Firms' Portfolio Choices, Capital Reallocation and Monetary Policy*

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Abstract

This paper builds a micro-founded model featuring frictional asset market and frictional capital market, to address the interaction between firms' portfolio choices and capital reallocation/accumulation, and the effects of monetary policy. Analytical results show there are different types of general equilibrium, depending on the cash and asset constraints. Two types of monetary policy are discussed: one is conventional policy, i.e., changing inflation rate; the other is unconventional policy, i.e., asset purchase/sale by the central bank. The latter is only valid when the asset constraint is binding. Quantitative work is used to address: (i) increasing cash holdings of U.S. corporations; (ii) general effects of monetary policy.

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1 Introduction

This paper builds a micro-founded model with frictional asset market and frictional capital market, to study the interaction between firms' portfolio choices and capital reallocation/accumulation, and the effects of monetary policy. *On one hand*, it is widely known that public corporations in the U.S. have steadily increased cash holdings over the past decades. Bates et al. (2009) show that the average cash-to-asset ratio of publicly listed industrial firms in the U.S. increased from 10.5% in 1980 to 23.2% in 2006. Except cash and checking deposits, Azar et al. (2016) show nowadays U.S. firms also hold other interest-bearing liquid assets (e.g., savings and time deposits, mutual funds, and money market funds), which is around 80% in the total holdings of cash and liquid assets by 2013. This cash holding issue has attracted attention from researchers, investors and policy makers. Fed economists even argued that "understanding this phenomenon...may help us tease out the reasons for the slow recovery from the Great Recession" (Sanchez and Yurdagul, 2013).

On the other hand, it is well known key components to achieve efficient output and growth are capital accumulation and reallocation. The former is about getting the right amount of aggregate investment, and the latter is about getting the capital available at a point in time into the hands of those best able to use it (Wright et al., 2018a). Since the Great Recession, firms' real investment has recovered quite slowly in the U.S. and other advanced economies (Ottonello, 2015; Sockin, 2015). Looking back, during the Great Recession, major advanced economies including the U.S., Japan and some European countries conducted several rounds of unconventional monetary policy (i.e., Quantitative Easing), which aimed to lower interest rates to stimulate capital accumulation and job creation. But this does not happen as policy makers expected.

This paper is motivated by the above two interesting phenomena, and try to address these research questions: How do firms' portfolio choices in asset market affect capital reallocation/accumulation in capital market, and vice versa? How does monetary policy, including conventional and unconventional policy, affect asset return, then portfolio choices of firms, and furthermore capital accumulation and reallocation across firms?

To answer these questions, I build a micro-founded model featuring frictional asset market and frictional capital market, by the approach of New Monetarism (see the surveys of Lagos et al. 2017 and Nosal and Rocheteau 2017). There are two types of agents in the model: entrepreneur holding portfolios of money and private assets, and suppliers producing and carrying capital. An entrepreneur combines a business and capital to produce (of course, he also need to hire labor, but this is not the focus of this paper). Money is the only medium of exchange in this economy, due to limited commitment and information frictions.

There are i.i.d. shocks in the beginning of every period, which determines the business ideas of entrepreneurs work with some probability, or not with the rest probability. For simplicity, the former is called type-1 entrepreneurs, and the latter type-0 entrepreneurs. There are three markets in a representative period. In the first subperiod, there is a frictional asset market (AM): after the shocks are realized, type-1 entrepreneurs want to acquire more cash by selling private assets to type-0 in AM, which is like an OTC market with search and bargaining frictions. In the second subperiod, there is a frictional capital market (KM): type-1 entrepreneurs have pairwise meetings with suppliers in a used capital market with search and bargaining frictions. In the last subperiod, there is a frictionless centralized market (CM): all agents work, produce, consume, or rebalance portfolio holdings, and the government is only active in this subperiod.

The bargaining solutions for AM and KM show these two markets are closely linked to each other. The portfolio choices of entrepreneurs in AM determine if they can get the first best capital reallocation in KM (secondary market of capital) and first best capital accumulation in CM (primary market of capital). Hence, depending on the cash and asset constraints of entrepreneurs, there are three cases for general equilibrium analysis. Then, two types of monetary policy are discussed: one is conventional policy i.e., changing inflation rate; the other is unconventional policy, i.e., asset purchase/sale by the central bank. Given type-1 entrepreneurs make take-it-or-leave-it offer in KM, the results show lower inflation will increase capital reallocation in KM, capital accumulation in CM, and real balance of cash holdings by firms. The last result is consistent with the empirical results from Azar et al. (2016): with the cost of carrying cash decreasing since 1980s, U.S. public corporations have steadily increased cash holdings.

As for unconventional policy, it only matters when the asset constraint is binding. Given T-I-O-L-I ("take it or leave it") offer in KM, the results show that asset purchase by central bank has opposite effects on two scenarios of capital reallocation for type-1 entrepreneurs, depending on the asset trading in AM. If type-1 trade assets in AM and get extra liquidity, asset purchase by the central bank will decrease capital reallocation in KM. The mechanism is as follows: the asset constraint is binding to type-1 entrepreneurs in AM, so they sell all of the assets, but still cannot get enough liquidity to achieve the first best in KM; when the central bank purchases assets from the public, the asset constraint will be further binding for type-1 entrepreneurs, so that they will get less extra liquidity from AM, and, in the end, capital reallocation will decrease. If type-1 do not match with trading partners to get extra liquidity, asset purchase by the central bank will increase capital reallocation. And the mechanism is as follows: in this scenario, type-1 just use the original cash holdings to acquire capital in KM; when the central bank purchases asset to inject money, the real balance of cash holding by type-1 will increase, so will capital reallocation in KM. However, the results also show the average capital reallocation still decreases with asset purchase by the central bank, which may be due to the dominating decreasing effect from the first scenarios. This also provides interesting policy implications: when the asset constraint is binding, unconventional policy (asset purchase by the central bank) turns out to decrease the average capital reallocation across firms. And the binding asset market, or the scarcity of safe assets or collateral, is an empirically relevant phenomenon in the Great Recession and its aftermath (See Andolfato and Williamson 2015 for more discussion). Of course, it will be more interesting to consider a general bargaining solution for KM, and the subsequent general results for equilibrium analysis.

This part is under work in progress. I also plan to collect data and do quantitative work for the general model.

In terms of literature, this paper is closely related to three lines of research. The first line of literature gets involved with micro-founded models of assets and liquidity, including Duffie et al. (2005), Lagos & Zhang (2015), Han (2015), Geromichalos and Herrenbrueck (2016), , Rocheteau et al. (2018), Dong and Xiao (2018), etc. These papers build micro-founded models for multiple assets, including money and other assets like government bonds, private assets modelled as Lucas trees, etc. But none of them incorporate capital to the model, to address the effects of monetary policy on capital reallocation/accumulation across firms.

The second line of literature gets involved with frictional capital reallocation, including Eisfeldt and Rampini (2006), Gavazza (2010, 2011a, b), Cao and Shi (2014), Ottonello (2015), Cooper and Schott (2016), Dong et al. (2016), Kurman and Rabinovich (2016), Lanteri (2016) and Wright et al. (2018a, b). All of these papers argue capital reallocation is not frictionless: Eisfeldt and Rampini (2006) study how capital reallocation and capital liquidity vary over the business cycle, and focus on the frictions such as informational and contractual frictions which inhibit capital reallocations. Gavazza (2010, 2011a, b) focuses on the used market for aircraft, and find used sales are three times of new sales. This supports the search friction for used capital market. And Cao and Shi (2014), Ottonello (2015), Dong et al. (2016), Kurman and Rabinovich (2016) and Wright et al. (2018a, b) all use a search approach to model used capital market. This paper is particularly related to Wright et al. (2018a), which build a model with a frictional capital market (secondary market of capital) and a frictionless centralized market (primary market of capital), and analyze the effects of monetary and fiscal policy. But Wright et al. (2018a) do not model asset market to address the liquidity need of entrepreneurs, and do not address the interaction between firms' portfolio choices in asset market and capital reallocation/accumulation in capital market. In addition, Wright et al. (2018a) only discuss one type of monetary policy: changing inflation rate, since money is the only asset in the model, while this paper discusses both conventional and unconventional monetary policy by incorporating private assets to the model.

The third line of literature is about firms' cash holdings and portfolio choices in the U.S., including Bates et al. (2009), Falato et al. (2013), Azar et al.(2016), Ma et al. (2018) and relevant corporate finance literature surveyed by these papers. Particularly, to answer the question why U.S. firms hold so much more cash than they used to, Azar et al.(2016) point out, corporate finance literature only focuses on the increase in corporate liquid assets since 1980, but leaves unexplained the even more pronounced decrease from 1945-1980, as well as international variation in the level of corporate liquid asset holdings. They then collect data on the cash and liquid portfolio holdings of public corporations in the U.S. and abroad, for the period 1945-2013, and find changes in the cost of carry can explain the dynamics of corporate cash holdings both in the U.S. and another six advanced economies. In the empirical analysis, they measure the cost of carry as the spread between the risk-free rate and three-month T-bill rate, which can be reduced to a fraction of the three-month T-bill rate in the end. In contrast, in my current paper, as in other micro-founded monetary models, at steady states, inflation rate π, or nominal interest rate ι can be used to measure the cost of carrying cash. In the quantitative work in Section 6, I may consider the dynamics of inflation rate, when measuring the cost of carrying cash for firms.

The rest of the paper is organized as this: Section 2 describes the environment; Section 3 describes the model; Section 4 provides bargaining solutions for AM and KM, then analyze the general equilibrium for three cases; Section 5 analyzes effects of monetary policy; Section 6 provides quantitative work; Section 7 provides discussion and concludes.

2 Environment

Time is discrete and continues forever. There are three subperiods in every period: an decentralized frictional asset market (AM) in the first subperiod, a decentralized frictional capital market (KM) in the second subperiod, and a frictionless centralized market (CM) in the last subperiods. There are two types of firms in the economy:

entrepreneurs e and suppliers s. The measure of e is normalized to 1 while the measure of s is not crucial for any of the results. The production technology is $F(k,h) = \varepsilon^i f(k) + h$ for entrepreneurs, $G(k,h) = g(k) + h$ for suppliers, where k is capital, h is labor hours workers provide. There is also a government, which is a consolidated fiscal and monetary authority. And all government activities take place only in CM.

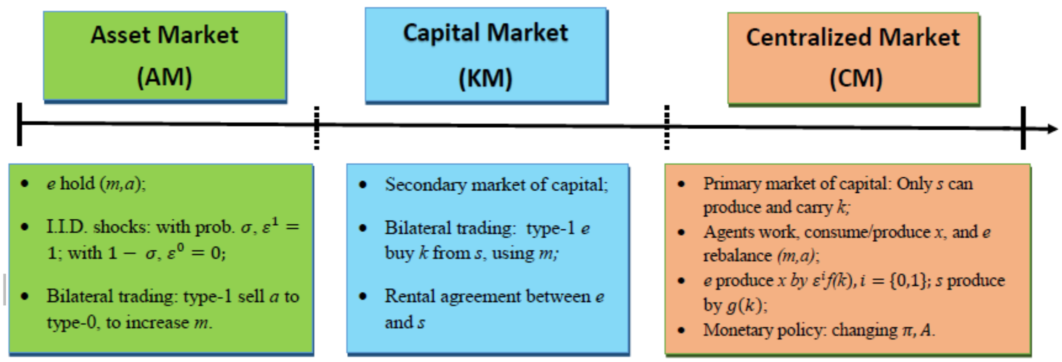

Figure 1: Timeline for a Representative Period

The timeline for a representative period is illustrated in Figure 1. Entrepreneurs need to combine a business idea and capital (acquired in KM) to produce output in CM, by the technology $\varepsilon^i f(k)$, $i = \{0, 1\}$. And the technology for suppliers is $g(k)$. In the beginning of every period, i.i.d. shocks determine if an business idea of entrepreneurs will work or not. With probability σ, $0 < \sigma < 1$, the business idea works and entrepreneurs can produce by $\varepsilon^1 f(k) = f(k)$; with the rest probability $1-\sigma$, the business idea does not work so that $\varepsilon^0 f(k) = 0$. For simplicity, I label them as type-1 (successful) or type-0 (unsuccessful) entrepreneurs.

In the beginning of a representative period, an entrepreneurs hold a portfolio of money m and private assets a. Due to limited commitment and information frictions, money is the only medium of exchange in the economy. And it is costly to hold money due to inflation. The price of money is ϕ_m, measured by numeraire goods x produced in CM. Private assets are like Lucas trees, issued with the price ϕ_a and generating a dividend γ every period. Again ϕ_a and γ are measured by x. Suppose the aggregate supply of assets is constant at $\bar{A}$, $\bar{A} > 0$. And, $\bar{A} = A + A_c$, where A is assets held by the public, and A_c is assets purchased and held by the central bank (the subscript "c" represents the central bank). Since $\bar{A}$ is constant, when A decreases, it represents A_c increases, i.e., the central bank purchases private assets to inject money.[1]

In the first subperiod, after the i.i.d shocks are realized, type-1 entrepreneur want to sell a to type-0, to increase his liquidity position in KM later. And type-0 entrepreneur is willing to buy a to decrease money holding, since he already knows the business idea does not work, and it is costly to hold money. Hence, type-1 and type-0 entrepreneurs search and match in the decentralized AM. If they match, they bargain to determine the terms of trade, (χ, ψ), where χ is assets traded, ψ is the payment, $\chi \leq \hat{a}$, $\psi \leq \hat{m}$. After AM trading, type-1 will participate in KM, while type-0 will directly go to CM.

In the second subperiod, type-1 entrepreneurs enter KM, with portfolio holding $(\ddot{m}, \ddot{a})$, but can only use cash $\ddot{m}$, $\ddot{m} \in (m, m+\psi)$, to acquire capital from suppliers. I use $(\ddot{m}, \ddot{a})$ to distinguish the portfolio holdings of type-1 from (m, a), the portfolio holdings in the beginning, due to the asset trading in AM. The terms of trade in KM is (q, p). Entrepreneurs and suppliers randomly search and match with each other in KM. If matched, suppliers transfer q units of capital to type-1, and get cash payment p. Here, I suppose type-1 entrepreneurs acquire the capital from suppliers in KM, but will return it to suppliers by the end of every period. There are different interpretations on this assumption. On one hand, due to the i.i.d. shocks in the beginning of every period, it is hard for type-1 entrepreneurs and suppliers to form long-term relationships. Hence, this assumption can be regarded as a rental agreement between them. On the other hand, this can also be interpreted as type-1 entrepreneurs need different types of capital, e.g., fixed at different locations, every period.

In the last subperiod, all agents are active in a frictionless Walras' market (CM): they work, produce or consume numeraire goods x and rebalance their portfolio hold-

[1] Decreasing A resembles private assets purchase by Bank of Japan (since the end of 1990s to now), by the Federal Reserve Bank of the U.S. (during 2008 - 2015, mainly agency debt and morgtage-back-securities (MBS)), and other central banks in the advanced economies during the Great Recession. Until nowadys, although Quantitaive Easing is over in most advanced economies, central banks such as Bank of Japan and the Fed, still hold plent of private assets.

ings. Particularly, type-1 entrepreneurs can produce x by the technology $\varepsilon^1 f(k) = f(k)$, if they match with suppliers, and get capital in KM. Suppliers can produce x by $g(k)$, also produce new capital, and carry to next period. The depreciation rate of capital is δ. Here the production technology satisfies, $f(0) = g(0) = 0$, $f'(k) > 0$, $f''(k) < 0$, $g'(k) > 0$, $g''(k) < 0$ and the Inada conditions. Also I suppose $f(k), g(k)$ exhibit decreasing returns to scale, which is to capture type-1 entrepreneurs want to acquire some but not all of the capital from suppliers in KM. If assuming constant returns to scale, given labor is hired in the frictionless CM, there would be no reason to transfer capital from suppliers to type-1 entrepreneurs if they are equally productive, and if one is more productive it would be efficient to allocate all the capital to them.[2]

Furthermore, it is necessary to point out entrepreneurs and suppliers have different costs to produce and carry capital. Suppliers have no cost while entrepreneurs has prohibitive cost to do so. Hence, in the model, entrepreneurs choose not to produce or carry new capital at all, instead just using money to acquire used capital from suppliers in KM. Suppliers can produce new capital, one-to-one with CM goods. In addition, CM can be regarded as the primary market of capital, while KM is the secondary market of capital.

The government is only active in CM. Suppose fiscal policy is passive and can accommodate any changes from monetary policy. This is also to focus on monetary policy. Two types of monetary policy are discussed: one is about changing inflation rate π, and the other is about changing asset holding by central bank A_c, which is equivalent of changing A as discussed before. Suppose money growth rate is μ, so $M_+ = (1+\mu)M$, where M is money supply for the current period. It is imposed $\mu > \beta - 1$, which is equivalent in steady state to $\iota > 0$. But $\iota \to 0$ is also considered, which is Friedman rule. At steady states, $\mu = \pi$, where π is inflation rate. By Fisher equation, $1 + \iota = (1+\pi)/\beta$, where $1/\beta = 1 + r$. Therefore, changing ι is equivalent

[2] The latter looks like a merger or acquisition, which may be interesting but is not the focus of this paper. See Wright et al. (2018a,b) for more discussion.

of changing π. The government budget constraint is,

$$G + T + \phi_a A_{c+} = \phi_m (M_+ - M) + (\phi_a + \gamma) A_c, \tag{1}$$

where G is government expenditure, T is lump-sum transfers minus taxes. To ease notations, I ignore the time scripts for all variables, and use hat or $+$ to show variables next period. For example, $\hat{m}, \hat{a}, \hat{k}$ are money, assets and capital carried to next period, ϕ_m^+ is the price of money next period, A_c^+ is asset holding of central bank next period, etc.

3 Model

The value functions for agents in AM, KM and CM are $U^j(m, a, k), V^j(m, a, k)$ and $W^j(m, a, k)$, $j = \{0, 1, s\}$ representing type-0 entrepreneurs, type-1 entrepreneurs and suppliers. In the beginning of the period, for type-0 and type-1 entrepreneurs, $k = 0$, while for suppliers, $m = a = 0$.

I start from CM in the current period. For type i entrepreneurs, $i = \{0, 1\}$,

$$\begin{aligned} W^i(m, a, k) &= \max_{x,h,\hat{m},\hat{a}} \{u(x) - h + \beta \mathbb{E} U^i(\hat{m}, \hat{a}, 0)\} \\ \text{s.t. } x + \phi_m \hat{m} + \phi_a \hat{a} &= wh + \phi_m m + (\phi_a + \gamma) a + \varepsilon^i f(k) + T, \end{aligned} \tag{2}$$

where $\mathbb{E}U^i(\hat{m}, \hat{a}, 0)$ is the expected value function for entrepreneurs next period AM, since they will experience shocks and may succeed or fail. The LHS of the budget constraint (2) is the expenditure: consumption on CM goods x, and the portfolio of money and assets $(\hat{m}, \hat{a})$ carried to next period. The RHS is revenues: labor income wh, where $w \equiv 1$; portfolio holdings for the current period, $\phi_m m + (\phi_a + \gamma) a$; production $\varepsilon^i f(k)$ plus lump sum transfers minus taxes T. Substituting h from the budget constraint, I have,

$$W^i(m, a, k) = \phi_m m + (\phi_a + \gamma) a + \varepsilon^i f(k) + T + u(x) - x + \max_{\hat{m},\hat{a}} \{\beta \mathbb{E} U^i(\hat{m}, \hat{a}, 0) - \phi_m \hat{m} - \phi_a \hat{a}\}.$$

The envelope conditions are,

$$
\begin{aligned}
W^i_m &= \phi_m && (3)\\
W^i_a &= \phi_a + \gamma && (4)\\
W^i_k &= \varepsilon^i f'(k). && (5)
\end{aligned}
$$

Always, the first best for x is $u'(x) = 1$. The FOCs against $\hat{m}, \hat{a}$ are,

$$
\begin{aligned}
\phi_m &= \beta \frac{\partial \mathbb{E} U^i(\hat{m}, \hat{a}, 0)}{\partial \hat{m}} && (6)\\
\phi_a &= \beta \frac{\partial \mathbb{E} U^i(\hat{m}, \hat{a}, 0)}{\partial \hat{a}} && (7)
\end{aligned}
$$

For suppliers,

$$
\begin{aligned}
W^s(m, 0, k) &= \max_{x,h,\hat{k}} \{u(x) - h + \beta V^s(0, 0, \hat{k})\}\\
\text{s.t. } x + \hat{k} &= h + \phi_m m + g(k) + (1-\delta)k + T && (8)
\end{aligned}
$$

which shows suppliers may get some money from KM trading, but $a = 0$, since money is the only medium of exchange for capital trading. Here, comparing the value functions of entrepreneurs and suppliers, particularly the budget constraints (2) and (8), it shows the difference between entrepreneurs and suppliers: firstly, only entrepreneurs carry the portfolio $(\hat{m}, \hat{a})$ to next period, while only suppliers produce and carry capital $\hat{k}$ to next period; secondly, there is a rental agreement between entrepreneurs and suppliers, therefore the net value of capital after depreciation only shows up in (8) by $(1 - \delta)k$.

Similarly, I have,

$$
W^s(m, 0, k) = \phi_m m + g(k) + (1-\delta)k + T + u(x) - x + \max_{\hat{k}} \{\beta V^s(0, 0, \hat{k}) - \hat{k}\}.
$$

Hence, the envelope conditions are,

$$W_m^s = \phi_m,\ W_k^s = 1 - \delta + g'(k).$$

And the FOC against $\hat{k}$ is,

$$1 = \beta \frac{\partial V^s(0, 0, \hat{k})}{\partial \hat{k}}. \tag{9}$$

For AM next period, in the beginning, entrepreneurs carry the same portfolio of money and assets $(\hat{m}, \hat{a})$, which is used to capture the holdings of cash and liquid asset by the U.S. firms nowadays. Then the i.i.d shocks determine if the business ideas of entrepreneurs will work or not. With probability σ, the idea works, and entrepreneurs succeed, labelled as type-1. With the rest probability, it does not work, and entrepreneurs do not succeed, labelled as type-0. Based on the law of large number, the measure of type-1 and type-0 will just be σ and $1-\sigma$. Suppose the total measure of matches in AM is $\mathcal{M}$, $0 < \mathcal{M} \leq \min\{\sigma, 1-\sigma\}$. Then the matching rate for type-1 and type-0 is $\mathcal{M}/\sigma$, $\mathcal{M}/(1-\sigma)$.

After the shocks are realized, type-1 want to sell assets to type-0, to get extra money for capital trading in KM. Type-0 are willing to buy assets to provide liquidity (money) since they will just skip KM, and directly go to CM. In the beginning of AM, the portfolio holding of entrepreneurs is $(\hat{m}, \hat{a}, 0)$, with $\hat{k} = 0$. Depending on the shocks, the value function for entrepreneurs is,

$$\mathbb{E}U^i(\hat{m}, \hat{a}, 0) = \sigma U^1(\hat{m}, \hat{a}, 0) + (1-\sigma) U^0(\hat{m}, \hat{a}, 0).$$

And,

$$\begin{aligned} U^1(\hat{m}, \hat{a}, 0) &= V^1(\hat{m}, \hat{a}, 0) + \frac{\mathcal{M}}{\sigma}[V^1(\hat{m}+\psi, \hat{a}-\chi, 0) - V^1(\hat{m}, \hat{a}, 0)] & (10) \\ U^0(\hat{m}, \hat{a}, 0) &= W^0(\hat{m}, \hat{a}, 0) + \frac{\mathcal{M}}{1-\sigma}[W^0(\hat{m}-\psi, \hat{a}+\chi, 0) - W^0(\hat{m}, \hat{a}, 0)], & (11) \end{aligned}$$

where χ is assets traded, ψ is the payment, $\chi \leq \hat{a}$, $\psi \leq \hat{m}$.

For KM next period, suppose type-1 entrepreneurs enter KM with the portfolio $(\ddot{m}, \ddot{a})$, and search suppliers to acquire capital. The terms of trade in KM is (q, p), $q \leq \hat{k}, p \leq \ddot{m}$. It means, to get q units of capital from suppliers, type-1 need to pay p by cash. To be precise, $q = q(\ddot{m})$, $p = p(\ddot{m})$ since the cash holdings of type-1, $\ddot{m}$, determine the quantity and payment of capital traded in KM.

For type-1 entrepreneurs,

$$\begin{aligned} V^1(\ddot{m}, \ddot{a}, 0) &= W^1(\ddot{m}, \ddot{a}, 0) + \alpha_1\{W^1[\ddot{m} - p(\ddot{m}), \ddot{a}, q(\ddot{m})] - W^1(\ddot{m}, \ddot{a}, 0)\} \\ &= W^1(\ddot{m}, \ddot{a}, 0) + \alpha_1[f(q(\ddot{m})) - \phi_m^+ p(\ddot{m})], \end{aligned} \tag{12}$$

where α_1 is the probability type-1 match with suppliers. For suppliers,

$$\begin{aligned} V^s(0, 0, \hat{k}) &= W^s(0, 0, \hat{k}) + \alpha_s\{W^s[p(\ddot{m}), 0, \hat{k} - q(\ddot{m})] - W^s(0, 0, \hat{k})\} \\ &= W^s(0, 0, \hat{k}) + \alpha_s\{\phi_m^+ p(\ddot{m}) + g[\hat{k} - q(\ddot{m})] - g(\hat{k})\}, \end{aligned}$$

where α_s is the probability suppliers match with type-1 entrepreneurs. Here $\alpha_i, i = \{1, s\}$, can also be interpreted as capturing the difficulty to find trading partners in a used capital market, either due to information frictions or capital specificity (Gavazza, 2011a).

In addition, it is obvious that the portfolio holdings of type-1, $(\ddot{m}, \ddot{a})$, depend on the asset trading with type-0 in AM. Furthermore, in KM, the capital reallocated to type-1 entrepreneurs from suppliers, q, also depends on $(\ddot{m}, \ddot{a})$, particularly the cash holdings $\ddot{m}$. Hence, the two markets, AM and KM, are closely linked to each other.

4 Equilibrium

After describing the model in Section 3, I proceed to the bargaining solutions for AM and KM by backward induction, and then go to equilibrium analysis. Backward induction means firstly analyzing the bargaining problem in KM, to figure out how much money is needed to achieve the first best capital reallocation q and capital accumulation $\hat{k}$; then I go back to AM, to analyze the bargaining problem of type-1 and type-0 entrepreneurs, and to figure out the optimal portfolio choices of entrepreneurs in AM.

4.1 Bargaining Solutions for KM

For simplicity, firstly I focus on a special case: type-1 entrepreneurs make take-it-or-leave-it offers. Hence, the bargaining problem of type-1 is,

$$\max_{q(\ddot{m}),p}\{f[q(\ddot{m})] - \phi_m^+ p(\ddot{m})\}$$

$$\text{s.t. } \phi_m^+ p(\ddot{m}) + g(\hat{k} - q(\ddot{m})) - g(\hat{k}) = 0$$

$$q(\ddot{m}) \leq \hat{k}, p(\ddot{m}) \leq \ddot{m}$$

From the first constraint, I have,

$$\phi_m^+ p(\ddot{m}) = g(\hat{k}) - g[\hat{k} - q(\ddot{m})]. \tag{13}$$

Since $q(\ddot{m}) \leq \hat{k}$ never binds because of the Inada conditions, given $\ddot{m}$, we only need to consider two cases: (1) $p(\ddot{m}) \leq \ddot{m}$ is binding; (2) $p(\ddot{m}) \leq \ddot{m}$ is not binding. Define m^* as the amount of money which enables type-1 to get first best q^*. The first case means $\ddot{m} < m^*$, so type-1 do not get enough money from the asset trading in AM. Hence, the solution is $p(\ddot{m}) = \ddot{m}$, $q(\ddot{m}) < q^*$, and satisfies $g[\hat{k} - q(\ddot{m})] = g(\hat{k}) - \phi_m^+ \ddot{m}$.

The second case means, $\ddot{m} \geq m^*$. With plentiful liquidity, type-1 will get the first best in KM, i.e., $q(\ddot{m}) = q^*$. Substitute (13) to the maximization problem, we have,

$$\max_{q(\ddot{m})}\{f[q(\ddot{m})] + g[\hat{k} - q(\ddot{m})] - g(\hat{k})\}$$

Then $q(\ddot{m}) = q^*$ satisfies $f'[q(\ddot{m})] = g'[\hat{k} - q(\ddot{m})]$. And we have, $p(\ddot{m}) = m^* = [g(\hat{k}) - g(\hat{k} - q^*)]/\phi_m^+$.

Given the take-it-or-leave-it offer, I describe the bargaining solutions for KM as follows.

Lemma 1 *Define the amount of money that allows type-1 entrepreneurs get* q^* *as,*

$$m^* = \frac{[g(\hat{k}) - g(\hat{k} - q^*)]}{\phi_m^+}.$$

And $q(\ddot{m}) = q^*$ *satisfies* $f'[q(\ddot{m})] = g'[\hat{k} - q(\ddot{m})]$*. Then the bargaining solutions for KM is as follows,*

$$q(\ddot{m}) = \begin{cases} \hat{k} - g^{-1}[g(\hat{k}) - \phi_m^+ \ddot{m}], & \text{if } \ddot{m} < m^* \\ q^*, & \text{if } \ddot{m} \geq m^* \end{cases}$$

$$p(\ddot{m}) = \begin{cases} \ddot{m}, & \text{if } \ddot{m} < m^* \\ m^*, & \text{if } \ddot{m} \geq m^* \end{cases}$$

Secondly, I consider a general case: type-1 entrepreneurs and suppliers do Kalai bargaining, and the bargaining power of type-1 is θ. Then the bargaining problem of type-1 is as follows,

$$\max_{q(\ddot{m}),p} \{f[q(\ddot{m})] - \phi_m^+ p(\ddot{m})\}$$

$$\text{s.t. } f[q(\ddot{m})] - \phi_m^+ p(\ddot{m}) = \theta\{f[q(\ddot{m})] + g[\hat{k} - q(\ddot{m})] - g(\hat{k})\}$$

$$q(\ddot{m}) \leq \hat{k}, p(\ddot{m}) \leq \ddot{m}$$

Similarly, from the first constraint,

$$\phi_m^+ p(\ddot{m}) = (1-\theta) f[q(\ddot{m})] + \theta\{g(\hat{k}) - g[\hat{k} - q(\ddot{m})]\}.$$

Again I need to consider two cases: (1) $p(\ddot{m}) \leq \ddot{m}$ is binding; (2) $p(\ddot{m}) \leq \ddot{m}$ is not binding. If $p(\ddot{m}) \leq \ddot{m}$ is binding, I have, $\ddot{m} < m^*$, $\phi_m^+ p(\ddot{m}) = \phi_m^+ \ddot{m} = \{(1-\theta) f[q(\ddot{m})] + \theta[g(\hat{k}) - g(\hat{k} - q(\ddot{m}))]\}$, and $q(\ddot{m}) < q^*$. Here I define $Q[q(\ddot{m})] \equiv (1-\theta) f[q(\ddot{m})] + \theta\{g(\hat{k}) - g[\hat{k} - q(\ddot{m})]\}$, then I can rewrite $q(\ddot{m})$ as,

$$q(\ddot{m}) = Q^{-1}(\phi_m^+ \ddot{m})$$

If $p(\ddot{m}) \leq \ddot{m}$ is not binding, I have, $\ddot{m} \geq m^*$ and $q(\ddot{m}) = q^*$. And the bargaining problem of type-1 becomes,

$$\max_{q(\ddot{m})} \theta\{f(q(\ddot{m})) + g[\hat{k} - q(\ddot{m})] - g(\hat{k})\}.$$

So again, $q(\ddot{m}) = q^*$ satisfies $f'[q(\ddot{m})] = g'[\hat{k} - q(\ddot{m})]$, and $p(\ddot{m}) = m^* = Q(q^*)/\phi_m^+$

Using Kalai bargaining, the bargaining solutions for KM are as follows,

Lemma 2 *Define the amount of money that allows type-1 entrepreneurs get q^*,*

$$m^* = \frac{Q(q^*)}{\phi_m^+},$$

where $Q(q^*) \equiv (1-\theta)f(q^*) + \theta[g(\hat{k}) - g(\hat{k} - q^*)]$, and $q(\ddot{m}) = q^*$ satisfies $f'[q(\ddot{m})] = g'[\hat{k} - q(\ddot{m})]$. Then the bargaining solutions for KM is as follows,

$$q(\ddot{m}) = \begin{cases} Q^{-1}(\phi_m^+ \ddot{m}), & \text{if } \ddot{m} < m^* \\ q^*, & \text{if } \ddot{m} \geq m^* \end{cases}$$

$$p(\ddot{m}) = \begin{cases} \ddot{m}, & \text{if } \ddot{m} < m^* \\ m^*, & \text{if } \ddot{m} \geq m^* \end{cases}$$

4.2 Bargaining Solutions for AM

After solving the bargaining problem in KM, now I go back to analyze the bargaining solutions in AM. Suppose type-1 and type-0 entrepreneurs do Kalai bargaining in AM, and the bargaining power of type-1 entrepreneurs is η. Although the portfolio holdings of type-1 and type-0 are ex ante homogenous, to distinguish the two types in asset trading, I label the portfolio holding of type-1 entrepreneurs as $(\hat{m}, \hat{a})$ and that of type-0 as $(\tilde{m}, \tilde{a})$. Then the bargaining problem is as follows,

$$\max_{\chi,\psi}\{V^1(\hat{m}+\psi, \hat{a}-\chi, 0) - V^1(\hat{m}, \hat{a}, 0)\}$$
$$\text{s.t. } V^1(\hat{m}+\psi, \hat{a}-\chi, 0) - V^1(\hat{m}, \hat{a}, 0) = \frac{\eta}{1-\eta}[W^0(\tilde{m}-\psi, \tilde{a}+\chi, 0) - W^0(\tilde{m}, \tilde{a}, 0)]$$
$$-\tilde{a} \le \chi \le \hat{a},\ -\hat{m} \le \psi \le \tilde{m}.$$

By Kalai bargaining, the objective is to maximize the surplus of type-1, given the surplus of type-1 as a fixed proportional η of the total surplus, which is shown by the first constraint. Since type-1 already know he has successful real investment projects later, he expects to proceed to KM with extra cash, ψ, and a reduced asset holding, $\hat{a}-\chi$. And type-0 will directly go to CM. Using (3)-(5), and (12), we can rewrite the bargaining problem as,

$$\max\{\phi_m^+\psi - (\phi_a^+ + \gamma)\chi + \alpha_1[f(\grave{q}) - f(q) - \phi_m^+(\grave{p} - p)]\} \tag{14}$$

$$\text{s.t. } \phi_m^+\psi - (\phi_a^+ + \gamma)\chi + \alpha_1[f(\grave{q}) - f(q) - \phi_m^+(\grave{p} - p)]\} = \frac{\eta}{1-\eta}[(\phi_a^+ + \gamma)\chi - \phi_m^+\psi] \tag{15}$$

$$-\tilde{a} \le \chi \le \hat{a},\ -\hat{m} \le \psi \le \tilde{m}, \tag{16}$$

where $\grave{q} \equiv q(\hat{m}+\psi), \grave{p} \equiv p(\hat{m}+\psi), q \equiv q(\hat{m}), p \equiv p(\hat{m})$, and $q(\cdot)$, $p(\cdot)$ come from the KM bargaining solution in Lemma 2. For example, $\grave{q}$ is the quantity of capital traded in KM when the cash holding of type-1 is $\hat{m}+\psi$, while q is the quantity of capital traded when the cash holding of type 1 is $\hat{m}$. More precisely, $\grave{q}$ represents capital reallocated in KM, when type-1 entrepreneurs match with type-0 and get extra liquidity ψ from AM; q represents capital reallocated in KM, when type-1 do not get extra liquidity from AM.

And I can rewrite the constraint in (15) as,

$$(\phi_a^+ + \gamma)\chi = \phi_m^+\psi + (1-\eta)\alpha_1[f(\grave{q}) - f(q) - \phi_m^+(\grave{p} - p)]. \tag{17}$$

This result shows the value of assets traded in AM should equal the real balance of the payment type-1 gives up, $\phi_m^+\psi$, plus a fraction, $(1-\eta)$, of the net surplus created when a monetary transfer, ψ, is made to type-1 entrepreneurs. This net surplus, $\alpha_1[f(\grave{q}) - f(q) - \phi_m^+(\grave{p} - p)]$, is also embodied as the gap of the expected gain of capital reallocation, i.e., $\grave{q}$ to q, for type-1 in KM.

Now I move on to solve the bargaining problem in AM. Suppose the trade in AM takes place only when a strictly positive surplus is generated. This is to rule out the cases where type-1 and type-0 just swap money for assets, but no surplus is generated. For example, if $\hat{m} \geq m^*$, type-1 carry enough money in the very beginning, and can always get first best q^*, no matter it matches or not with type-0 in AM. Hence, the asset and cash constraints in (16) can be simplified to, $\chi \leq \hat{a}$, $\psi \leq \tilde{m}$. I also suppose $\hat{m}, \tilde{m} \leq m^*$, since it is costly to carry money. Before I solve the bargaining problems, there are two important issues to consider. *Firstly*, I need to ask if type-0 can provide enough cash so that $\hat{m} + \tilde{m} \geq m^*$. It means, if these two types of entrepreneurs pool their cash together, will it be possible that type-1 can purchase q^* in KM? *Secondly*, I also need to ask if type-1 carry enough assets to compensate type-0 for the transfer of liquidity. If the asset constraint $\chi \leq \hat{a}$, is not binding, I will always have $\psi = \min\{m^* - \hat{m}, \tilde{m}\}$. Therefore, there are two cases to consider as follows.

Case 1: $\hat{m} + \tilde{m} \geq m^*$

In this case, if the asset constraint is not binding, I have, $\psi = m^* - \hat{m}$. Define the critical level of assets that type-1 should carry to compensate type-0 for $m^* - \hat{m}$ units of money as $\bar{a}(\hat{m}, \tilde{m})$. Substitute $\psi = m^* - \hat{m}$ to (17), I can derive,

$$\bar{a}(\hat{m}, \tilde{m}) = \frac{(1-\Delta)\phi_m^+(m^* - \hat{m}) + \Delta[f(q^*) - f(q)]}{\phi_a^+ + \gamma},$$

where $\Delta \equiv (1-\eta)\alpha_1$. If $\hat{a} \geq \bar{a}(\hat{m}, \tilde{m})$, the asset constraint is not binding, I have, $\chi = \bar{a}(\hat{m}, \tilde{m})$, and $\psi = m^* - \hat{m}$, since $\tilde{m} \geq m^* - \hat{m}$. Then the price of assets traded is $(m^* - \hat{m})/\bar{a}(\hat{m}, \tilde{m})$. If $\hat{a} < \bar{a}(\hat{m}, \tilde{m})$, type-1 does not have enough assets to acquire the first best level of money m^*. In this case, he will sell up all of the assets he holds, i.e., $\chi = \hat{a}$. By substituting $\chi = \hat{a}$ to (17), I can derive the real balance of monetary transfer,

$$\bar{\psi} = \frac{(\phi_a^+ + \gamma)\hat{a} - \Delta[f(\grave{q}) - f(q)]}{\phi_m^+(1-\Delta)}. \tag{18}$$

Case 2: $\hat{m} + \tilde{m} < m^*$

In this case, if the asset constraint is not binding, I have, $\psi = \tilde{m}$. Again I can derive the critical level of assets that type-1 should carry to afford $\tilde{m}$ by substituting $\psi = \tilde{m}$ to (17). That is,

$$\bar{a}(\hat{m}, \tilde{m}) = \frac{(1-\Delta)\phi_m^+ \tilde{m} + \Delta[f(\grave{q}) - f(q)]}{\phi_a^+ + \gamma}.$$

If $\hat{a} \geq \bar{a}(\hat{m}, \tilde{m})$, the asset constraint is not binding, I have, $\chi = \bar{a}(\hat{m}, \tilde{m})$, and $\psi = \tilde{m}$, since $\tilde{m} < m^* - \hat{m}$. Then the price of assets traded is $\tilde{m}/\bar{a}(\hat{m}, \tilde{m})$. If $\hat{a} < \bar{a}(\hat{m}, \tilde{m})$, type-1 trades away all of the assets he holds, $\chi = \hat{a}$. Similarly, I can derive the real balance of monetary transfer as in (18).

Using Kalai bargaining, I can summarize the bargaining solutions for AM as follows.

Lemma 3 *Define the cutoff level of asset holdings,*

$$\bar{a}(\hat{m}, \tilde{m}) = \begin{cases} \frac{1}{\phi_a^+ + \gamma}\{(1-\Delta)\phi_m^+ \tilde{m} + \Delta[f(\grave{q}) - f(q)]\}, & \text{if } \hat{m} + \tilde{m} < m^* \\ \frac{1}{\phi_a^+ + \gamma}\{(1-\Delta)\phi_m^+(m^* - \hat{m}) + \Delta[f(q^*) - f(q)]\}, & \text{if } \hat{m} + \tilde{m} \geq m^* \end{cases}$$

Then the bargaining solutions are as follows,

$$\begin{aligned} \chi(\hat{m}, \tilde{m}, \hat{a}) &= \begin{cases} \bar{a}(\hat{m}, \tilde{m}), & \text{if } \hat{a} \geq \bar{a}(\hat{m}, \tilde{m}) \\ \hat{a}, & \text{if } \hat{a} < \bar{a}(\hat{m}, \tilde{m}) \end{cases} \\ \psi(\hat{m}, \tilde{m}, \hat{a}) &= \begin{cases} \min\{m^* - \hat{m}, \tilde{m}\}, & \text{if } \hat{a} \geq \bar{a}(\hat{m}, \tilde{m}) \\ \bar{\psi}, & \text{if } \hat{a} < \bar{a}(\hat{m}, \tilde{m}) \end{cases}, \end{aligned}$$

where,

$$\bar{\psi} = \frac{(\phi_a^+ + \gamma)\hat{a} - \Delta[f(\grave{q}) - f(q)]}{\phi_m^+(1-\Delta)}.$$

4.3 General Equilibrium

After characterizing the bargaining solutions in KM and AM, I can proceed to the equilibrium analysis. In Section 4.2, I use $(\hat{m}, \hat{a})$ and $(\tilde{m}, \tilde{a})$ to distinguish portfolio holdings of type-1 and type-0, but these two types are ex ante homogenous in the model, i.e., $\tilde{m} = \hat{m}$, $\tilde{a} = \hat{a}$. For equilibrium analysis, I focus on the stationary equilibrium, and define as follows.

Definition 1 *Given policy parameters $\{G, T, \pi, A_c\}$, a stationary general equilibrium is a list $\{\chi, \psi, \grave{q}, q, \hat{k}, \phi_m, \phi_a\}$. And the equilibrium features,*

1. *The terms of trade, (χ, ψ), and bargaining solutions for AM satisfy Lemma 3;*
2. *The terms of trade, $(\grave{q}, \grave{p})$, (q, p), and bargaining solutions for KM satisfy Lemma 2;*
3. *The government budget constraint (1), is satisfied;*
4. *Market clears: $\hat{m} = \tilde{m} = M$, $\hat{a} = \tilde{a} = A$.*

And I can prove,

Lemma 4 *A stationary monetary equilibrium $\{\chi, \psi, \grave{q}, q, \hat{k}, \phi_m, \phi_a\}$ exists and is unique.*

Proof: See the Appendix

Furthermore, based on Lemma 2 and 3, there are three cases for the general equilibrium analysis.

Case I: $\hat{m} \in (m^* - \tilde{m}, m^*)$ and $\hat{a}$, $\tilde{a} > \bar{a}(\hat{m}, \tilde{m})$

In this region, the money holdings of type-1 and type-0 entrepreneurs allow type-1 to bring the first best m^* into KM. And the asset holdings of type-1 are enough to compensate type-0 for the monetary transfer. Hence, the bargaining solutions for AM are,

$$\begin{aligned}
\chi(\hat{m}, \tilde{m}, \hat{a}) &= \bar{a}(\hat{m}, \tilde{m}) = \frac{1}{\phi_a^+ + \gamma}\{(1-\Delta)\phi_m^+(m^* - \hat{m}) + \Delta[f(q^*) - f(q)]\} \\
\psi(\hat{m}, \tilde{m}, \hat{a}) &= m^* - \hat{m}
\end{aligned}$$

Subsequently, in KM, type-1 entrepreneurs get the first best $\grave{q} = q^*$, where $\grave{q}$ satisfies $f'(\grave{q}) = g'(\hat{k} - \grave{q})$, and $\grave{p}(\ddot{m}) = m^* = Q(q^*)/\phi_m^+$. Since agents get the first best m^* and q^*, monetary policy is not valid in this region.

Case II: $\hat{m} < m^* - \tilde{m}$**, and** $\hat{a}$, $\tilde{a} > \bar{a}(\hat{m}, \tilde{m})$

In this region, type-1 can not acquire enough money from AM to bring m^* into KM. In AM trading, type-1 get all of the cash holding of type-0, since the asset constraint is not binding. Hence, the bargaining solutions for AM are,

$$\begin{aligned} \chi(\hat{m}, \tilde{m}, \hat{a}) &= \bar{a}(\hat{m}, \tilde{m}) = \frac{(1-\Delta)\phi_m^+ \tilde{m} + \Delta[f(\grave{q}) - f(q)]}{\phi_a^+ + \gamma} && (19) \\ \psi(\hat{m}, \tilde{m}, \hat{a}) &= \tilde{m}. && (20) \end{aligned}$$

Subsequently, in KM, I substitute (12) to (10),

$$\begin{aligned} U^1(\hat{m}, \hat{a}, 0) &= (1 - \frac{\mathcal{M}}{\sigma})\{W^1(\hat{m}, \hat{a}, 0) + \alpha_1[W^1(\hat{m} - p, \hat{a}, q) - W^1(\hat{m}, \hat{a}, 0)]\} \\ &\quad + \frac{\mathcal{M}}{\sigma}\{W^1(\hat{m} + \psi, \hat{a} - \chi, 0) + \alpha_1[W^1(\hat{m} + \psi - \grave{p}, \hat{a} - \chi, \grave{q}) - W^1((\hat{m} + \psi, \hat{a} - \chi, 0)]\} \\ &= W^1(\hat{m}, \hat{a}, 0) + \frac{\mathcal{M}}{\sigma}[\phi_m^+\psi - (\phi_a^+ + \gamma)\chi] + \alpha_1 \frac{\mathcal{M}}{\sigma}[f(\grave{q}) - \phi_m^+ \grave{p}] + \alpha_1(1 - \frac{\mathcal{M}}{\sigma})[f(q) - \phi_m^+ p]. \end{aligned}$$

Similarly,

$$U^0(\hat{m}, \hat{a}, 0) = W^0(\hat{m}, \hat{a}, 0) + \frac{\mathcal{M}}{1-\sigma}[(\phi_a^+ + \gamma)\chi - \phi_m^+\psi].$$

Hence, I can rewrite,

$$\mathbb{E}U^i(\hat{m}, \hat{a}, 0) = \sigma W^1(\hat{m}, \hat{a}, 0) + (1-\sigma)W^0(\hat{m}, \hat{a}, 0) + \alpha_1\{\mathcal{M}[f(\grave{q}) - \phi_m^+\grave{p}] + (\sigma - \mathcal{M})[f(q) - \phi_m^+ p]\}. \tag{21}$$

In addition, from Lemma 2,

$$\begin{aligned} \phi_m^+ \grave{p} &= (1-\theta)f(\grave{q}) + \theta[g(\hat{k}) - g(\hat{k} - \grave{q})] && (22) \\ \phi_m^+ p &= (1-\theta)f(q) + \theta[g(\hat{k}) - g(\hat{k} - q)]. && (23) \end{aligned}$$

Since $\hat{m} = \tilde{m}$, $\psi(\hat{m}, \tilde{m}, \hat{a}) = \tilde{m}$, I have, $\grave{p} = \hat{m} + \tilde{m} = 2\hat{m}$, and $p = \hat{m}$. Then, using (22) and (23),

$$\frac{\partial \grave{q}}{\partial \hat{m}} = \frac{2\phi_m^+}{(1-\theta)f'(\grave{q}) + \theta g'(\hat{k} - \grave{q})} > 0 \tag{24}$$

$$\frac{\partial \grave{q}}{\partial \hat{k}} = \frac{\theta[g'(\hat{k} - \grave{q}) - g'(\hat{k})]}{(1-\theta)f'(\grave{q}) + \theta g'(\hat{k} - \grave{q})} > 0 \tag{25}$$

$$\frac{\partial q}{\partial \hat{m}} = \frac{\phi_m^+}{(1-\theta)f'(q) + \theta g'(\hat{k} - q)} > 0 \tag{26}$$

$$\frac{\partial q}{\partial \hat{k}} = \frac{\theta[g'(\hat{k} - q) - g'(\hat{k})]}{(1-\theta)f'(q) + \theta g'(\hat{k} - q)} > 0. \tag{27}$$

With $\partial \grave{q}/\partial \hat{m} > 0$, $\partial q/\partial \hat{m} > 0$, it means: the more cash holdings of type-1 entrepreneurs, the more capital reallocated $\grave{q}$ and q in KM. Also, with $\partial \grave{q}/\partial \hat{k} > 0$, $\partial q/\partial \hat{k} > 0$, it means: the more capital accumulation in the primary market from suppliers, the more capital reallocated in KM. Furthermore, using (22) and (23), I can rewrite (21) as,

$$\begin{aligned} \mathbb{E}U^i(\hat{m}, \hat{a}, 0) &= \sigma W^1(\hat{m}, \hat{a}, 0) + (1-\sigma)W^0(\hat{m}, \hat{a}, 0) + \alpha_1 \mathcal{M}\theta[f(\grave{q}) + g(\hat{k} - \grave{q}) - g(\hat{k})] \\ &\quad + \alpha_1(\sigma - \mathcal{M})\theta[f(q) + g(\hat{k} - q) - g(\hat{k})]. \end{aligned} \tag{28}$$

Using (28), (24) and (26), we can rewrite (6) as,

$$\frac{\iota}{\alpha_1} = \mathcal{M}\frac{2\theta[f'(\grave{q}) - g'(\hat{k} - \grave{q})]}{(1-\theta)f'(\grave{q}) + \theta g'(\hat{k} - \grave{q})} + (\sigma - \mathcal{M})\frac{\theta[f'(q) - g'(\hat{k} - q)]}{(1-\theta)f'(q) + \theta g'(\hat{k} - q)}. \tag{29}$$

The LHS of (29) is the marginal cost of spending one more units of money, ι, discounted by the matching probability in KM, while the RHS is the marginal benefit, shown by the expected gain of capital reallocation in KM. Furthermore, the expected gain is a weighted sum of gains in two cases: the first term in the RHS refers to, the gain for type-1 matched in AM, with the measure $\mathcal{M}$, getting extra liquidity to acquire $\grave{q}$ units of capital in KM, while the second term refers to, that for the rest type-1 not matched in AM, with the measure, $\sigma - \mathcal{M}$, only using the original cash holding to acquire q in KM. Obviously, $\grave{q} > q$, since $\partial \grave{q}/\partial \hat{m} > 0$, $\partial q/\partial \hat{m} > 0$. The more cash type-1 carry to KM, the more capital it can acquire from suppliers.

On the other hand, since $\hat{a}$, $\tilde{a} > \bar{a}(\hat{m}, \tilde{m})$, and $\chi(\hat{m}, \tilde{m}, \hat{a}) = \bar{a}(\hat{m}, \tilde{m})$, $\psi(\hat{m}, \tilde{m}, \hat{a}) = \tilde{m}$, we can see the third term in (21) is irrelevant to the choice of assets for entrepreneurs. Hence, we can rewrite (7) as,

$$1 + \iota = 1 + \iota_a, \tag{30}$$

where $1 + \iota_a \equiv (\phi_a^+ + \gamma)\phi_m/(\phi_a \phi_m^+) = (1+\pi)(\phi_a^+ + \gamma)/\phi_a$, and ι_a is the nominal rate of return for private assets. Since the asset constraint is not binding, the nominal return of assets just equals with the nominal interest rate, ι.

Lastly, I focus on the maximization problem of suppliers. Depending on the matching status of type-1 entrepreneurs in AM, they may acquire $\grave{q}$ or q from suppliers. Symmetrically, I can rewrite the value function of suppliers as,

$$\begin{aligned} V^s(0,0,\hat{k}) &= (1-\alpha_s)W^s(0,0,\hat{k}) + \alpha_s[\frac{\mathcal{M}}{\sigma}W^s(\grave{p},0,\hat{k}-\grave{q}) + (1-\frac{\mathcal{M}}{\sigma})W^s(p,0,\hat{k}-q)] \\ &= W^s(0,0,\hat{k}) + \alpha_s\{\frac{\mathcal{M}}{\sigma}[\phi_m^+\grave{p} + g(\hat{k}-\grave{q}) - g(\hat{k})] + (1-\frac{\mathcal{M}}{\sigma})[\phi_m^+ p + g(\hat{k}-q) - g(\hat{k})]\}. \end{aligned}$$

Hence, using the above results, $\grave{p} = 2\hat{m}$, $p = \hat{m}$ and (25), (27), I can rewrite (9) as,

$$r + \delta = (1-\alpha_s)g'(\hat{k}) + \alpha_s[\frac{\mathcal{M}}{\sigma}g'(\hat{k}-\grave{q})\Phi(\grave{q},\hat{k}) + (1-\frac{\mathcal{M}}{\sigma})g'(\hat{k}-q)\Phi(q,\hat{k})], \tag{31}$$

where

$$\Phi(\grave{q},\hat{k}) = \frac{[(1-\theta)f'(\grave{q}) + \theta g'(\hat{k})]}{(1-\theta)f'(\grave{q}) + \theta g'(\hat{k}-\grave{q})},$$

and $\Phi(q,\hat{k})$ has the same expression, except that q replaces $\grave{q}$ in $\Phi(\grave{q},\hat{k})$. Notice $\Phi(\grave{q},\hat{k}) < 1$, $\Phi(q,\hat{k}) < 1$ if $\theta > 0$. The LHS of (31) is the marginal cost of using one more unit of capital. And the RHS is a weighted sum of marginal product of capital: with probability $1-\alpha_s$, there is no matching for suppliers in KM, then they carry the same $\hat{k}$ to CM, with the marginal product of capital as $g'(\hat{k})$; with probability

α_s, suppliers match with type-1 entrepreneurs in KM, then they may carry $\hat{k} - \grave{q}$ or $\hat{k} - q$ to CM. That explains why the second term in the RHS is a weighted sum of marginal product of capital $g'(\hat{k} - \grave{q})$ and $g'(\hat{k} - q)$, but with two wedge terms $\Phi(\grave{q}, \hat{k})$, $\Phi(q, \hat{k})$. However, the wedge terms determines the whole second term, in fact, is less than $(\mathcal{M}/\sigma)g'(\hat{k} - \grave{q}) + (1 - \mathcal{M}/\sigma)g'(\hat{k} - q)$. This means, when $\theta > 0$, as sellers, suppliers do not get all of the surplus in the secondary market of capital (KM), which motivates them to invest more capital in the primary market (CM), and hence lowers the marginal value of capital. All of this captures the option value of investing in the primary market.

Case III: $\hat{m} < m^* - \tilde{m}$**, and** $\hat{a}$,$\tilde{a} < \bar{a}(\hat{m}, \tilde{m})$

In this region, type-1 does not carry enough assets to trade in AM. When matched in AM, he sells all of the assets but still can't get enough money to achieve the first best in KM. The bargaining solutions for AM are,

$$\begin{aligned} \chi(\hat{m}, \tilde{m}, \hat{a}) &= \hat{a} \\ \psi(\hat{m}, \tilde{m}, \hat{a}) &= \frac{(\phi_a^+ + \gamma)\hat{a} - \Delta[f(\grave{q}) - f(q)]}{\phi_m^+(1 - \Delta)}. \end{aligned}$$

As for KM, $\grave{p} = \hat{m} + \psi$, $p = \hat{m}$. Then the bargaining solution on $(\grave{q}, \grave{p})$ can be rewritten as,

$$\phi_m^+ \hat{m} + \frac{(\phi_a^+ + \gamma)\hat{a} - \Delta[f(\grave{q}) - f(q)]}{1 - \Delta} = (1 - \theta)f(\grave{q}) + \theta[g(\hat{k}) - g(\hat{k} - \grave{q})].$$

Hence,

$$\begin{aligned} \frac{\partial \grave{q}}{\partial \hat{m}} &= \frac{\phi_m^+}{[1/(1 - \Delta) - \theta]f'(\grave{q}) + \theta g'(\hat{k} - \grave{q})} \\ \frac{\partial \grave{q}}{\partial \hat{a}} &= \frac{\phi_a^+ + \gamma/(1 - \Delta)}{[1/(1 - \Delta) - \theta]f'(\grave{q}) + \theta g'(\hat{k} - \grave{q})} \\ \frac{\partial \grave{q}}{\partial \hat{k}} &= \frac{\theta[g'(\hat{k} - \grave{q}) - g'(\hat{k})]}{[1/(1 - \Delta) - \theta]f'(\grave{q}) + \theta g'(\hat{k} - \grave{q})}. \end{aligned}$$

The bargaining solutions on (q, p) remain the same as in Case II, so do $\partial q/\partial \hat{m}$, $\partial q/\partial \hat{k}$.

Similarly, as in Case II, I can rewrite (6), (7) and (9) as,

$$\frac{\iota}{\alpha_1} = \frac{\mathcal{M}\theta[f'(\grave{q}) - g'(\hat{k} - \grave{q})]}{[1/(1-\Delta) - \theta]f'(\grave{q}) + \theta g'(\hat{k} - \grave{q})} + \frac{(\sigma - \mathcal{M})\theta[f'(q) - g'(\hat{k} - q)]}{(1-\theta)f'(q) + \theta g'(\hat{k} - q)} \quad (32)$$

$$s = \frac{\alpha_1 \theta \mathcal{M}[f'(\grave{q}) - g'(\hat{k} - \grave{q})]}{[1 - \theta(1-\Delta)]f'(\grave{q}) + \theta(1-\Delta)g'(\hat{k} - \grave{q})} \quad (33)$$

$$r + \delta = g'(\hat{k}) + \alpha_s \Phi', \quad (34)$$

where

$$s = \frac{\iota - \iota_a}{1 + \iota_a}$$

$$\Phi' = \frac{\mathcal{M}}{\sigma}\frac{(1-\theta)f'(\grave{q})[g'(\hat{k} - \grave{q}) - g'(\hat{k})]/(1-\Delta)}{[1/(1-\Delta) - \theta]f'(\grave{q}) + \theta g'(\hat{k} - \grave{q})} + (1 - \frac{\mathcal{M}}{\sigma})\frac{(1-\theta)f'(q)[g'(\hat{k} - q) - g'(\hat{k})]}{(1-\theta)f'(q) + \theta g'(\hat{k} - q)}.$$

Notice s is the spread between the nominal returns of illiquid bonds and private assets a, and represents the cost of liquidity services provided by a.

Compared to Case II, the main difference is that assets a provide liquidity premium, as shown by (33). This is because type-1 entrepreneurs do not have enough assets to get all of the cash holdings of type-0, the asset constraint does bind, and assets provide liquidity premium.

5 Effects of Monetary Policy

Following the equilibrium analysis in Section 4, now I move on to analyze the effects of monetary policy, which includes two types of policy tools: one is the conventional tool, i.e., changing inflation rate; the other is the unconventional one, i.e., asset purchase by the central bank. At steady states, changing inflation rate, π, is equivalent of changing ι; asset purchase by the central bank is changing the asset supply, A. As explained in Section 2, since $\bar{A} = A + A_c$ is constant, changing A_c is equivalent of changing A: when a central bank purchases private assets, it decreases the private assets held by the public, so A decreases; alternatively, when a central bank sells private assets, it increase the assets held by the public, so A increases.

To be precise, based on Section 4.3, conventional policy matters in Case II and III, but the unconventional policy only matters in Case III. For simplify, I start from a special case: type-1 entrepreneurs make take-it-or-leave-it offers in KM, i.e., $\theta = 1$.

5.1 Conventional Policy: Changing ι

For Case II, $\hat{m} \in (m^* - \tilde{m}, m^*)$ and $\hat{a}$, $\tilde{a} > \bar{a}(\hat{m}, \tilde{m})$, hence, the cash constraint is binding, but not the asset constraint When $\theta = 1$, the equilibrium conditions are reduced to,

$$\frac{\iota}{\alpha_1} = 2\mathcal{M}\frac{f'(\grave{q}) - g'(\hat{k} - \grave{q})}{g'(\hat{k} - \grave{q})} + (\sigma - \mathcal{M})\frac{f'(q) - g'(\hat{k} - q)}{g'(\hat{k} - q)} \tag{35}$$

$$1 + \iota = 1 + \iota_a \tag{36}$$

$$r + \delta = g'(\hat{k}) \tag{37}$$

$$\hat{z} = g(\hat{k}) - g(\hat{k} - q) \tag{38}$$

$$2\hat{z} = g(\hat{k}) - g(\hat{k} - \grave{q}) \tag{39}$$

$$\chi = \frac{(1 - \Delta)\hat{z} + \Delta[f(\grave{q}) - f(q)]}{\phi_a^+ + \gamma} = \bar{a} \tag{40}$$

$$\phi_m^+ \psi = \hat{z}, \tag{41}$$

where $\hat{z} \equiv \phi_m^+ \hat{m}$, and $\phi_m^+ p = \phi_m^+ \hat{m} = \hat{z}$ for (38), $\phi_m^+ \grave{p} = \phi_m^+ (\hat{m} + \psi) = 2\hat{z}$ for (39). From (36), it is straightforward that $\iota_a = \iota$: since the asset constraint is not binding, the nominal return of private assets are the same as that of illiquid bonds. From (37), I can let $\hat{k} \equiv \bar{k}$, where $\bar{k}$ is constant, since the marginal product of capital is exactly equal to the real interest rate plus the depreciation rate of capital, due to the take-it-or-leave-it-offer setting. Once $\hat{k} = \bar{k}$ is derived, the equilibrium conditions are reduced to three equations, (35), (38), (39) for three variables $(q, \grave{q}, \hat{z})$. And χ can be derived by (40), once $(q, \grave{q}, \hat{z})$ are derived.

Hence, for the effects of changing ι, the comparative statics are,

$$\begin{aligned}
\frac{\partial \grave{q}}{\partial \iota} &= -\frac{2g'(\bar{k}-q)}{D_1} < 0 \\
\frac{\partial q}{\partial \iota} &= -\frac{g'(\bar{k}-\grave{q})}{D_1} < 0 \\
\frac{\partial \hat{z}}{\partial \iota} &= -\frac{g'(\bar{k}-q)g'(\bar{k}-\grave{q})}{D_1} < 0,
\end{aligned}$$

where $D_1 = -4\mathcal{M}\alpha_1 g'(\bar{k}-q)[f''(\grave{q})g'(\bar{k}-\grave{q}) + f'(\grave{q})g''(\bar{k}-\grave{q})]/g'^2(\bar{k}-\grave{q})$
$-(\sigma-\mathcal{M})\alpha_1 g'(\bar{k}-\grave{q})[f''(q)g'(\bar{k}-q)+f'(q)g''(\bar{k}-q)]/g'^2(\bar{k}-q) > 0$. Here I define the average trading quantities in KM as $q_{KM} \equiv (\mathcal{M}/\sigma)\grave{q} + (1-\mathcal{M}/\sigma)q$. Then,

$$\frac{\partial q_{KM}}{\partial \iota} = -(1+\frac{\mathcal{M}}{\sigma})\frac{g'(\bar{k}-q)}{D_1} < 0$$

The above results show higher inflation leads to lower capital reallocation in KM, i.e., lower $\grave{q}$, q, also q_{KM}, and lower real balance of money $\hat{z}$ as well. With the cash constraint being binding, these results quite make sense. Higher inflation has no impacts on r_a since the asset constraint is not binding, and assets are priced at the fundamental level. It also has no impact on $\hat{k}$ since $\hat{k}$ is constant due to the take-it-or-leave-it pricing protocol.

For Case III, $\hat{m} < m^* - \tilde{m}$, and $\hat{a}$,$\tilde{a} < \bar{a}(\hat{m}, \tilde{m})$, neither the cash or asset constraints are binding. When $\theta = 1$, the equilibrium conditions are reduced to,

$$
\begin{aligned}
\frac{\iota}{\alpha_1} &= \mathcal{M}\frac{f'(\grave{q}) - g'(\hat{k} - \grave{q})]}{\Delta f'(\grave{q})/(1-\Delta) + g'(\hat{k} - \grave{q})} + (\sigma - \mathcal{M})\frac{f'(q) - g'(\hat{k} - q)}{g'(\hat{k} - q)} &(42)\\
s &= \alpha_1 \mathcal{M}\frac{f'(\grave{q}) - g'(\hat{k} - \grave{q})}{\Delta f'(\grave{q}) + (1-\Delta) g'(\hat{k} - \grave{q})} &(43)\\
r + \delta &= g'(\hat{k}) &(44)\\
\hat{z} &= g(\hat{k}) - g(\hat{k} - q) &(45)\\
\hat{z} + \phi_m^+ \psi &= g(\hat{k}) - g(\hat{k} - \grave{q}) &(46)\\
\chi &= \hat{a} = A &(47)\\
\phi_m^+ \psi &= \frac{(\phi_a^+ + \gamma)\chi - \Delta[f(\grave{q}) - f(q)]}{(1-\Delta)} &(48)
\end{aligned}
$$

Again $\hat{k} \equiv \bar{k}$, since $g'(\hat{k}) = r + \delta$. And the system can be reduced to two equations for two variables, $(\grave{q}, q)$, i.e., (42) plus,

$$
(\phi_a^+ + \gamma)A = \Delta[f(\grave{q}) - f(q)] + (1-\Delta)[g(\bar{k} - q) - g(\bar{k} - \grave{q})], \tag{49}
$$

which uses (45), (46) and (48). Then, taking full derivation against (42) and (49),

$$
\begin{bmatrix} \alpha_1 \mathcal{M} \Phi_1 & \alpha_1(\sigma - \mathcal{M})\Phi_2 \\ \Delta f'(\grave{q}) + (1-\Delta)g'(\bar{k} - \grave{q}) & -\Delta f'(q) - (1-\Delta) g'(\bar{k} - q) \end{bmatrix} \begin{bmatrix} d\grave{q} \\ dq \end{bmatrix} = \begin{bmatrix} d\iota \\ (\phi_a^+ + \gamma) dA \end{bmatrix},
$$

where $\Phi_1 = (1-\Delta)[[f''(\grave{q})g'(\bar{k} - \grave{q}) + f'(\grave{q})g''(\bar{k} - \grave{q})]/[\Delta f'(\grave{q}) + (1-\Delta)g'(\bar{k} - \grave{q})]^2 < 0$, $\Phi_2 = [f''(q)g'(\bar{k} - q) + f'(q)g''(\bar{k} - q)]/g'^2(\bar{k} - q) < 0$.

Hence, for the effects of changing ι,

$$
\begin{aligned}
\frac{\partial \grave{q}}{\partial \iota} &= -\frac{\Delta f'(q) + (1-\Delta)g'(\bar{k}-q)}{D_2} < 0 \\
\frac{\partial q}{\partial \iota} &= -\frac{\Delta f'(\grave{q}) + (1-\Delta)g'(\bar{k}-\grave{q})}{D_2} < 0 \\
\frac{\partial q_{KM}}{\partial \iota} &= \frac{\mathcal{M}}{\sigma}\frac{\partial \grave{q}}{\partial \iota} + (1-\frac{\mathcal{M}}{\sigma})\frac{\partial q}{\partial \iota} < 0 \\
\frac{\partial \hat{z}}{\partial \iota} &= g'(\bar{k}-q)\frac{\partial q}{\partial \iota} < 0 \\
\frac{\partial s}{\partial \iota} &= \frac{\alpha_1 \mathcal{M}\Phi_1}{1-\Delta}\frac{\partial \grave{q}}{\partial \iota} > 0 \\
\frac{\partial \iota_a}{\partial \iota} &= \frac{1+s-(1+\iota)\partial s/\partial \iota}{(1+s)^2} \gtreqless 0,
\end{aligned}
$$

where $D_2 = -\alpha_1\mathcal{M}\Phi_1[\Delta f'(q) + (1-\Delta)g'(\bar{k}-q)] - \alpha_1(\sigma-\mathcal{M})\Phi_1[\Delta f'(\grave{q}) + (1-\Delta)g'(\bar{k}-\grave{q})] > 0$. The above results show that higher inflation leads to less capital reallocation in KM, i.e., lower q and $\grave{q}$, and entrepreneurs will choose to carry less real balance of money, either for AM or KM. Again higher inflation does not affect $\hat{k}$ since it is constant, due to the take-it-or-leave-it pricing protocol. Higher inflation also increases the spread between illiquid bonds and private assets, but its impact on the return of assets is ambiguous due to the mixture of Fisher effects and Mundell effects (Please refer to Rocheteau et al., 2018 for detailed discussion).

Now I summarize the effects of changing ι as in Proposition 1

Proposition 1 *Given type-1 entrepreneurs make take-it-or-leave-it offers in KM, (i) Case II, $\hat{m} < m^* - \tilde{m}$, and $\hat{a}$, $\tilde{a} > \bar{a}(\hat{m},\tilde{m})$, for the effects of changing ι,*

$$\frac{\partial \grave{q}}{\partial \iota} < 0, \frac{\partial q}{\partial \iota} < 0, \frac{\partial q_{KM}}{\partial \iota} < 0, \frac{\partial \hat{z}}{\partial \iota} < 0, \frac{\partial \hat{k}}{\partial \iota} = 0, \frac{\partial \iota_a}{\partial \iota} = 1.$$

(ii) Case III, $\hat{m} < m^* - \tilde{m}$, and $\hat{a}$,$\tilde{a} < \bar{a}(\hat{m},\tilde{m})$, for the effects of changing ι,

$$\frac{\partial \grave{q}}{\partial \iota} < 0, \frac{\partial q}{\partial \iota} < 0, \frac{\partial q_{KM}}{\partial \iota} < 0, \frac{\partial \hat{z}}{\partial \iota} < 0, \frac{\partial \hat{k}}{\partial \iota} = 0, \frac{\partial s}{\partial \iota} > 0, \frac{\partial \iota_a}{\partial \iota} \gtreqless 0.$$

Proposition 1 shows the real balances of cash holdings of entrepreneurs increases when the nominal interest rate ι decreases in both Case II and III; but the spread s decreases when ι decreases in Case III. If interpreting ι as the cost of carry cash, the former is consistent with the empirical results from Azar et al. (2016): changes in the cost of carry cash explain the increasing of corporate "cash" holdings in the U.S. during 1945 to 2013. If interpreting s as the cost of holding interest-bearing assets a, the latter indirectly explain why U.S. public firms also hold plenty of liquid assets.

5.2 Unconventional Policy: Changing A

As mentioned before, unconventional policy only matters in Case III of Section 4.3. Using the equilibrium conditions, (42) -(48), I can get the effects of changing A as follows,

$$\begin{aligned}
\frac{\partial \grave{q}}{\partial A} &= -\frac{\alpha_1 \Phi_2 (\sigma - \mathcal{M})(\phi_a^+ + \gamma)}{D_2} > 0 \\
\frac{\partial q}{\partial A} &= \frac{\alpha_1 \Phi_1 \mathcal{M}(\phi_a^+ + \gamma)}{D_2} < 0 \\
\frac{\partial q_{KM}}{\partial A} &= \frac{\alpha_1 \Phi_1 \Phi_3 \mathcal{M}(1 - \mathcal{M}/\sigma)(\phi_a^+ + \gamma)}{(1-\Delta) g'^2(\bar{k} - \grave{q}) \cdot D_2} > 0 \\
\frac{\partial \hat{z}}{\partial A} &= g'(\bar{k} - q)\frac{\partial q}{\partial A} < 0 \\
\frac{\partial s}{\partial A} &= \frac{\alpha_1 \mathcal{M} \Phi_1}{(1-\Delta)} \frac{\partial \grave{q}}{\partial A} < 0 \\
\frac{\partial \iota_a}{\partial A} &= -\frac{(1+\iota_a)^2}{1+\iota} \frac{\partial s}{\partial A} > 0.
\end{aligned}$$

where $\Phi_3 = \Delta(1-\Delta) g'(\bar{k} - \grave{q})[g'(\bar{k} - \grave{q}) - 2f'(\grave{q})] - \Delta^2 f'^2(\grave{q}) < 0$. Notice that $f'(\grave{q}) > g'(\hat{k} - \grave{q})$, from (43), given $s > 0$. Then $2f'(\grave{q}) > 2g'(\hat{k} - \grave{q}) > g'(\bar{k} - \grave{q}) \longrightarrow \Phi_3 < 0$.

As explained before, changing A represents unconventional policy, by which the central bank buys or sells private asset to change money supply. Since A represents private assets held by the public, decreasing A means asset purchase by the central bank to inject money. Alternatively, increasing A means the central bank sells private assets to withdraw money. This resembles the exit of unconventional policy, as what the Fed has done in the U.S. since 2015. Once this is clear, it is easy to interpret the above results: the asset constraint is binding in AM, so type-1 entrepreneur sell all of the assets he holds, but still cannot get enough liquidity to achieve the first best in KM. When the central bank purchases assets from the public (decreasing A), the asset constraint will be further binding for type-1 entrepreneurs, so that he will get less extra liquidity from AM, and, in the end, capital reallocation $\grave{q}$ will decrease. In contrast, when a type-1 entrepreneur does not match with type 0 in AM, he just use the original cash holdings to acquire capital in KM. Since decreasing A means increasing $\hat{z}$, type-1 has higher real balance of money, to acquire more in KM, i.e., q will increase. But the average capital reallocation, q_{KM}, still decreases when decreasing A. This may be due to the dominating effect of decreasing $\grave{q}$ (bigger than that of increasing q), through the asset market channel. To sum up, asset purchase by the central bank has opposite effects on $\grave{q}$ and q, but still decreases the average capital reallocation in KM, through the channel of asset market. Again, changing A has no impact on $\hat{k}$ since it is constant.

As for the effects on financial variables, decreasing A will increase the demand for private assets, hence the price of assets, ϕ_a, increases, then the nominal rate of return ι_a decreases, and the spread between illiquid bonds and private assets, s, will increase.

Now I summarize the effects of changing A as in Proposition 2:

Proposition 2 *Given type-1 entrepreneurs make take-it-or-leave-it offers in KM, unconventional policy only matters in Case III:* $\hat{m} < m^* - \tilde{m}$, *and* $\hat{a},\tilde{a} < \bar{a}(\hat{m}, \tilde{m})$*. That is, for the effects of changing* A*,*

$$\frac{\partial \grave{q}}{\partial A} > 0, \frac{\partial q}{\partial A} < 0, \frac{\partial q_{KM}}{\partial A} > 0, \frac{\partial \hat{z}}{\partial A} < 0, \frac{\partial \hat{k}}{\partial \iota} = 0, \frac{\partial s}{\partial A} < 0, \frac{\partial \iota_a}{\partial A} > 0.$$

It is obvious that, giving the T-I-O-L-I pricing protocol for KM, suppliers get zero surplus from capital reallocation. Hence, they have no motivation to supply capital more than the fundamental level, which means capital supply is fixed at the level that marginal product of capital is equal with $r + \delta$. If considering a more general bargaining protocol, e.g., Kalai bargaining, suppliers will get some surplus from the total surplus of capital reallocation in KM, so that they are willing to supply capital more than the fundamental level. The general equilibrium conditions will become more complicated (see the details in Section 4.3), but it may also delivery more interest results. This part is still under work in progress.

6 Quantitative Analysis

I plan to collect data on cash holdings, capital reallocation and investment of the U.S. firms, and do quantitative work to address: (i) increasing cash holdings of U.S. public corporations; (ii) general effects of monetary policy on asset return, capital accumulation and reallocation across firms.

7 Discussion and Conclusion

Appendix

Proof for Lemma 4: TBD

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