative equilibrium if each voter acquires information and votes according to the signal received if the voter's cost is below  $c^*$  and abstains otherwise*

2. *If  $\beta = 0$ , there are no other symmetric, neutral equilibria*

## Simple majority: an example with subjective beliefs

- ▶ **observable parameters:**  $b = 10$ ,  $q = 1/6$ ,  $c$  is distributed uniformly in  $[0, 1]$  and  $n = 3$  or  $n = 7$ , and the rule is majority as in the lab experiments below
- ▶ **subjective beliefs:** in addition, suppose

$$\epsilon = \begin{cases} 0 & \text{with probability } 1 - p & \dots \text{ unbiased voters} \\ -\beta & \text{with probability } p/2 & \dots \text{ biased for } B \\ \beta & \text{with probability } p/2 & \dots \text{ biased for } A \end{cases}$$

- ▶  $\beta \geq 3/10$  and  $p \in [0, 1)$

## Simple majority: an example with subjective beliefs

		$p = 0$	$p = 1/2$
$n = 3$	Pr of Info Acquisition	0.5569	0.3778
	Pr of Vote A if Uninformed	0	0.25
	Pr of Vote B if Uninformed	0	0.25
	Pr of Vote A if signal $s_A$	1	1
	Pr of Vote B if signal $s_B$	1	1
	Pr of Correct Decision	0.6650	0.5954
$n = 7$	Pr of Info Acquisition	0.3870	0.2404
	Pr of Vote A if Uninformed	0	0.25
	Pr of Vote B if Uninformed	0	0.25
	Pr of Vote A if signal $s_A$	1	1
	Pr of Vote B if signal $s_B$	1	1
	Pr of Correct Decision	0.7063	0.5153

# Hypothesis under majority rule

H1 voters follow cutoff strategies

H2 members of smaller committees acquire more information

H3 informed voters follow their signals

\*H4 uninformed voters abstain

\*H5 larger committees perform better

\*\*H6 unbiased voters acquire information & abstain if uninformed

\*\*H7 biased voters do not acquire information & vote

(\*) Bayesian equilibrium

(\*\*) subjective beliefs equilibrium

**Note:** cursed voters could vote if uninformed, but would buy more, not less information

## Unanimity rule: symmetric Bayesian equilibria

- ▶ no equilibria in which voters acquire information with positive probability, vote according to the signal received, and abstain if uninformed . . . best responding voter would rather abstain than vote for  $A$  after signal  $s_A$  (swing voter's curse)
- ▶ no equilibria in which voters acquire information with positive probability, vote for  $B$  after signal  $s_B$ , and abstain otherwise . . . a best responding voter would rather vote for  $A$  after signal  $s_A$  than abstain
- ▶ there is a mixed strategy equilibrium in which voters randomize between voting for  $A$  and abstaining after signal  $s_A$
- ▶ there are also mixed strategy equilibria in which voters randomize when uninformed between voting for  $B$  and abstaining

## Theorem

*Under unanimity rule, if  $\beta = 0$ ,*

- 1. There are some  $c, y$  such that there is a symmetric, informative equilibrium, in which each voter acquires information if the voter's cost is below  $c$ , votes for  $B$  after receiving signal  $s_B$ , votes for  $A$  with probability  $y$  after receiving signal  $s_A$ , and abstains otherwise*
- 2. There is some  $c$  and a continuum of values of  $z$  such that there is a symmetric, informative equilibrium, in which each voter acquires information if the voter's cost is below  $c$ , votes for  $A$  after receiving signal  $s_A$ , abstains with probability  $z$  if uninformed, and votes for  $B$  otherwise*
- 3. There are no other symmetric, informative equilibria*

# Unanimity: an example with subjective beliefs

- ▶ **observable parameters:**  $b = 10$ ,  $q = 1/6$ ,  $c$  is distributed uniformly in  $[0, 1]$  and  $n = 3$  or  $n = 7$ , and the rule is majority as in the lab experiments below
- ▶ **subjective beliefs:** in addition, suppose

$$\epsilon = \begin{cases} 0 & \text{with probability } 1 - p & \dots \text{ unbiased voters} \\ -\beta & \text{with probability } p/2 & \dots \text{ biased for } B \\ \beta & \text{with probability } p/2 & \dots \text{ biased for } A \end{cases}$$

- ▶  $\beta \geq 0.14$  and  $p \in [0, 1)$

## Unanimity rule: an example with subjective beliefs

		$p = 0$		$p = 1/2$
$n = 3$	Pr of Info Acquisition	0.4622	0.4434	0.2226
	Pr of Vote A if Uninformed	0	0	0.25
	Pr of Vote B if Uninformed	0	[0.07, 1]	[0.25, 0.75]
	Pr of Vote A if signal $s_A$	0.5000	1	1
	Pr of Vote B if signal $s_B$	1	1	1
	Pr of Correct Decision	0.6398	0.6347	0.5455
$n = 7$	Pr of Info Acquisition	0.2514	0.2225	0.0750
	Pr of Vote A if Uninformed	0	0	0.25
	Pr of Vote B if Uninformed	0	[0.08, 1]	[0.25, 0.75]
	Pr of Vote A if signal $s_A$	0.4528	1	1
	Pr of Vote B if signal $s_B$	1	1	1
	Pr of Correct Decision	0.6417	0.6290	0.5115

# Hypothesis under unanimity rule

- H1 voters follow cutoff strategies
  - H2 members of smaller committees acquire more information
  - H8 there is less information acquisition under unanimity than majority
  - \*H9 informed voters for  $B$  vote for  $B$
  - \*H10 informed voters for  $A$  abstain or vote for  $A$
  - \*H11 uninformed voters abstain or vote for  $B$
  - \*H12 larger committees perform worse
  - \*\*H13 unbiased voters acquire information & abstain or vote for  $B$  if uninformed
  - \*\*H14 biased voters do not acquire information & vote
- (\*) Bayesian equilibrium  
(\*\*) subjective beliefs equilibrium

# Experiment design, 1

- ▶ Condorcet jury “jar” interface introduced by Guarnaschelli et al. (2000) and Battaglini et al. (2010)
- ▶ states of the world are represented as a red jar and a blue jar; red jar contains 8 red balls and 4 blue balls, blue jar the opposite
- ▶ master computer tosses a fair coin to select the jar
- ▶ each committee member is assigned an integer-valued signal cost drawn uniformly over  $0, 1, \dots, 100$
- ▶ each committee member chooses whether to pay their signal cost in order to privately observe the color of one of the balls randomly drawn
- ▶ each committee member votes for Red, for Blue, or Abstains
- ▶ if the committee choice is correct each committee member receives 1000 points, less whatever the private cost

## Experiment design, 2

- ▶ each committee decision is a single experimental round, then committees were randomly re-matched and new jars and private observation costs were drawn independently from the previous rounds
- ▶ all experimental sessions (21 subjects each, except for a single 15-subject session with three member committees deciding by majority rule) consisted of 25 rounds of the same treatment
- ▶ number of sessions

		Voting rule	
		<i>majority</i>	<i>unanimity</i>
Committee size	<i>three</i>	4	3
	<i>seven</i>	3	3

## Experimental results: information acquisition

- ▶ voters seem to follow cutoff strategies
- ▶ less information acquisition than Bayesian equilibrium prediction
- ▶ more information acquisition under majority than under unanimity
- ▶ ... no effect of committee size:

<b>Treatment:</b>	<b>3M</b>	<b>7M</b>	<b>3U</b>	<b>7U</b>
Data	0.33	0.33	0.27	0.27
Bayesian equilibrium	0.56	0.39	(0.44, 0.46)	(0.22, 0.25)

## Experimental results: voting

- ▶ striking feature: frequent uninformed voting under majority
- ▶ voters follow their signals (except for  $A$  under unanimity)
- ▶ more uninformed voting under unanimity for  $B$

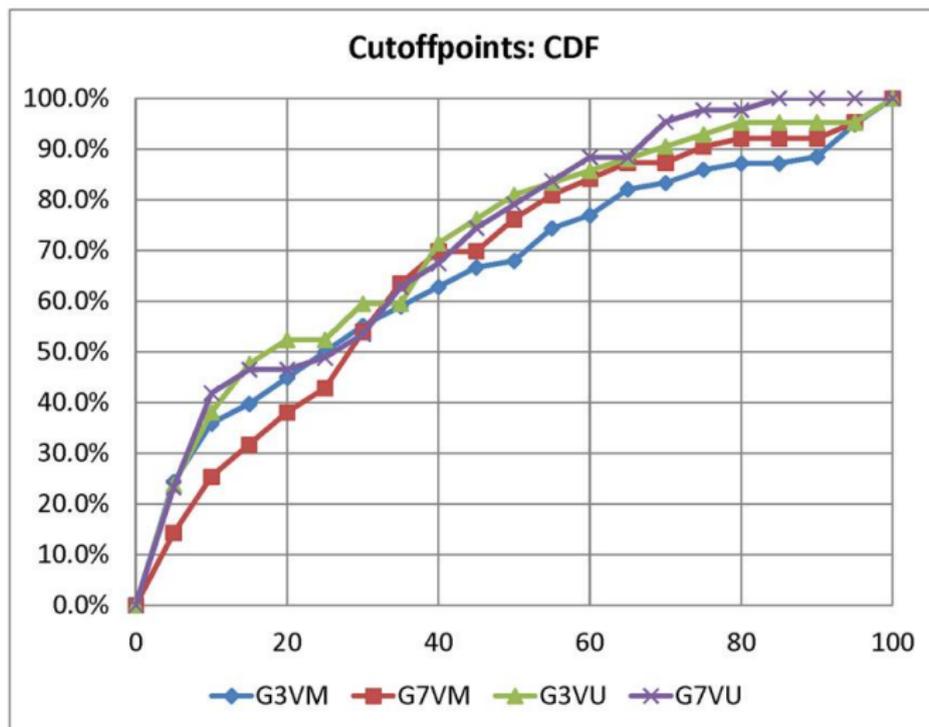
Voter information	Vote decision	<b>3M</b>	<b>7M</b>	<b>3U</b>	<b>7U</b>
Red signal ( $B$ )	Red	0.97	0.93	0.94	0.97
	Blue	0.03	0.06	0.03	0.00
	Abstain	0.00	0.02	0.04	0.03
Blue signal ( $A$ )	Red	0.04	0.02	0.04	0.03
	Blue	0.96	0.96	0.83	0.81
	Abstain	0.00	0.02	0.13	0.17
No signal	Red	0.37	0.28	0.35	0.35
	Blue	0.39	0.33	0.29	0.21
	Abstain	0.24	0.39	0.37	0.45

## Experimental results: information aggregation

- ▶ frequency of successful decision below Bayesian equilibrium
- ▶ majority better than unanimity
- ▶ majority improves with committee size

<b>Treatment:</b>	<b>3M</b>	<b>7M</b>	<b>3U</b>	<b>7U</b>
Data	0.58	0.62	0.54	0.55
Bayesian equilibrium	0.67	0.71	(0.63, 0.64)	(0.63, 0.64)

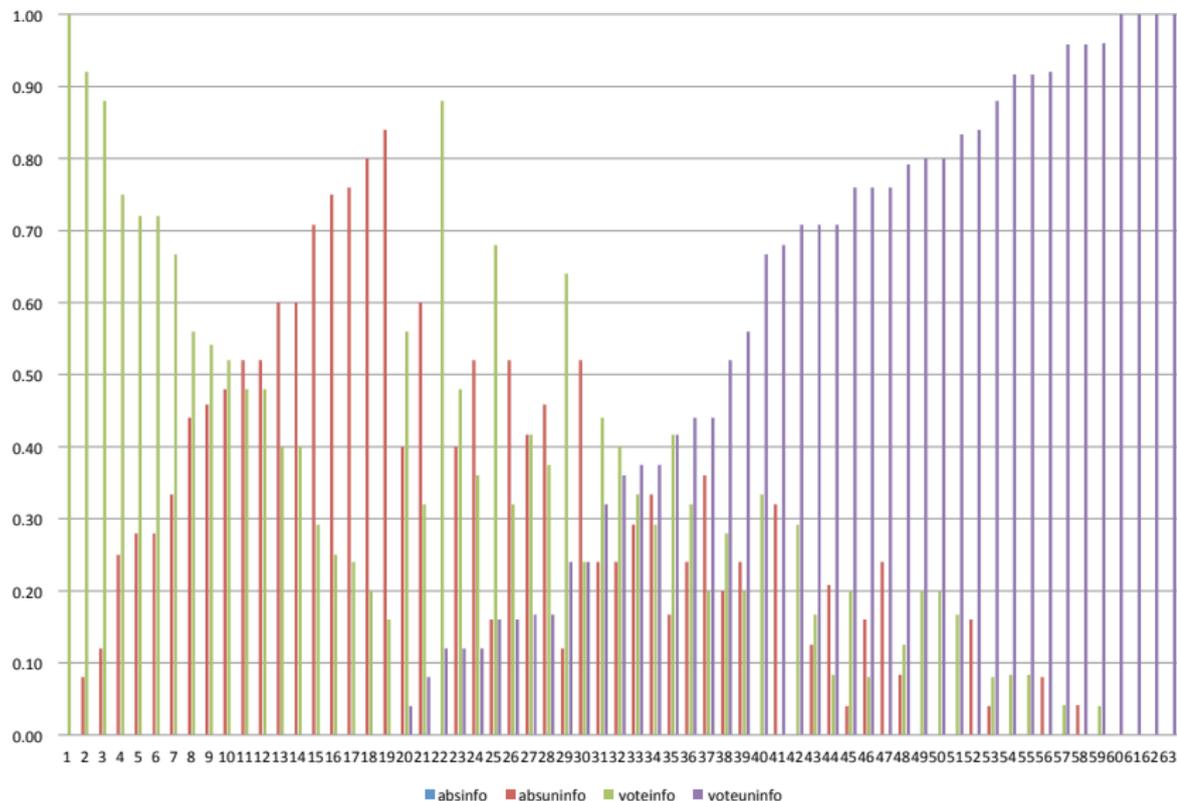
# Experimental results: individual heterogeneity



variation in individual cutoffs, correlated with voting behavior

# Experimental results: individual heterogeneity

Voting: group of 7 and majority rule



## Experimental results: individual heterogeneity

Behavioral Type	<b>3M</b>	<b>7M</b>	<b>3U</b>	<b>7U</b>
Guesser	0.57	0.49	0.43	0.45
Informed	0.34	0.27	0.29	0.19
Mixed	0.09	0.24	0.29	0.36
N	77	63	42	42

## Structural estimation ( $p, Q$ )

- ▶ we estimate using maximum likelihood a version of the subjective beliefs equilibrium model
- ▶  $\beta$  large enough for biased voters not to acquire information
- ▶  $p$ : probability of a biased voter
- ▶ in each round, a subject acts according to the theoretical equilibrium behavior given their type with probability  $Q$ , and randomizes over actions with probability  $1 - Q$
- ▶ nonequilibrium behavior: become informed with probability  $1/2$ , vote for  $A$ , for  $B$  or abstain with probability  $1/3$  regardless of signal

## Structural estimation: majority rule, 3 member committee

action: acquired signal, vote

$$p = 0.4, Q = 0.75, i(p, Q) = 0.74$$

action	mean actual	predicted
AA	0.158	0.188
AB	0.005	0.021
A0	0.001	0.021
BA	0.006	0.021
BB	0.159	0.188
B0	0.001	0.021
0A	0.250	0.192
0B	0.258	0.192
00	0.162	0.156

## Structural estimation: majority rule, 7 member committee

action: acquired signal, vote

$$p = 0.4, Q = 0.8, i(p, Q) = 0.49$$

action	mean actual	predicted
AA	0.182	0.134
AB	0.007	0.017
A0	0.003	0.017
BA	0.003	0.017
BB	0.170	0.135
B0	0.003	0.017
0A	0.158	0.193
0B	0.187	0.193
00	0.277	0.277

## Structural estimation: unanimity rule, 3 member committee

action: acquired signal, vote

$p = 0.39$ ,  $Q = 0.81$ ,  $z = 0.8$  (unbiased voter abstains),  
 $i(p, Q) = 0.47$

action	mean actual	predicted
AA	0.130	0.133
AB	0.006	0.016
A0	0.020	0.016
BA	0.004	0.016
BB	0.137	0.133
B0	0.006	0.016
0A	0.172	0.190
0B	0.260	0.242
00	0.266	0.240

## Structural estimation: unanimity rule, 7 member committee

action: acquired signal, vote

$p = 0.14$ ,  $Q = 0.78$ ,  $z = 0.8$  (unbiased voter abstains),  
 $i(p, Q, z) = 0.21$

action	mean actual	predicted
AA	0.112	0.089
AB	0.004	0.018
A0	0.022	0.018
BA	0.000	0.018
BB	0.128	0.089
B0	0.004	0.018
0A	0.176	0.091
0B	0.207	0.197
00	0.347	0.460

## Final remarks

- ▶ we still need to understand behavioral biases that are important in the actual performance of institutions such as committees under different rules
- ▶ potential for surprises in the lab that may tell us about actual behavior (e.g. extent of uninformed, “opinionated” voting)
- ▶ we need both theory and experiments to make progress in understand actual performance and in designing institutions