Expectations-Based Loss Aversion in Common-Value Auctions: Extensive vs. Intensive Risk*

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Abstract

We analyze the behavior of expectations-based loss-averse bidders in first-price and second-price common-value auctions. Highlighting the distinction between the uncertainty bidders face over whether they win the auction (extensive risk) and that over the value of the prize conditional on winning (intensive risk), we show that loss-averse bidders react differently to these different kinds of risk. In particular, the intensive risk pushes bidders to behave less aggressively in a common-value environment compared to one with private values. Yet, despite this "precautionary bidding" effect, in equilibrium bidders can be exposed to the "winner's curse". We consider two alternative specifications for how bidders assess outcomes as either gains or losses. Under narrow bracketing, bidders experience gains and losses separately over whether they receive the prize and how much they pay. Under broad bracketing, instead, bidders assess gains and losses over their net surplus. With narrow bracketing, first-price auctions expose bidders to less intensive risk and yield a higher expected revenue than second-price auctions, while the opposite result might hold with broad bracketing.

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1 Introduction

In many auctions the value of the good for sale is subject to ex-post risk and bidders will learn its true value only after the auction is over. A preeminent example of such auctions are so-called "common-value" auctions where bidders share the same value for the good up for sale, but at the time of the auction each bidder is only partially informed about this value. In particular, a bidder participating in a common-value auction is exposed to two different kinds of risk. First, there is the uncertainty regarding losing and winning the auction, and if so at what price. Second, uncertainty does not fully resolve for the winner because the value of the good is unknown and affected by the information held by the other bidders. Hence, common-value auctions are intrinsically risky for bidders. In such auctions, therefore, bidders' attitudes towards risk play a crucial role in determining their bidding strategies and in turn the auction's performance in terms of revenue.

In this paper, we analyze first-price (FPA) and second-price (SPA) common-value auctions with bidders who are expectations-based loss-averse à la Kőszegi and Rabin (2006, 2007). For each auction format, we derive the unique symmetric equilibrium. We also provide a thorough characterization of the impact of loss aversion on bidding, highlighting how the equilibrium strategy of loss-averse bidders differs from that of risk-neutral as well as from that of risk-averse bidders. Moreover, we show that revenue equivalence between the FPA and the SPA fails even if bidders have independent private signals about the good's common value.

Our paper is the first to study the role of loss aversion in common-value auctions where, differently from the case of private values, bidders are exposed to two sources of risk. First, when submitting his bid a bidder faces a lottery between losing and winning the auction; we call the risk associated with this lottery the "extensive risk". The extensive risk captures the strategic uncertainty as it stems from the uncertainty about whether a bidder submitted the highest bid. Moreover, a bidder is also exposed to uncertainty over the good's true value and, depending on the auction format, the price to pay; we call the risk associated with this lottery the "intensive risk". The intensive risk relates to the uncertainty to which a bidder is exposed even conditional on the fact that he submitted the largest bid. We show that loss-averse bidders react differently to these two sources of risk. In particular, equilibrium bids might be increasing or decreasing in extensive risk, but they are always decreasing in intensive risk.

When analyzing models of reference-dependent preferences, it is crucial to specify what are the dimensions of utility over which an individual experiences gains and/or losses. We consider two alternative specifications for how bidders evaluate outcomes relative to reference points. The first specification, *narrow bracketing of gains and losses*, posits that bidders feel gains and losses separately in each dimension of consumption utility so that, for instance, winning the auction and acquiring a good at a particular price entails a gain in the good dimension and a loss in the money dimension. This specification builds on the concept of *mental accounting* (Thaler 1985, 1999) and is consistent with the *endowment effect* observed in many laboratory trade experiments

(Kahneman *et al.* 1990, 1991). Moreover, several studies in finance argue that well-known puzzles like the *disposition effect* and the evidence that people are averse to small, independent gambles, even when actuarially favorable, can be explained by narrow bracketing/framing; see, for instance, Barberis and Huang (2001), Barberis *et al.* (2006) and Barberis and Xiong (2009). The second specification, *broad bracketing of gains and losses*, posits that bidders evaluate gains and losses over their net consumer surplus; i.e., the good's value minus its price. For example, this specification is appropriate for most experimental auctions in the lab. Moreover, it also applies to those real-world auctions where the goods for sale are sought after by bidders not for their consumption value, but rather for commercial purposes, e.g. a production or a resale motive. Both specifications are sensible from a theoretical perspective. For instance, auctions are a popular means for selling residential real estate in Australia, New Zealand and Singapore.[1] Both prospective first-home buyers as well as investors, who are interested in acquiring the property for sale in order to either rent it out or "flip" it, participate in these auctions. Even though both types of bidders can conceivably be loss averse, narrow bracketing is likely to apply to prospective first-home buyers who do not possess a home yet and are interested in using the property for housing; by contrast, broad bracketing seems more appropriate for investors who look at the property as an asset and are mainly interested in its returns.

The distinction between narrow and broad bracketing is also crucial to identify which auction format fetches a higher revenue for the seller. If bidders use symmetric strategies, all formats lead to the same allocation of the good and, hence, expose bidders to the same extensive risk. Thus, the format that induces less intensive risk will yield a higher revenue. In the SPA the winner pays the second-highest bid, whereas in the FPA he pays his own bid. Therefore, the winner's payment is deterministic in the FPA but stochastic in the SPA. This, in turn, implies that under narrow bracketing, where bidders evaluate uncertainty over the good's value and money separately, the FPA results in less intensive risk than the SPA. By contrast, under broad bracketing bidders focus on the uncertainty in their net surplus. Therefore, it may well be that the SPA is the less risky format in this case. Hence, under narrow bracketing the FPA yields a higher revenue than the SPA while the opposite ranking might hold under broad bracketing.

Section 2 introduces the auction environment, the bidders' preferences, and the solution concept. We focus on an environment with pure common values where bidders receive independent private signals as in Klemperer (1998)'s "Wallet Game". This formulation of the common value preserves revenue equivalence under risk neutrality; hence, any difference in the expected revenue between the two auction formats will be driven by the bidders' preferences and not by correlation in the bidders' signals or values. Following Kőszegi and Rabin (2006), we posit that in addition to classical *consumption utility*, a bidder also derives *gain-loss utility* from comparing his consumption outcomes to a reference point equal to his lagged expectations regarding the same outcomes,

[1] For empirical evidence on residential real estate auctions, see Mayer (1995, 1998), Dotzour *et al.* (1998) and Chow *et al.* (2015).

with losses being more painful than equal-sized gains are pleasant. The solution concept is Choice Acclimating Personal Equilibrium (*CPE)* as defined in Kőszegi and Rabin (2007). According to this solution concept, bidders are fully aware of how a change in their bids affects the distribution of outcomes and take this into account when forming their reference point so that, in equilibrium, the distribution of the reference point and the distribution of outcomes coincide.

Section 3 analyzes the FPA and the SPA with loss-averse bidders who bracket gains and losses narrowly. For each auction format we derive the unique symmetric equilibrium and characterize how the behavior of loss-averse bidders differs from that of risk-neutral bidders. In both formats, loss aversion has a "bifurcating" effect whereby bidders with high signals overbid relative to risk-neutral bidders playing a Bayesian Nash equilibrium, while those with low signals underbid. Hence, in equilibrium bidders with high signals might be exposed to the "winner's curse".[2] Moreover, we also identify a "precautionary bidding" effect — akin to the one identified by Eső and White (2004) for bidders displaying decreasing absolute risk aversion (DARA) — that pushes bidders to bid less aggressively in a common-value environment compared to a private-value one. The reason is that loss-averse bidders dislike the uncertainty over the good's value and, in turn, react by shading their bids. Yet, differently from the effect identified by Eső and White (2004), the precautionary bidding effect in our model is entirely driven by the intensive risk and, therefore, it vanishes as the number of bidders in the auction grows large. Intuitively, with many bidders the Law of Large Numbers applies. Hence, the average signal realization converges to its expected value so that there is no uncertainty regarding the good's common value and therefore the intensive risk disappears. Regarding revenue, we show that the FPA fetches a higher expected revenue than the SPA. The reason is that in the FPA, conditional on winning, a bidder knows what he will pay; i.e., his bid. In the SPA, instead, bidders are exposed to risk in their monetary outcomes even conditional on winning. As loss-averse bidders dislike the additional intensive risk with respect to their payment ingrained in the SPA, their expected bids are larger in the FPA than in the SPA.

Section 4 analyzes the FPA and the SPA with loss-averse bidders who bracket gains and losses broadly. We show that, as under narrow bracketing, the intensive risk induces a precautionary bidding effect that leads bidders to behave less aggressively compared to the case of private values. Yet, under broad-bracketing loss-averse bidders react differently to the extensive risk than under narrow bracketing. Indeed, the extensive risk creates an upward pressure on the bidding strategy of broad-bracketing bidders. This happens because the bidders evaluate gains and losses with respect to their overall net surplus rather than separately in each dimension; hence, the extensive risk induces a hedging motive which pushes bidders to be more aggressive than under narrow bracketing. However, differently from the case of narrow bracketing, winning the auction might be

[2] We say that a bidder is exposed to the "winner's curse" if he overbids compared to the risk-neutral (Bayesian) Nash Equilibrium. Some researchers, like Kagel and Levin (1986) and Eyster and Rabin (2005), use a more stringent definition: that the winning bidder obtains a negative payoff. Under this alternative definition, a loss-averse bidder is never exposed to the "winner's curse". Yet, we think our weaker definition corresponds more closely to the deviations from the risk-neutral (and risk-averse) equilibrium that are the main focus of our paper.

bad news ex-post for a loss-averse bidder who brackets broadly; i.e., the winning price is higher than the realized value of the prize. In this case, the hedging motive switches sign and bidders react by bidding less aggressively compared to a situation when winning is always good news. Moreover, this reduction in bids increases bidders' equilibrium utility. Indeed, whether winning is good news for a given bidder depends on his type, and the fact that winning can be bad news ex-post increases his deviation payoff when mimicking a bidder with a lower type, as winning is more often good news for him than for the type he mimics. Hence, bidders with high types now garner an additional information rent. In terms of revenue, we now find that the SPA might yield a higher expected revenue than the FPA. The reason is that, under broad bracketing, a loss-averse bidder is concerned only with the risk in his overall net surplus, rather than the risk in each separate utility dimension. Therefore, the SPA can give rise to fewer intensive risk and be a less risky format than the FPA. The reason is that the risk in the payment embedded in the SPA counterbalances the risk in the value of the prize, thereby reducing the intensive risk in the bidder's overall surplus conditional on winning.

In section 5 we discus the differences between narrow and broad bracketing, and compare the behavior of loss-averse bidders to that of bidders with preferences displaying constant absolute risk aversion (CARA) and decreasing absolute risk aversion (DARA). In both auction formats, a loss-averse bidder's expected payment is higher under broad bracketing than under narrow bracketing if winning the auction is always good news. Indeed, under broad bracketing bidders can hedge against the extensive risk, and this induces them to bid more aggressively. We also discuss how extensive and intensive risks shape the behavior of CARA bidders, and argue that their reaction to these two different kinds of risk is similar to that of broad-bracketing loss-averse bidders when winning is always good news. However, while CARA bidders always bid less than risk-neutral ones, loss-averse bidders instead might bid more aggressively than risk-neutral ones. Finally, if winning the auction can be bad news ex post, we show that the behavior of loss-averse bidders who bracket broadly resembles that of DARA bidders.

Section 6 concludes the paper by recapping its results and discussing possible avenues for future research. All proofs are relegated to Appendix A. The remainder of this section discusses the two strands of literature most closely related to our paper.

Common-Value Auctions with Risk-Averse Bidders As recognized already by Milgrom and Weber (1982), for models that include both risk aversion and common values the FPA and SPA cannot generally be ranked by their expected revenues. Hence, subsequent papers in this literature have restricted attention to specific cases. Eső and White (2004) identify a "precautionary bidding" effect whereby symmetric DARA bidders prefer bidding in a common-value setting to bidding in a private-value one; hence, the potential for a "winner's curse" can be a blessing for rational DARA bidders.[3] The intuition for this effect is that DARA bidders prefer a higher income when they

[3] Conducting experimental auctions in the laboratory, Kocher *et al.* (2015) find strong evidence for precautionary bidding. Moreover, the authors report that, although their study was inspired by Eső and White (2004), their results

win the auction and so they reduce their bids by more than the appropriate risk premium; in other words, DARA bidders are "prudent" (see Kimball, 1990 and Eeckhoudt *et al.*, 1996). Our precautionary bidding effect, instead, is driven by the intensive risk and has a first-order nature in the sense that it does not depend on the curvature of the utility function.[4] Moreover, while we also identify a precautionary bidding effect, in our model we have bidder-payoff equivalence between an environment with common values and one with private values (up to a scaling factor) under narrow bracketing as well as, if winning the auction never leads to a loss, broad bracketing. Thus, with loss-averse preferences increasing the risk regarding the good's value, by moving from a private-value environment to a common-value one, does not affect the bidders' equilibrium payoffs but reduces the seller's revenue. Menicucci (2004) analyzes first-price auctions with CARA bidders and shows that, differently from the private-value case, with common values risk aversion may reduce the seller's revenue compared to the risk-neutral benchmark. In contrast, we find that loss aversion might increase the seller's revenue compared to risk neutrality. Murto and Valimäki (2015) consider common-value auctions with a large number of risk-averse bidders and show that if bidders have CARA preferences, in the limit as the number of bidders increases towards infinity, the SPA yields a higher expected revenue than the FPA. This is line with our results under broad bracketing where the SPA might be the less risky format; under narrow bracketing, instead, we obtain the opposite result.

Expectations-Based Loss Aversion Next to expected utility theory (EUT), Kahneman and Tversky's (1979, 1991) Prospect Theory has arguably become the most prominent approach for modeling risk preferences. Together with probability weighting and diminishing sensitivity, the central building blocks of Prospect Theory are reference dependence and loss aversion. In a series of influential papers, Kőszegi and Rabin (2006, 2007, 2009; henceforth, KR) have developed a model of reference-dependent preferences and loss aversion where "gain–loss utility" is derived from standard "consumption utility" and the reference point is determined endogenously within the model by rational expectations. The KR model has found many fruitful applications in different areas of economics, including firms' pricing (Heidhues and Köszegi, 2008, 2014; Spiegler, 2012; Herweg and Mierendorff, 2013; Karle and Peitz, 2014; Rosato, 2016) and advertising strategies (Karle and Peitz, 2017; Karle and Schumacher, 2017), incentives' provision (Herweg *et al.*, 2010; Eliaz and Spiegler, 2015; Macera, 2018; Daido and Murooka, 2016), rank-order tournaments (Gill and Stone, 2010; Dato *et al.*, 2017a; Gül Mermer, 2017), asset pricing (Pagel, 2016), life-cycle consumption (Pagel, 2017), and bilateral negotiations (Rosato, 2017a; Benkert, 2017; Herweg *et al.*, 2018). In particular, there have been several theoretical contributions analyzing the role of expectations-based loss aversion in auctions. Lange and Ratan (2010) study FPA and SPA with

can also be explained by loss aversion.

[4]The difference between our precautionary bidding effect and the one identified by Eső and White (2004) for DARA bidders is similar to the difference between the expected-utility-of-wealth theories of precautionary savings that rely on prudence and the first-order precautionary-savings motive that induces loss-averse consumers to increase their savings in response to an in increase in background risk; see also Kőszegi and Rabin (2009) and Pagel (2017).

independent private values and show that the two formats are not revenue equivalent. Furthermore, they also show that the predictions of the KR model vary greatly depending on whether the auction analyzed is a real-object one or an induced-value one. This implies that transferring qualitative behavioral findings from induced-value laboratory experiments to the field may be problematic if subjects are expectations-based loss-averse. Eisenhuth (2018) shows that with expectations-based loss averse bidders who bracket narrowly and have independent private values, the all-pay auction yields the highest revenue among sealed-bid formats, while Ehrhart and Ott (2014) show that the Dutch auction yields a higher expected revenue than the English auction. von Wangenheim (2017) compares the English auction with the second-price one, showing that the latter yields a higher expected revenue. Rosato (2017b) studies sequential sealed-bid auctions of multiple objects and shows that expectations-based loss aversion can explain the afternoon effect — the puzzling yet robust empirical phenomenon whereby prices of identical goods tend to decline between rounds. Moreover, he also shows that sequential and simultaneous auctions are not revenue equivalent anymore. All these previous contributions, however, restrict attention to auctions where bidders have independent private values. Hence, our paper is the first one to study the role of expectations-based loss aversion in auctions with common values.

2 The Model

2.1 Environment

A seller auctions off an item to $N \geq 2$ bidders via a sealed-bid auction. Each bidder $i \in \{1, 2, ..., N\}$ observes a private signal t_i independently and identically distributed on the support $[\underline{t}, \overline{t}]$, with $\underline{t} \geq 0$ and $\overline{t} > \underline{t}$, according to the cumulative distribution function F. We assume that F is continuously differentiable, with positive density f on its support. The value of the object for sale is the same for all bidders and is given by $V = \sum_{i=1}^{N} t_i/N$. This structure is a re-scaled version of the "Wallet Game" (Klemperer, 1998; Bulow and Klemperer, 2002).[5] The normalization $1/N$ allows us to study how loss-averse bidders react to an arbitrary increase in the number of competitors. We consider two canonical selling mechanisms: the first-price auction (FPA) and the second-price auction (SPA). Both auctions have a zero reserve price.[6] We restrict attention to symmetric equilibria in increasing strategies.

[5] An advantage of this formulation is that it preserves revenue equivalence under risk neutrality. An alternative formulation is one where the common value V has some known prior distribution and signals are drawn conditional on a particular realization of V. We point out that both formulations have similar qualitative features. First, the object is worth the same to all bidders. Second, in both formulations bidders should realize that winning means that their signal is likely to be too optimistic; hence, to avoid the "winner's curse", bidders must shade their bids.

[6] See Rosenkranz and Schmitz (2007) for an analysis of reserve prices as reference points in auctions.

2.2 Bidders' Preferences

Bidders have reference-dependent preferences as formulated by Kőszegi and Rabin (2006). A bidder's utility function has two components. First, if he wins the auction at price p, a bidder experiences *consumption utility* $V - p$, which represents the classical notion of outcome-based utility. Second, the bidder also derives utility from comparing his actual consumption to a reference consumption outcome given by his recent expectations (probabilistic beliefs).[7] Hence, for a deterministic outcome (V, p) and deterministic reference point $\left(r^V, r^p\right)$, a bidder's total utility is

$$U\left[(V,p) \mid \left(r^V, r^p\right)\right] = V - p + \mu\left(V - r^V\right) + \mu\left(r^p - p\right), \tag{1}$$

where

$$\mu(x) = \begin{cases} \eta x & \text{if} \quad x \geq 0 \\ \eta\lambda x & \text{if} \quad x < 0 \end{cases}$$

is *gain-loss utility,* with $\eta > 0$ and $\lambda > 1$. The parameter η captures the relative weight a consumer attaches to gain-loss utility while λ is the coefficient of loss aversion. Importantly, the marginal utility from gains is constant and lower than the constant marginal disutility from losses. Thus, the employed formulation captures Prospect Theory's loss aversion, but without its diminishing sensitivity.[8] Moreover, according to (1), a bidder assesses gains and losses separately over each dimension of consumption utility. For instance, if his reference point is *not getting the good and paying nothing*, then he evaluates getting the good and paying for it as a gain in the good dimension and a loss in the money dimension rather than as a single gain or loss. This is consistent with much of the experimental evidence commonly interpreted in terms of loss aversion.[9]

In our setting, reference points are random variables. Hence, following Kőszegi and Rabin (2006), we allow for the reference point to be stochastic. Let H^V and H^p denote the reference points' distributions; then, a bidder's overall utility from the outcome (V, p) can be written as

$$U\left[(V,p) \mid \left(H^V, H^p\right)\right] = V - p + \int_{r^V} \mu\left(V - r^V\right) dH^V\left(r^V\right) + \int_{r^p} \mu\left(r^p - p\right) dH^p\left(r^p\right).$$

In words, for each utility dimension a bidder compares the realized outcome with each possible outcome in the reference lottery. Moreover, the weight on the loss (resp. gain) in the overall experience is equal to the probability with which he was expecting to win (resp. lose) the auction.

[7] Recent experimental evidence lends support to Kőszegi and Rabin's (2006) expectations-based model of reference-dependent preferences and loss aversion; see, for instance, Abeler *et al.* (2011), Ericson and Fuster (2011), Gill and Prowse (2012), Karle *et al.* (2015) and Sprenger (2015). More pertinently, Banerji and Gupta (2014), Eisenhuth and Grunewald (2018) and Rosato and Tymula (2018) provide experimental evidence supporting the KR model in the context of sealed-bid auctions with independent private values.

[8] Kőszegi and Rabin (2006) allow for gain-loss utility to be non-linear to capture diminishing sensitivity. For simplicity, we only focus on loss aversion.

[9] This feature can rationalize the endowment effect observed in many laboratory experiments (see Kahneman *et al.* 1990, 1991). The common explanation of the endowment effect is that owners perceive giving up an object as a painful loss that counts more than the money received in exchange.

For example, if he expected to win the auction and pay price $p \geq 0$ with probability $q \geq 0$ and to lose the auction and pay nothing with complementary probability, then winning the auction feels like a gain of $\eta V(1-q)$ in the item dimension and a loss of $\eta\lambda p(1-q)$ in the money dimension; similarly, losing the auction results in a loss of $\eta\lambda V q$ and a gain of $\eta q p$. Finally, notice that when submitting his bid, a bidder has only an estimate of the good's value which is based on his signal, and knows that if he wins the true value will be revealed only some time after the auction. Hence, as the value of the good is subject to ex-post risk, a loss-averse bidder is also exposed to feelings of loss and/or gain that stem from comparing the actual realized value of the good to all the possible values the good could have taken with positive probability.

2.3 Solution Concept

A bidder learns his signal (or type) before submitting a bid and, therefore, maximizes his interim expected utility. If the distribution of the reference points is $\mathbf{H} = \left(H^V, H^p\right)$ and the distribution of consumption outcomes is $\mathbf{G} = \left(G^V, G^p\right)$, a bidder's interim expected utility is

$$EU\left[\mathbf{G}|\mathbf{H}\right] = \int_{\{V,p\}} \int_{\{r^V, r^p\}} U\left[(V,p) \mid \left(r^V, r^p\right)\right] d\mathbf{H}\left(r^V, r^p\right) d\mathbf{G}(V,p).$$

In a sealed-bid auction setting, uncertainty is resolved *after* all bids are submitted. Thus, holding his opponents' strategies fixed, a bidder's strategy affects the distribution over the final consumption outcome. As pointed out by Kőszegi and Rabin (2007), it is natural to assume that the bidder is aware of this relation so that, in equilibrium, $\mathbf{G} = \mathbf{H}$. Hence, we employ Köszegi and Rabin (2007)'s Choice Acclimating Personal Equilibrium (*CPE)* as solution concept. Thus, bidders are forward looking, correctly anticipate how their actions affect the distribution over basic outcomes, and hold reference points that depend on their rational expectations both on and off the equilibrium path. A strategy for bidder i is a function $\beta_i : [\underline{t}, \overline{t}] \to \mathbb{R}_+$. Fixing all other bidders' strategies, $\boldsymbol{\beta}_{-i}$, the bid of bidder i with type t, $\beta_i(t)$, induces a distribution over the set of final consumption outcomes. Let $\Gamma\left(\beta_i(t), \boldsymbol{\beta}_{-i}\right)$ denote this distribution.

Definition 1. *A strategy profile $\boldsymbol{\beta}^*$ constitutes a Choice Acclimating Personal Equilibrium (CPE) if for all i and for all t:*

$$EU\left[\Gamma\left(\beta_i^*(t), \boldsymbol{\beta}_{-i}^*\right) | \Gamma\left(\beta_i^*(t), \boldsymbol{\beta}_{-i}^*\right)\right] \geq EU\left[\Gamma\left(b, \boldsymbol{\beta}_{-i}^*\right) | \Gamma\left(b, \boldsymbol{\beta}_{-i}^*\right)\right]$$

for any $b \in \mathbb{R}_+$.[10]

Let $\eta^m > 0$ and $\lambda^m > 1$ be the relative weight a consumer attaches to gain-loss utility and the coefficient of loss aversion for the money dimension, respectively. Similarly, let $\eta^g > 0$ and

[10] As shown my Dato *et al.* (2017b), focusing on equilibria in pure strategies is without loss of generality.

$\lambda^g > 1$ be the the relative weight a consumer attaches to gain-loss utility and the coefficient of loss aversion over the good dimension, respectively.[11] The following assumption, maintained for the remainder of the paper, guarantees that all bidders participate in the auction for any realization of their type, and that the equilibrium bidding functions derived in the next sections are strictly increasing:

Assumption 1. *(No dominance of gain-loss utility in the item dimension)* $\eta^g (\lambda^g - 1) \leq 1$.

For given λ^g, Assumption 1 places an upper bound on η^g (and vice versa).[12] This bound ensures that a bidder's expected utility is increasing in his type by imposing that the weight he puts on gain-loss utility does not (strictly) exceed the weight he puts on consumption utility.[13] Finally, notice that risk neutrality is embedded in the model as a special case (for either $\eta^g = \eta^m = 0$ or $\lambda^g = \lambda^m = 1$).

3 Narrow Bracketing

3.1 First-Price Auctions

We focus on symmetric pure-strategy equilibria which feature bidding functions that are increasing in bidders' types. To begin, we take the point of view of bidder i with type t_i, and consider the order statistics associated with the types of the other bidders. Let $t_1^{(N-1)} \equiv t_1$ be the highest of $N-1$ values, $t_2^{(N-1)} \equiv t_2$ be the second-highest and so on. Also, let F_1 and F_2 be the distributions of t_1 and t_2 respectively, with corresponding densities f_1 and f_2. We claim the existence of a symmetric equilibrium and then verify the claim. Consider bidder i with type $t_i = t$ who plans to bid as if his type were $\tilde{t}$ when all other $N-1$ bidders follow the posited equilibrium strategy $\beta_I^*(\cdot)$. This bidder faces a lottery $X_{t_i}^{\tilde{t}} = (V, p) \in \mathbb{R}^2$ which realizes as $(0,0)$ if $t_1 > \tilde{t}$, and as $\left(\frac{t+\sum_{i=1}^{N-1} t_i}{N}, \beta_I^*(\tilde{t})\right)$ otherwise. The support of the lottery therefore is $(0,0) \cup \left\{\left(x, \beta_I^*(\tilde{t})\right) | x \in \left[\left(t + (N-1)\,\underline{t}\right)/N, \left(t + (N-1)\,\tilde{t}\right)/N\right]\right\}$. Moreover, let $\tilde{F}(\cdot|\tilde{t})$ denote the

[11] We allow for different parameters of gain-loss utility and loss aversion in the good and money dimensions, because the two have different implications for bidding in auctions. In particular, our formulation is rich enough to capture situations where bidders are loss-averse only regarding the consumption dimension. Such case applies if bidders' income is subject to large background risk, as argued by Köszegi and Rabin (2009); in a similar vein, Novemsky and Kahneman (2005) argue that money given up in purchases is generally not subject to loss aversion.

[12] We relax this assumption Appendix B, where we show that most of our results continue to hold qualitatively. The main difference is that, if Assumption 1 does not hold, the equilibrium entails partial pooling at the bottom whereby some bidders bid zero in order to lose the auction for sure and avoid any risk.

[13] Herweg *et al.* (2010) first introduced Assumption 1 and referred to it as "no dominance of gain-loss utility". This assumption, which has been used also by Lange and Ratan (2010), Eisenhuth (2018) and Rosato (2017b), ensures that a loss-averse agent does not select first-order stochastically-dominated options (see also Masatlioglu and Raymond, 2016). Using data from first-price and all-pay auctions, Eisenhuth and Grunewald (2018) obtain an estimate for Λ of 0.42 (with a standard error of 0.16); similarly, using data from a BDM-like auction, Banerji and Gupta (2014) obtain an estimate for Λ of 0.283 (with a standard error of 0.08).

distribution of the sum of i's opponents' signals, $\sum_{j\neq i} t_j$, conditional on $t_1 \leq \tilde{t}$ and $\tilde{f}(\cdot|\tilde{t})$ denote its corresponding density. Then, the bidder's expected payoff is

$$\begin{aligned} EU\left(\tilde{t},t\right) &= F_1\left(\tilde{t}\right)\left[\frac{t+(N-1)\int_{\underline{t}}^{\tilde{t}} xf\left(x|\tilde{t}\right)dx}{N} - \beta_I^*\left(\tilde{t}\right)\right] \\ &+F_1\left(\tilde{t}\right)\left\{\eta^g\left[\frac{t+(N-1)\int_{\underline{t}}^{\tilde{t}} xf\left(x|\tilde{t}\right)dx}{N} - 0\right] + \eta^m\lambda^m\left[0-\beta_I^*\left(\tilde{t}\right)\right]\right\}\left[1-F_1\left(\tilde{t}\right)\right] \\ &+\left[1-F_1\left(\tilde{t}\right)\right]\left\{\eta^g\lambda^g\left[0-\frac{t+(N-1)\int_{\underline{t}}^{\tilde{t}} xf\left(x|\tilde{t}\right)dx}{N}\right] + \eta^m\left[\beta_I^*\left(\tilde{t}\right)-0\right]\right\}F_1\left(\tilde{t}\right) \\ &+F_1(\tilde{t})\left\{\eta^g\int_{\underline{t}(N-1)}^{\tilde{t}(N-1)}\int_{\underline{t}(N-1)}^{x}\left(\frac{x-y}{N}\right)\tilde{f}(y|\tilde{t})dy\tilde{f}(x|\tilde{t})dx + \eta^g\lambda^g\int_{\underline{t}(N-1)}^{\tilde{t}(N-1)}\int_{x}^{\tilde{t}(N-1)}\left(\frac{x-y}{N}\right)\tilde{f}(y|\tilde{t})dy\tilde{f}(x|\tilde{t})dx\right\}F_1(\tilde{t}) \quad (2) \end{aligned}$$

where $f\left(x|\tilde{t}\right) = f\left(x\right)/F\left(\tilde{t}\right)$, and $F_1\left(\tilde{t}\right) = F\left(\tilde{t}\right)^{N-1}$ denotes the probability that t_1, the highest signal among $N-1$, is less than $\tilde{t}$. The first term on the right-hand-side of (2) captures standard expected consumption utility. The other terms capture expected gain-loss utility and are derived as follows. A bidder of type t bidding as if his type were $\tilde{t}$ expects to win the auction with probability $F_1\left(\tilde{t}\right)$ and, conditional on winning, the expected value of the item is $\left[t+(N-1)\int_{\underline{t}}^{\tilde{t}} xf\left(x|\tilde{t}\right)dx\right]/N$ and the payment is $\beta_I^*\left(\tilde{t}\right)$. This outcome needs to be compared to the outcome of losing the auction, that is, not getting the good and paying nothing, which the bidder expects to happen with probability $\left[1-F_1\left(\tilde{t}\right)\right]$. Thus, winning the auction feels like a gain of $\eta^g\left\{\left[t+(N-1)\int_{\underline{t}}^{\tilde{t}} xf\left(x|\tilde{t}\right)dx\right]/N-0\right\}$ in the item dimension and like a loss of $\eta^m\lambda^m\left[0-\beta_I^*\left(\tilde{t}\right)\right]$ in the money dimension. Similarly, with probability $\left[1-F_1\left(\tilde{t}\right)\right]$ the bidder loses the auction in which case he gets nothing and pays nothing; thus, losing the auction entails a loss of $\eta^g\lambda^g\left\{0-\left[t+(N-1)\int_{\underline{t}}^{\tilde{t}} xf\left(x|\tilde{t}\right)dx\right]/N\right\}$ and a gain of $\eta^m\left[\beta_I^*\left(\tilde{t}\right)-0\right]$ compared to winning the auction, which the bidder expects to happen with probability $F_1\left(\tilde{t}\right)$. Finally, the last term on the right-hand-side of (2) captures the feelings of gain and loss in the good dimension when the bidder wins the auction and compares the realized value of the good to the other values it could have taken.[14] This last term arises from the fact that in common-value auctions a bidder's value is subject to ex-post risk which gives rise to additional feelings of gain and loss compared to the private-value case. The following lemma allows us to re-write expression (2) in a more compact form.

[14]Notice that as a bidder knows his own signal, the uncertainty is only with respect to the realizations of his competitors' signals.

Lemma 1. *Let $\Lambda^l := \eta^l \left(\lambda^l - 1\right)$ for $l \in \{g, m\}$ and define*

$$\begin{aligned}
\gamma(\tilde{t}) &:= \left(1-\Lambda^g\right) F_1\left(\tilde{t}\right) + \Lambda^g F_1\left(\tilde{t}\right)^2, \qquad q(\tilde{t}) := (N-1)\int_{\underline{t}}^{\tilde{t}} x f\left(x|\tilde{t}\right) dx, \\
\Omega(\tilde{t}) &:= F_1(\tilde{t})^2 \left[\int_{\underline{t}(N-1)}^{\tilde{t}(N-1)} \tilde{F}(x|\tilde{t}) dx - \int_{\underline{t}(N-1)}^{\tilde{t}(N-1)} \tilde{F}(x|\tilde{t})^2 dx\right], \\
\mathcal{T}_I(\tilde{t}) &:= F_1(\tilde{t})\beta_I^*(\tilde{t}) \left\{1 + \Lambda^m \left[1 - F_1(\tilde{t})\right]\right\}.
\end{aligned}$$

Then, $EU(\tilde{t}, t)$ as defined in (2) admits the following representation

$$EU(\tilde{t}, t) = \frac{\gamma(\tilde{t})}{N}\left[t + q(\tilde{t})\right] - \Lambda^g \frac{\Omega(\tilde{t})}{N} - \mathcal{T}_I(\tilde{t}). \tag{3}$$

Lemma 1 re-formulates the direct utility in a form that is suitable to apply the envelope theorem. Hence, in equilibrium $\tilde{t} = t$ maximizes (3) so that

$$\frac{dEU(t,t)}{dt} = \frac{\gamma(t)}{N} \Rightarrow EU(t,t) = \int_{\underline{t}}^{t} \frac{\gamma(s)}{N} ds + EU(\underline{t}, \underline{t}). \tag{4}$$

Moreover, $F_1(\underline{t}) = 0$ implies that $EU(\underline{t}, \underline{t}) = 0$. Combining (3) and (4) implies that a symmetric equilibrium bidding function satisfies

$$\frac{\gamma(t)\left[t + q(t)\right] - \Lambda^g \Omega(\tilde{t})}{N} - \mathcal{T}_I(t) = \int_{\underline{t}}^{t} \frac{\gamma(s)}{N} ds.$$

By re-arranging the above equation and solving for $\beta_I^*(t)$, we obtain the following proposition.

Proposition 1. *Suppose Assumption 1 holds. Then symmetric equilibrium strategies in a sealed-bid FPA are given by:*

$$\begin{aligned}
\beta_I^*(t) &= \frac{\int_{\underline{t}}^{t}\left[2s + \frac{(N-2)\int_{\underline{t}}^{s} x f(x) dx}{F(s)}\right] dF_1(s)}{N F_1(t)\left\{1 + \Lambda^m\left[1 - F_1(t)\right]\right\}} - \frac{\Lambda^g \int_{\underline{t}}^{t}\left\{2s + \frac{(N-2)\int_{\underline{t}}^{s} x f(x) dx}{F(s)} - F_1(s)\left[3s + \frac{(2N-3)\int_{\underline{t}}^{s} x f(x) dx}{F(s)}\right]\right\} dF_1(s)}{N F_1(t)\left\{1 + \Lambda^m\left[1 - F_1(t)\right]\right\}} \\
&\quad - \frac{\Lambda^g \Omega(t)}{N F_1(t)\left\{1 + \Lambda^m\left[1 - F_1(t)\right]\right\}}.
\end{aligned} \tag{5}$$

Condition (4) is only a necessary condition. In the proof of Proposition 1 in Appendix A we show that Assumption 1 is sufficient for (5) to constitute a symmetric equilibrium. Moreover, it is easy to verify that for $\Lambda^g = \Lambda^m = 0$, $\beta_I^*(t)$ reduces to the well-known risk-neutral bid.

To highlight the effect of the common-value risk on the behavior of a loss-averse bidder, it is useful to compare the bidding function in (5) with its private-value analogue in Lange and Ratan

(2010). With private values (i.e., $V_i = t_i$) the equilibrium strategy in the FPA is given by

$$b_I^*(t) = \frac{\int_{\underline{t}}^{t} s dF_1(s)}{F_1(t)\{1+\Lambda^m[1-F_1(t)]\}} - \frac{\Lambda^g \int_{\underline{t}}^{t} s[1-2F_1(s)]dF_1(s)}{F_1(t)\{1+\Lambda^m[1-F_1(t)]\}}. \tag{6}$$

The first terms on the right-hand-side of (5) and (6) are simply the risk-neutral bids re-scaled by $\{1+\Lambda^m[1-F_1(t)]\}$. The second term on the right-hand-side of (6) captures how the extensive risk affects a bidder's reference point. With private values, in a symmetric equilibrium a bidder with type t expects to win (resp. lose) the auction with probability $F_1(t)$ (resp. $1-F_1(t)$); hence, his expected gain-loss utility in the good dimension is given by $-\Lambda^g F_1(t)[1-F_1(t)]t$. Therefore, the second term in (6) captures how a bidder's deviation at the margin affects his reference point and hence his expected gain-loss utility. A similar intuition applies to the second term on the right-hand-side of (5). The only difference is that with common values a bidder's deviation from equilibrium affects not just his likelihood of winning but also the expected value of the good conditional on winning. Indeed, the integrand in the second term of (5) can be re-written as

$$\left[s + \frac{(N-1)\int_{\underline{t}}^{s} xf(x)dx}{F(s)}\right][1-2F_1(s)] + \left[s - \frac{\int_{\underline{t}}^{s} xf(x)dx}{F(s)}\right][1-F_1(s)]. \tag{7}$$

The first term in (7) is the common-value analogue of the second term on the right-hand-side of (6), but now the good's private value is replaced by the expectation of the good's common value (conditional on winning). Hence, as in the private-value case, this term captures how a change in the bid affects the likelihood of winning and losing the auction. The second term in (7), instead, captures how the estimate of the good's value changes if a bidder deviates from the symmetric equilibrium strategy. For example, if a bidder deviates by bidding as if his signal was lower than it actually is, he might win against rivals with lower signals and acquire a good with a lower expected value. Finally, compared to the private-value case, the bidding function in (5) contains an additional term. This last term arises because the bidder is unsure about the good's true value; hence, this term captures the impact of (ex-post) intensive risk on the bidder's equilibrium strategy. As a loss-averse bidder dislikes uncertainty in his consumption outcomes, the intensive risk creates a "precautionary bidding" effect that pushes bidders to behave less aggressively compared to the case of private values.

Next, we compare $\beta_I^*(t)$ to the risk-neutral bid $\beta_I^{RN}(t)$. We start with the following observation:

Observation 1. $\frac{\partial \beta_I^*(t)}{\partial \Lambda^m} \leq 0 \ \forall t$ *and the inequality is strict if* $t \in (0, \bar{t})$.

Intuitively, loss aversion over money lowers equilibrium bids compared to the risk-neutral benchmark, as loss-averse bidders dislike the extensive risk in monetary outcomes. Yet, the strategy of the bidder with the highest signal is not affected by loss aversion over money as in equilibrium he expects to win the auction and pay his bid with probability one. A similar argument applies to

the bidder with the lowest signal who expects to never win the auction and hence to never pay.

The effect of loss aversion in the good dimension is more intricate. We begin with an informal discussion, highlighting the different roles of extensive and intensive risk. Suppose first that $\Lambda^m = 0$, so that bidders are not loss-averse with respect to money. Then, re-arranging the first two terms in (5) yields a convex combination between the risk-neutral bid and a term that depends on the bidder's reference point: $(1-\Lambda^g)\int_{\underline{t}}^{t}\left[\frac{2s+(N-2)\int_{\underline{t}}^{s} xf(x|s)dx}{NF_1(t)}\right]dF_1(s)+\Lambda^g\int_{\underline{t}}^{t}F_1(s)\left[\frac{3s+(2N-3)\int_{\underline{t}}^{s} xf(x|s)dx}{NF_1(t)}\right]dF_1(s)$. In this convex combination, $(1-\Lambda^g)$ is the weight put on the risk-neutral bid. Hence, when Λ^g increases the weight on the second term becomes larger. Furthermore, notice that in this second term, the value of the good is scaled by $F_1(s)$ which, in equilibrium, represents the bidder's expectation of how likely he is to win the auction. Hence, this term relates to the extensive risk, whereby a bidder compares the expected consumption value from winning to that from not winning. For bidders with relatively high signals, the second term of the convex combination dominates the first term. Indeed, bidders with high signals expect to win with a high probability and are optimistic about the good's value; thus, they perceive losing the auction as a rather painful loss. Vice versa, bidders with low signals do not expect to win the auction and are pessimistic about the good's value; hence, they prefer to submit a rather low bid. Thus, the extensive risk in consumption leads loss-averse bidders with high signals to bid more aggressively compared to their risk-neutral counterparts, whereas the opposite holds for loss-averse bidders with low signals. Finally, notice that the larger Λ^g is, the larger is the weight on the third and last term in (5) which pushes all bidders to behave less aggressively compared to the risk-neutral benchmark. This additional effect, which is driven by the intensive risk in consumption, represents a form of precautionary bidding. Hence, the intensive risk pushes loss-averse bidders to reduce their bids compared to the risk-neutral benchmark, independently of their signal. The following proposition compares $\beta_I^*(t)$ to $\beta_I^{RN}(t)$ for any Λ^g and Λ^m.

Proposition 2. *Let t^m be such that $F_1(t^m) = 0.5$. Comparing $\beta_I^*(t)$ to $\beta_I^{RN}(t)$ we have:*

(i) If $t \leq t^m$, then $\beta_I^*(t) < \beta_I^{RN}(t)$.

(ii) There exists a $t' > t^m$ such that $\beta_I^*(t) \geq \beta_I^{RN}(t)\ \forall\ t \in [t', \bar{t}]$ if and only if

$$\int_{\underline{t}}^{t} F_1(x)\left[1-F_1(x)\right]dx - F_1(t)\left[1-F_1(t)\right]\left[t+q(t)\right]\Bigg|_{t=\bar{t}} \geq \underbrace{\int_{(N-1)\underline{t}}^{(N-1)\bar{t}} \widetilde{F}(x|\bar{t})\left[1-\widetilde{F}(x|\bar{t})\right]dx}_{\Omega(\bar{t})}. \quad (8)$$

Proposition 2 characterizes how the bidding behavior of loss-averse bidders differs from their risk-neutral counterparts. Condition (8) shows that whether a loss-averse bidder behaves more or less aggressively than a risk-neutral one depends on the relative magnitude of the extensive and intensive risk. In particular, while the intensive risk always pushes loss-averse bidders to behave

less aggressively compared to the risk-neutral benchmark, the effect of the extensive risk depends on the bidder's type. First, consider those bidders whose type is below t^m. These bidders have less than a 50% chance of winning the auction and always bid less than their risk-neutral counterparts. The intuition for this result is as follows. When comparing the outcome of winning (resp. losing) the auction to the counterfactual, a loss-averse bidder with type t experiences expected gain-loss disutility proportional to $-\Lambda^g F_1(t)[1-F_1(t)]$. Notice that $F_1(t)[1-F_1(t)]$ is maximized at $F_1(t)=0.5$, which is the point where the bidder faces the highest uncertainty between winning and losing the auction; that is, the point with the highest extensive risk. Bidders who expect to win with less than 50% probability do not feel attached to the good and therefore bid less aggressively to keep their expectations low and mitigate their disappointment if they lose. Hence, for these bidders both the intensive risk and the extensive risk have a negative effect on bids. In contrast, loss aversion induces bidders whose type is above t^m to increase their bids, as imitating a lower type would expose them to a fairly large extensive risk. Therefore, the extensive risk has positive effect on the strategy of those bidders who have more than a 50% chance of winning the auction. On the other hand, the intensive risk always has a negative effect on bids. Hence, the effect of the extensive risk must outweigh that of the intensive risk for these bidders to bid more aggressively than their risk-neutral counterparts. Consider, for instance, a type-$\bar{t}$ bidder. In equilibrium, this bidder expects to win with probability one. Hence, the only uncertainty this bidder faces is with respect to the value of the good; that is, with respect to the realization of his opponents' signals. The uncertainty over the good's value affects the bidding function via two separate channels. The first effect, which is positive, relates to how the expected value of the good affects the bidder's reference point: fixing the probability of winning, the higher the expected value of the good is, the higher the reference point and the more the bidder will tend to bid. The second effect, which is negative, relates to the bidder's dislike of intensive risk: the more uncertain the value of the good is, the lower the bid. When $N=2$, these two effects completely offset each other and condition (8) holds as an equality, as a change in the signal of the bidder's sole opponent affects the expected value of the good as well as the uncertainty over the good's value by exactly the same amount. Hence, when $N=2$, a loss-averse bidder with type $\bar{t}$ bids exactly the same as a risk-neutral one, whereas all other types bid less compared to the risk-neutral benchmark. For higher values of N it is not possible to say a priori which of the two aforementioned effects will dominate. However, condition (8) reveals that this depends on the comparison between the extensive and the intensive risk. If the condition holds as a strict inequality then a bidder with type $\bar{t}$ bids strictly more compared to the risk-neutral benchmark. In this case, therefore, there is a positive measure of types at the top of the types' distribution who bid more than in the risk-neutral benchmark. Thus, if the effect of the extensive risk outweighs that of the intensive risk, loss aversion has a "bifurcating"effect compared to the risk-neutral benchmark, inducing bidders with high signals to bid more aggressively and bidders with low signals to bid more conservatively.[15] Hence, bidders with high signals can be

[15] A similar result arises in Laohakunakorn *et al.* (2017) who consider common-value auctions where bidders have

exposed to the winner's curse in equilibrium. This prediction stands in contrast to the one made by Eyster and Rabin (2005)'s model of "cursedness" whereby bidders with low signals overbid while bidders with high signals underbid compared to the risk-neutral benchmark. Therefore, while "cursedness" predicts that those bidders who are more pessimistic about the item's value will overbid and be exposed to the winner's curse, loss aversion with expectations as the reference point yields the exact opposite prediction.

3.2 Second-Price Auctions

In this section we analyze the SPA. Consider bidder i with type $t_i = t$ who plans to bid as if his type were $\tilde{t}$ when all other $N-1$ bidders follow the posited symmetric equilibrium strategy β^*_{II}. His expected payoff is

$$EU\left(\tilde{t}, t\right) = \gamma(\tilde{t})/N\left[t + q(\tilde{t})\right] - \Lambda^g \Omega(\tilde{t})/N - \mathcal{T}_{II}(\tilde{t}), \tag{9}$$

with $\mathcal{T}_{II}(\tilde{t}) := \int_{\underline{t}}^{\tilde{t}} \beta^*_{II}(s) f_1(s) ds \left\{1 + \Lambda^m \left[1 - F_1(\tilde{t})\right]\right\} + \Lambda^m \int_{\underline{t}}^{\tilde{t}} \left(\int_{\underline{t}}^{x} \left(\beta^*_{II}(x) - \beta^*_{II}(v)\right) f_1(v)\, dv\right) f_1(x)\, dx$. Comparing (9) with (3), it is easy to see that the two expressions differ only in those terms related to the bidder's payment. Intuitively, as we are focusing on equilibria in increasing strategies, the two auction formats lead to the same allocation of the good. Yet, in the SPA a bidder faces uncertainty regarding his monetary payment when winning while this uncertainty is not present in the FPA. In particular, expression (9) contains an additional term, $-\Lambda^m \int_{\underline{t}}^{\tilde{t}} \left(\int_{\underline{t}}^{x} (\beta^*_{II}(x) - \beta^*_{II}(v)) f_1(v)\, dv\right) f_1(x)\, dx$. This term captures the expected gain-loss (dis)utility in the money dimension arising from the intensive risk in the payment that is ingrained in the SPA. The following proposition describes the symmetric equilibrium strategies for the SPA.

Proposition 3. *Suppose Assumption 1 holds. Then symmetric equilibrium strategies in the sealed-bid SPA are given by:*

$$\beta^*_{II}(t) = \frac{(\gamma(t)q(t))' + \gamma'(t)t - \Lambda^g \Omega'(t)}{N(1+\Lambda^m) f_1(t)} + \frac{2\Lambda^m \int_{\underline{t}}^{t} \left[(\gamma(v)q(v))' - \Lambda^g \Omega'(v) + \gamma'(v)v\right] e^{\frac{2\Lambda^m [F_1(t) - F_1(v)]}{1+\Lambda^m}} dv}{N(1+\Lambda^m)^2}. \tag{10}$$

Again, it is easy to verify that for $\Lambda^g = \Lambda^m = 0$, $\beta^*_I(t)$ reduces to the well-known risk-neutral bid. Moreover, it is insightful to compare the bidding function in (10) with its private-value analogue derived by Lange and Ratan (2010). With private values (i.e., $V_i = t_i$) the equilibrium strategies in the SPA are given by

$$b^*_{II}(t) = \frac{\gamma'(t)t}{(1+\Lambda^m) f_1(t)} + \frac{2\Lambda^m}{(1+\Lambda^m)^2} \int_{\underline{t}}^{t} \gamma'(v) v e^{\frac{2\Lambda^m [F_1(t) - F_1(v)]}{1+\Lambda^m}} dv. \tag{11}$$

ambiguity over the joint information structures generating their signals. Their model differs from ours as they consider an environment with affiliated and discrete signals, and restrict attention to auctions with two bidders.

Comparing expression (10) with expression (11), it is easy to see that the former differs from the latter only for the presence of $(\gamma(t)q(t))' - \Lambda^g \Omega'(t)$. Indeed, as these terms capture how bidder i's value depends on the signals of his competitors, they do not appear in the private-value case.

Next, we compare β^*_{II} to the risk-neutral bid, β^{RN}_{II}. Suppose first that $\Lambda^m = 0$, so that bidders are not loss-averse with respect to money. Then, $\beta^*_{II}(t)$ reduces to

$$\begin{aligned} \frac{(\gamma(t)q(t))' + \gamma'(t)t - \Lambda^g \Omega'(t)}{N f_1(t)} &= (1-\Lambda^g)\left[\frac{2t + (N-2)\int_{\underline{t}}^{t} x f(x|t)dx}{N}\right] \\ &\quad + \Lambda^g F_1(t)\left[\frac{3t + (2N-3)\int_{\underline{t}}^{t} x f(x|t)dx}{N}\right] - \Lambda^g \Omega'(t)/N. \end{aligned} \tag{12}$$

Notice that the first two terms on the right-hand-side of expression (12) represent a convex combination between the risk-neutral bid and a term that depends on the bidder's reference point with weights $(1-\Lambda^g)$ and Λ^g, respectively. The second term of the convex combination captures the extensive risk and dominates the first term for a bidder with a relatively high signal. Hence, similar to the FPA discussed in Section 3.1, the extensive risk pushes loss-averse bidders with high signals to bid more aggressively than their risk-neutral counterparts whereas the opposite holds for loss-averse bidders with low signals. Finally, notice that the larger Λ^g is, the larger is the weight on the disutility from the intensive risk which pushes bidders to behave less aggressively compared to the risk-neutral benchmark. This last effect is a precautionary bidding effect akin to that described in Section 3.1 for the FPA. The following proposition compares $\beta^*_{II}(t)$ to $\beta^{RN}_{II}(t)$ for any Λ^g and Λ^m.

Proposition 4. *Let t^m be such that $F_1(t^m) = 0.5$. Comparing $\beta^*_{II}(t)$ to $\beta^{RN}_{II}(t)$ we have:*

(i) If $t \leq t^m$, then $\beta^*_{II}(t) < \beta^{RN}_{II}(t)$ for any Λ^m .

(ii) There exists a $\hat{\Lambda}^m > 0$ and a t' such that if $\Lambda^m < \hat{\Lambda}^m$ then $\beta^*_{II}(t) \geq \beta^{RN}_{II}(t)$ $\forall t \in [t', \bar{t}]$ if and only if

$$\underbrace{\left(\int_{\underline{t}}^{t} F_1(x)\left[1 - F_1(x)\right]dx - F_1(t)\left[1 - F_1(t)\right]\left[t + q(t)\right]\right)'\Bigg|_{t=\bar{t}}}_{f_1(\bar{t})[\bar{t}+q(\bar{t})]} > \Omega'(\bar{t}). \tag{13}$$

Proposition 4 describes how the bidding behavior of loss-averse bidders differs from that of risk-neutral ones. As in the FPA, loss aversion has a bifurcating effect. First, loss-averse bidders who have less than a 50% chance of winning the auction bid less than their risk-neutral counterparts; this holds true irrespective of the strength of loss aversion over money. Second, when loss aversion over money is not too strong, those bidders with relatively high signals might overbid compared to the risk-neutral benchmark. This happens if and only if condition (13) is satisfied. Notice that condition (13) results from differentiating both sides of (8), the condition determining whether in

the FPA a loss-averse bidder overbids compared to the risk-neutral benchmark. In the FPA the sign of the difference in the bids is determined by the difference in the expected payments; in the SPA, instead, it is the derivative of the difference in expected payments that determines whether a loss-averse bidder overbids compared to the risk-neutral case. The reason why money loss aversion cannot be too strong for this effect to hold is that the SPA exposes bidders to intensive risk also in the money dimension. Hence, if loss aversion over money is strong enough, bidders will reduce their bid compared to the risk-neutral benchmark, irrespective of their signals.

3.3 FPA vs. SPA under Narrow Bracketing

We have seen that a loss-averse bidder reacts differently to extensive risk than intensive risk. In particular, the extensive risk creates an upward (resp. downward) pressure on the bid of a loss-averse bidder who expects to win the auction with probability larger (resp. smaller) than 0.5. In contrast, the intensive risk unambiguously creates a downward pressure on bids. In equilibrium, both auction formats lead to the same allocation of the good and thus expose the bidders to the same extensive risk. However, because of its payment rule, the SPA exposes bidders to more intensive risk. This difference in intensive risk implies that loss-averse bidders have a lower direct utility in the SPA than in the FPA. In equilibrium, bidders react by appropriately shading down their bids in the SPA and, as a result, enjoy the same indirect utility in both auction formats. The seller, however, bears the costs of the additional risk in the SPA. Hence, we have the following result.

Proposition 5. *In equilibrium, bidders attain the same utility in both auction formats. However, the expected payment of type-t bidder, for $t > \underline{t}$, is strictly larger in the FPA than that in the SPA if $\Lambda^m > 0$.*

The following corollary is an immediate consequence of Proposition 5.

Corollary 1. *Let $\Lambda^g > 0$. The expected revenue in the SPA is the same as in the FPA if $\Lambda^m = 0$ and it is strictly lower if $\Lambda^m > 0$.*

Corollary 1 extends Lange and Ratan (2010)'s revenue-ranking result under narrow bracketing for independent private-value auctions to the case of common values. The intuition for this result is that a bidder's equilibrium utility is the same in either auction format; that is, even though the two auctions are not revenue equivalent for the seller, they are payoff equivalent for the bidders. If $\Lambda^m = 0$, the difference in the expected payoffs between the two auction formats coincides with the difference between the expected payments and is zero. If $\Lambda^m > 0$, as bidders dislike the additional uncertainty that the SPA entails for their monetary outcomes, the expected payments are larger in the FPA than in the SPA for every bidder's type.

Recall that with risk-neutral bidders the revenue equivalence theorem applies to the common-value framework considered in this paper because signals are independent. Moreover, in Milgrom

and Weber (1982)'s general symmetric model with risk-neutral bidders and affiliated signals, the SPA yields a higher expected revenue than the FPA. Our analysis instead shows that if bidders are loss averse and bracket narrowly, revenue-maximizing sellers should always favor the FPA over the SPA.

4 Broad Bracketing

In this section we study auctions for monetary prizes. In this case, there is only one dimension of consumption utility, namely money; hence, a loss-averse bidder's gains and losses are defined with respect to the value of his net surplus, i.e. $V - p$. This broad-bracketing formulation is appropriate for those auctions where the items for sale are not sought after by the buyers for their consumption value, but rather for their commercial (or resale) value such as commercial licenses, mineral rights, production equipment, real estate for commercial (or investment) purposes, etc.[16]

4.1 First-Price Auctions

We begin by deriving the symmetric equilibrium of the FPA. Let $\Lambda^g = \Lambda^m = \Lambda \leq 1$. The expected payoff of a type-t bidder who plans to bid as if his type were $\widetilde{t}$ when all other $N-1$ bidders follow the posited equilibrium strategy β_I^{BB} is given by

$$\begin{aligned} EU\left(\widetilde{t}, t\right) &= F_1\left(\widetilde{t}\right)\left[\frac{t + q\left(\widetilde{t}\right)}{N} - \beta_I^{BB}\left(\widetilde{t}\right)\right] \\ &\quad -\Lambda F_1\left(\widetilde{t}\right)\left[1 - F_1\left(\widetilde{t}\right)\right]\int_{\underline{t}}^{\tilde{t}} \ldots \int_{\underline{t}}^{t_{N-2}} \left|\frac{t + \sum_{j\neq i} t_j}{N} - \beta_I^{BB}\left(\widetilde{t}\right)\right| f_{N-1}(t_{N-1}|t_{N-2})dt_{N-1}\ldots f_1(t_1|\tilde{t})dt_1 \\ &\quad -\Lambda\Omega_I(\widetilde{t})/N \end{aligned} \tag{14}$$

where $q\left(\widetilde{t}\right)$ and $\Omega_I\left(\widetilde{t}\right) = \Omega\left(\widetilde{t}\right)$ are defined as in Section 3.

Comparing (14) with its narrow-bracketing analogue (3) in Section 3.1, it is easy to see that the two expressions differ in the expected gain-loss (dis)utility term that captures the comparison between the feelings of gain and loss arising from winning the auction and those arising when losing the auction; that is, the extensive risk. Indeed, under narrow bracketing, winning the auction entails a gain in consumption and a loss in money compared to the outcome of losing the auction; similarly, losing the auction entails a loss in consumption but a gain in money compared to winning the auction. Therefore, bidders are simultaneously exposed to gains and losses in both utility dimensions. Under broad bracketing, however, there is only one relevant utility dimension.

[16] The broad-bracketing framework is also the most appropriate one for analyzing loss aversion in experimental auctions as the majority of laboratory experiments use an induced-value procedure whereby the auction's prize is a voucher that the subjects can redeem from the experimenter in exchange of money.

Hence, winning (resp. losing) the auction feels like a gain (resp. loss) if and only if the value of the prize exceeds the price. Thus, the extensive risk term under broad bracketing is different than that under narrow bracketing. By contrast, the intensive risk is the same under both narrow and broad bracketing.

When analyzing the impact of the extensive risk on the equilibrium bid under broad bracketing, it is crucial to distinguish whether winning the auction unambiguously leads to a gain compared to losing the auction. Let $\tilde{\beta}_I^{BB}$ be a candidate equilibrium bidding strategy that is strictly increasing. Then, winning the auction unambiguously leads to a gain if and only if $\forall t$ the following condition holds

$$N\tilde{\beta}_I^{BB}(t) \leq t + (N-1)\underline{t}\ . \tag{$*$}$$

When condition $(*)$ is satisfied, the realized value of the prize always exceeds the price paid by the winning bidder. In this case, winning the auction always leads to a feeling of gain whereas losing the auction always generates a feeling of loss. Next, define $\gamma^{BB}(\tilde{t}) := (1-\Lambda)F_1\left(\tilde{t}\right)+\Lambda F_1\left(\tilde{t}\right)^2$. If condition $(*)$ holds then, as in Section 3, we can re-write (14) in a more compact form:

$$EU\left(\tilde{t},t\right) = \gamma^{BB}\left(\tilde{t}\right)\left[\frac{t+q\left(\tilde{t}\right)}{N} - \tilde{\beta}_I^{BB}\left(\tilde{t}\right)\right] - \Lambda\Omega(\tilde{t})/N. \tag{15}$$

Taking the first-order condition yields

$$\left(\gamma^{BB}\left(\tilde{t}\right)\left[\frac{t+q\left(\tilde{t}\right)}{N}\right]\right)'|_{\tilde{t}=t} - \Lambda\Omega'(\tilde{t})/N|_{\tilde{t}=t} = \left(\gamma^{BB}\left(\tilde{t}\right)\tilde{\beta}_I^{BB}\left(\tilde{t}\right)\right)'|_{\tilde{t}=t}.$$

The solution to this differential equation then reads

$$\tilde{\beta}_I^{BB}(t) = \frac{t+q(t)}{N} - \frac{\int_{\underline{t}}^{t}\gamma^{BB}(s)ds + \Lambda\Omega(t)}{\gamma^{BB}(t)N}. \tag{16}$$

If $\tilde{\beta}_I^{BB}(t)$ satisfies condition $(*)$, then it is the equilibrium bid. Moreover, it is easy to verify that $\tilde{\beta}_I^{BB}(t)$ coincides with the risk-neutral bid when $N=2$, in which case condition $(*)$ is satisfied. However, if condition $(*)$ does not hold for all types, then winning the auction might be bad news ex-post for some types. Indeed, suppose that $N\tilde{\beta}_I^{BB}(t) > t + (N-1)\underline{t}$ for *some* bidder i with type t. Then, if this bidder behaves as if his type were $\tilde{t}$, for $\tilde{t}$ in a neighborhood of t, the term capturing the extensive risk in (14) can be rewritten as

$$\begin{aligned}
&\int_{\underline{t}}^{\tilde{t}}\cdots\int_{\underline{t}}^{t_{N-2}}\left|\frac{t+\sum_{j\neq i}t_j}{N} - \beta_I^{BB}\left(\tilde{t}\right)\right| f_{N-1}(t_{N-1}|t_{N-2})dt_{N-1}...f_1(t_1|\tilde{t})dt_1 \\
= &\int_{N\beta_I^{BB}(\tilde{t})-t}^{(N-1)\tilde{t}}\left[\frac{t+x}{N} - \beta_I^{BB}(\tilde{t})\right]\tilde{f}(x|\tilde{t})dx - \int_{(N-1)\underline{t}}^{N\beta_I^{BB}(\tilde{t})-t}\left[\frac{t+x}{N} - \beta_I^{BB}(\tilde{t})\right]\tilde{f}(x|\tilde{t})dx,
\end{aligned} \tag{17}$$

where $\tilde{f}(\cdot|\tilde{t})$ is the density of $\sum_{j\neq i} t_j$ conditional on $t_1 \leq \tilde{t}$. The first term on the LHS of (17) captures the cases where winning the auction leads to a gain, whereas the second term captures the cases where the bidder obtains a negative intrinsic payoff from winning so that winning the auction leads to a loss. Applying partial integration to (17) and substituting into (14) yields

$$EU(\tilde{t},t) = \gamma^{BB}(\tilde{t})\left[\frac{t+q(\tilde{t})}{N} - \beta_I^{BB}(\tilde{t})\right] - \Lambda\Omega(\tilde{t}) - 2\Lambda F_1(\tilde{t})\left[1-F_1(\tilde{t})\right]\left[\int_{(N-1)\underline{t}}^{N\beta_I^{BB}(\tilde{t})-t} \frac{\tilde{F}(x|\tilde{t})}{N}dx\right].$$

In the expression above, the first two terms represent the direct utility if winning the auction were always good news, whereas the last term captures the disutility from the possibility of being disappointed ex post from winning. We now derive a unified representation for the direct utility which is applicable independently of whether condition $(*)$ holds. Define the increasing and differentiable function $S : \mathbb{R} \mapsto \mathbb{R}$ as follows: $S(x) = \int_{(N-1)\underline{t}}^{x} \frac{\tilde{F}(s|\tilde{t})}{N}ds$ if $x \geq (N-1)\underline{t}$, and $S(x) = 0$ otherwise. Now consider the following modified expression for the direct utility

$$EU(\tilde{t},t) = \gamma^{BB}(\tilde{t})\left[\frac{t+q(\tilde{t})}{N} - \beta_I^{BB}(\tilde{t})\right] - \Lambda\Omega(\tilde{t}) - 2\Lambda F_1(\tilde{t})(1-F_1(\tilde{t}))S\left(N\beta_I^{BB}(\tilde{t}) - t\right). \tag{18}$$

The first-order condition of (18) defines a differential equation. In the appendix we show that this equation has an increasing solution. This solution is the equilibrium bid, β_I^{BB}. If $N\tilde{\beta}_I^{BB}(t) < t + (N-1)\underline{t}$, then $S(N\beta_I^{BB}(t) - t) = 0$ and winning is always good news for a type-t bidder. Otherwise, this bidder might be disappointed from winning. As a result, he will shade his bid below $\tilde{\beta}_I^{BB}$, and so will do all bidders with a higher type than his. By a small abuse of notation, let $EU_{BB}(t)$ be type-t bidder's indirect equilibrium utility under broad bracketing and $EU_{NB}(t)$ that under narrow bracketing. Then, we have the following result.

Proposition 6. *A unique equilibrium in strictly increasing strategies exists, and has the following properties:*

1. $\beta_I^{BB}(t) = \tilde{\beta}_I^{BB}(t)$ *if and only if* $N\tilde{\beta}_I^{BB}(t') \leq t' + (N-1)\underline{t}$ *for all* $t' \leq t$.
2. $\beta_I^{BB}(t) < \tilde{\beta}_I^{BB}(t)$ *for all* $t > t'$ *if and only if* $N\tilde{\beta}_I^{BB}(t') > t' + (N-1)\underline{t}$ *for some* t'.
3. $EU_{BB}(t) > EU_{NB}(t)$ *for all* $t > t'$ *if and only if* $N\tilde{\beta}_I^{BB}(t') > t' + (N-1)\underline{t}$ *for some* t'.

The first part of Proposition 6 states the equilibrium bidding strategy when winning the auction always leads to a feeling of gain. We can re-write $\tilde{\beta}_I^{BB}(t)$ as follows

$$\tilde{\beta}_I^{BB}(t) = \frac{t + (N-1)\int_{\underline{t}}^{t} xf(x|t)\,dx}{N} - \frac{\int_{\underline{t}}^{t} F_1(s)\left\{1-\Lambda\left[1-F_1(s)\right]\right\}ds}{N\left[(1-\Lambda)F_1(t) + \Lambda F_1(t)^2\right]} - \frac{\Lambda\Omega_I(t)}{N\left[(1-\Lambda)F_1(t) + \Lambda F_1(t)^2\right]}.$$

It is easy to verify that for $\Lambda = 0$, β_I^{BB} reduces to the risk-neutral bid. Moreover, it is also easy to see that the first two terms in $\tilde{\beta}_I^{BB}(t)$ resemble the private-value bidding strategy in Lange and

Ratan (2010) — with the bidder's conditional estimate of the good's common value in place of the bidder's private value. With common values, however, the bidding function has an additional term capturing how bidders shade their bids to account for the intensive risk regarding the good's value. Hence, also under broad bracketing a loss-averse bidder is less aggressive in a common-value auction than in a private-value auction. The second part of Proposition 6 points out that a type-t bidder reduces his bid if and only if for some type below him winning the auction can be bad news ex post. Notice that this is true independently of whether winning the auction with the bid $\tilde{\beta}_I^{BB}(t)$ is unambiguously good news. As long as there is some type below type t, say t', for whom bidding $\tilde{\beta}_I^{BB}(t')$ leads to a possible loss from winning compared to losing, type t decreases his bid. Indeed, note first that type t' decreases his bid because he values winning less due to the possibility of making an ex-post loss. The reason why type t also reduces his bid can be best explained in combination with the last part of Proposition 6. This part states that type t, $t > t'$, achieves a higher utility under broad bracketing than under narrow bracketing. Indeed, by the envelope theorem we have that the indirect utility satisfies the following condition

$$EU_{BB}(t) = \int_{\underline{t}}^{t} \frac{\partial EU(s,s)}{\partial t} ds = \int_{\underline{t}}^{t} \left[\gamma^{BB}(s)/N + 2\Lambda F_1(s)\left[1 - F_1(s)\right] S'(N\beta_I^{BB}(s) - s)\right] ds. \quad (19)$$

Thus, under broad bracketing loss-averse bidders receive an additional information rent and therefore enjoy a higher equilibrium utility than under narrow bracketing. The intuition is that by shading his bid below $\tilde{\beta}_I^{BB}(t)$ and imitating a type t', a type-t bidder can reduce the probability that he will be disappointed from winning the auction. That is, imitating a lower type does not only reduces a bidder's expected payment but it also makes it more likely that he will experience a gain when winning the auction. Thus, having a high type is intrinsically more valuable under broad bracketing than under narrow bracketing.

In the remainder of this section, we restrict attention to cases where winning the auction is always good news. This happens if N is small and/or low types are relatively likely. In such cases, a bidder expects that the good's common value is rather low and thus bids relatively little; hence, he is never disappointed when comparing winning to losing.[17] The next proposition compares the equilibrium strategy of loss averse bidders who bracket broadly with that of risk-neutral bidders.

Proposition 7. *Assume condition* $(*)$ *holds* $\forall t$. *Then,* $\beta_I^{BB}(t) \geq \beta_I^{RN}(t)$ *if and only if*

$$\int_{\underline{t}}^{t} F_1(x|t)\left[1 - F_1(x|t)\right] dx \geq \underbrace{\int_{(N-1)\underline{t}}^{(N-1)t} \widetilde{F}(x|t)\left[1 - \widetilde{F}(x|t)\right] dx}_{\Omega_I(t)/F_1(t)^2}. \quad (20)$$

With private values, Lange and Ratan (2010) showed that broad-bracketing loss-averse bidders

[17] This holds, for example, when types are distributed on $[0,1]$ according to $F(t) = t^a$, with a being sufficiently small, independently of N.

always bid more than risk-neutral ones. Yet, condition (20) reveals that this result does not necessarily extend to the case of common values, even when winning is always good news. The reason is that loss-averse bidders react differently to intensive and extensive risk. Indeed, the left-hand-side of (20) captures the effect of the extensive risk on the equilibrium bid whereas the right-hand-side captures the effect of the intensive risk. Under broad bracketing, the extensive risk pushes every type of a bidder to bid more aggressively than in the risk-neutral case. Yet, as under narrow bracketing, the intensive risk pushes bids down. Hence, whether a loss-averse bidder behaves more or less aggressively than a risk-neutral one depends on which effect dominates. Moreover, notice the difference between the result in Proposition 7 and that in part (ii) of Proposition 2. Under narrow bracketing, the fact that the extensive risk dominates the intensive risk leads to a more aggressive bidding behavior only for (some of) those types above the median; under broad bracketing, instead, all types bid more aggressively (compared to risk neutrality) when the extensive risk dominates the intensive risk. This stems from the difference in when bidders experience gains and losses. Consider a bidder whose type is below the median. Under narrow bracketing, imitating a bidder with a lower type leads to less extensive risk than bidding according to his true type. Hence, pretending to have a lower type is relatively more attractive than under risk neutrality. Therefore, a bidder who expects to win the auction with less than 50% probability bids less compared to the risk-neutral benchmark. Under broad bracketing instead, when condition $(*)$ holds, even a bidder who expects to win the auction with a fairly small probability overbids compared to the risk-neutral benchmark. This happens because a bidder can reduce the expected loss from not winning by increasing his bid. That is, instead of mimicking a lower type to reduce the extensive risk, under bracketing the bidder hedges to reduce the loss from not winning. Therefore, under broad bracketing of gains and losses all bidders can be exposed to the winner's curse.

4.2 Second-Price Auctions

We now turn to the analysis of the SPA. As before, let $\Lambda^g = \Lambda^m = \Lambda \leq 1$ and consider bidder i with type $t_i = t$ who plans to bid as if his type were $\tilde{t}$ when all other $N-1$ bidders follow the posited equilibrium strategy β_{II}^{BB}. His expected payoff is

$$\begin{aligned} EU\left(\tilde{t}, t\right) &= F_1\left(\tilde{t}\right)\left[\frac{t+q\left(\tilde{t}\right)}{N}\right] - \int_{\underline{t}}^{\tilde{t}} \beta_{II}^{BB}(t_1) f_1(t_1) dt_1 \\ &\quad -\Lambda F_1\left(\tilde{t}\right)\left[1-F_1\left(\tilde{t}\right)\right]\int_{\underline{t}}^{\tilde{t}} ... \int_{\underline{t}}^{t_{N-2}} \left|\frac{t+\sum_{j\neq i} t_j}{N} - \beta_{II}^{BB}\left(t_1\right)\right| f_{N-1}(t_{N-1}|t_{N-2}) dt_N ... f_1(t_1|\tilde{t}) dt_1 \\ &\quad -\Lambda \Omega_{II}(\tilde{t})/N \end{aligned} \tag{21}$$

where

$$\Omega_{II}(\tilde{t}) = F_1(\tilde{t})^2/2 \int_{\underline{t}}^{\tilde{t}} \int_{\underline{t}}^{t_1} ... \int_{\underline{t}}^{t_{N-2}} \int_{\underline{t}}^{\tilde{t}} ... \int_{\underline{t}}^{y_1} \int_{\underline{t}}^{y_{N-2}} \left| \sum_{j \neq i} (t_j - y_j) - N \left[\beta_{II}^{BB}(t_1) - \beta_{II}^{BB}(y_1) \right] \right| \times$$
$$f_{N-1}(t_{N-1}|t_{N-2}) dt_N ... f_1(t_1|\tilde{t}_1) dt_1 f_{N-1}(y_{N-1}|y_{N-2}) dy_N ... f_1(y_1|\tilde{t}) dy_1.$$

Notice that, differently from the SPA under narrow bracketing, the intensive risk is now defined with respect to the overall net surplus. Moreover, differently from the FPA under broad bracketing, the net surplus depends on the bid of i's toughest competitor, $\tilde{\beta}_{II}^{BB}(t_1)$, which is a random variable. Hence, whether the intensive risk in the SPA is larger or smaller than that in the FPA now depends on the sign of $N\tilde{\beta}_{II}^{BB}(t_1) - t_1$. That is, it depends on whether the uncertainty in the price dominates the uncertainty in the prize's value.

We begin by focusing on a two-bidder auction and show that in this case, like for the FPA, winning is always good news; i.e., winning (resp. losing) the auction always leads to a feeling of gain (resp. loss). Let us reformulate the term capturing intensive risk as follows:

$$\Omega_{II}(\tilde{t}) = \underbrace{-\int_{\underline{t}}^{\tilde{t}} \int_{\underline{t}}^{t_1} (t_1 - y_1) f_1(y_1) dy_1 f_1(t_1) dt_1}_{=\Omega_I} + \int_{\underline{t}}^{\tilde{t}} \int_{\underline{t}}^{t_1} \left[\beta_{II}^{BB}(t_1) - \beta_{II}^{BB}(y_1) \right] f_1(y_1) dy_1 f_1(t_1) dt_1. \tag{22}$$

This formulation highlights how the risk regarding the value of the prize is counterbalanced by that regarding the payment. This pushes the bidders to bid more aggressively compared to the narrow-bracketing case. By substituting (22) into the expression for the direct utility and evaluating the first-order condition at $\tilde{t} = t$, we arrive at the following result:

Proposition 8. *Suppose $N = 2$. Then winning is always good news and the equilibrium bidding function in the SPA takes the form $\beta_{II}^{BB}(t) = t$.*

Hence, when $N = 2$, loss-averse bidders who bracket broadly behave the same as risk-neutral bidders. As in the FPA, the extensive risk creates a hedging motive inducing bidders to increase their bids; by contrast, the intensive risk creates a downward pressure on the bids. In the equilibrium described in Proposition 8 these two effects exactly off-set each other as for $N = 2$ the upward pressure on the equilibrium bid induced by the extensive risk has the same magnitude as the downward pressure created by the intensive risk. Moreover, notice that for $N = 2$ the intensive risk, which is conditional on winning, stems from the randomness in the difference between the type of the bidder's competitor, $t_1/2$, and the winning payment $\beta_{II}^{BB}(t_1)$. Thus, the intensive risk is generated by the randomness of the variable $t_1/2 - \beta_{II}^{BB}(t_1) = -t_1/2$. This random variable, however, is the mirror image of the random variable that generates the intensive risk in the FPA, $t_1/2$, where the uncertainty is solely about the value of the good conditional on winning. Thus, if $N = 2$ the FPA and the SPA lead to the same intensive risk. Therefore, as summarized in the next proposition, revenue equivalence holds in this situation.

Proposition 9. *If $N = 2$ the FPA is revenue equivalent to the SPA.*

Next, we turn to the general case and account for the possibility that winning the auction can sometimes lead to a loss. The analysis shares strong similarities with that of the FPA. Let $\tilde{\beta}_{II}^{BB}$ be the equilibrium bid in an auxiliary model where we ignore the fact that winning might be bad news.[18] We have the following result.

Proposition 10. *A unique equilibrium in strictly increasing strategies exists, and has the following properties:*

1. $\beta_{II}^{BB}(t) = \tilde{\beta}_{II}^{BB}(t)$ *if and only if for all* $t_1 \leq t' \leq t$ *we have* $N\tilde{\beta}_{II}^{BB}(t_1) \leq t' + t_1 + (N-1)\underline{t}$.
2. $\beta_{II}^{BB}(t) < \tilde{\beta}_{II}^{BB}(t)$ *for all* $t > t'$ *if and only if there exist a* $t_1 \leq t'$ *such that* $N\tilde{\beta}_{II}^{BB}(t_1) > t' + t_1 + (N-1)\underline{t}$.
3. $EU_{BB}(t) > EU_{NB}(t)$ *for all* $t > t'$ *if and only if there exist a* $t_1 \leq t'$ *such that* $N\tilde{\beta}_{II}^{BB}(t_1) > t' + t_1 + (N-1)\underline{t}$.

Hence, as for the FPA, if winning can be bad news ex post, loss-averse bidders who bracket broadly will reduce their bids and achieve a higher equilibrium utility than under narrow bracketing. In the remainder of this section, we separately analyze the extensive and intensive risk and highlight how they affect the strategies in Proposition 10. Without loss of generality, we focus on the case where $N \geq 3$ and begin by deriving a tractable expression for the intensive risk. For a given bidder i, fix two realizations of his toughest competitor's signal, t_1, and y_1 with $t_1 \geq y_1$ and define $\tilde{D}_2 := y_1 - \beta_{II}^{BB}(y_1) - t_1 + \beta_{II}^{BB}(t_1)$. For given y_1 and t_1, whenever the random variable $D_2 \equiv \sum_{i\geq 2}(t_i - y_i)/N$ falls short of this threshold, a bidder experiences a loss when winning. Suppose that $y_1 - \beta_{II}^{BB}(y_1) - t_1 + \beta_{II}^{BB}(t_1) > \sum_{i\geq 2}(t_i - y_i)/N$ for some y_1 and t_1. Then the risk in the payment dominates that in the prize's value and the intensive risk has the following representation.

Lemma 2. *Suppose $N \geq 3$. If $y_1 - \beta(y_1) - t_1 + \beta(t_1) > \sum_{i\geq 2}(t_i - y_i)/N$ for some y_1 and t_1, then the intensive risk reads*

$$\Omega_{II}(\tilde{t}) = \int_{\underline{t}}^{\tilde{t}} \int_{\underline{t}}^{\tilde{t}} \left[2\Phi_2(\tilde{D}_2|t_1, y_1) - 1\right] \left[\beta_{II}^{BB}(t_1) - \beta_{II}^{BB}(y_1)\right] f_1(y_1)dy_1 f_1(t_1)dt_1 + \hat{\Omega}(\tilde{t}), \tag{23}$$

where $\Phi_2(\cdot|t_1, y_1)$ is the CDF of D_2 conditional on the realizations t_1 and y_1, and $\hat{\Omega}(\tilde{t})$ is some function of $\Omega_I(t)$.

It is easy to see that the intensive risk in the SPA under broad bracketing differs from that of the SPA under narrow bracketing as well as from that of the FPA under broad bracketing.

[18] Differently from the FPA, a closed-form solution for the bidding function is not available in the SPA when bidders bracket broadly.

The first term in (23) captures the uncertainty regarding the payment; that is, the uncertainty regarding the signal of bidder i's toughest competitor. This uncertainty is weighted by a measure of how likely it is that the uncertainty in the winning payment dominates that in the prize's value, $\left[2\Phi_2(\tilde{D}_2|t_1, y_1) - 1\right]$, since the more likely it is that D_2 falls short of the threshold $\tilde{D}_2$, the more likely it is for the bidder to experience a loss when facing an unexpectedly tough competitor. The second term in (23), $\hat{\Omega}(\tilde{t})$, instead captures the uncertainty in the prize's value conditional on winning.

Next, we focus on the extensive risk. Below we state a general formulation which unifies the case when winning is unambiguously good news and the case when winning might be bad news for some realizations of a bidder's toughest competitor's signal.

Lemma 3. *There exists a non-negative and differentiable function $\tilde{S} : \mathbb{R}^2 \mapsto \mathbb{R}^+$ such that*

$$\begin{aligned} & \int_{\underline{t}}^{\tilde{t}} ... \int_{\underline{t}}^{t_{N-2}} \left| \frac{t + \sum_{j \neq i} t_j}{N} - \beta_{II}^{BB}(t_1) \right| f_{N-1}(t_{N-1}|t_{N-2}) dt_{N-1} ... f_1(t_1|\tilde{t}) dt_1 \\ = \; & t + q(\tilde{t}) - \left[1 - F_1(\tilde{t})\right] \int_{\underline{t}}^{\tilde{t}} \beta_{II}^{BB}(t_1) f_1(t_1) dt_1 + 2 \int_{\underline{t}}^{\tilde{t}} \frac{\tilde{S}(\beta_{II}^{BB}(t_1) - t_1 - t, t_1)}{N} f_1(t_1) dt_1. \quad (24) \end{aligned}$$

Notice that the expression for the extensive risk in the SPA, (24), is similar to that in the FPA. That is, (24) unifies the direct utility for those cases when winning is always good news and those when sometimes winning is bad news. Substituting both (24) and (23) into the direct utility leads to the following representation

$$\begin{aligned} EU(\tilde{t}, t) \; = \; & \gamma^{BB}(\tilde{t}) \left[t + q(\tilde{t})\right] / N + \Lambda \tilde{\Omega}(\tilde{t}) / N \\ & -2\Lambda F_1(\tilde{t}) \left[1 - F_1(\tilde{t})\right] \int_{\underline{t}}^{\tilde{t}} \frac{\tilde{S}(\beta_{II}^{BB}(t_1) - t_1 - t, t_1)}{N} f_1(t_1) dt_1 - \frac{\gamma^{BB}(\tilde{t})}{F_1(\tilde{t})} \int_{\underline{t}}^{\tilde{t}} \beta_{II}^{BB}(x) f_1(x) dx \\ & - \int_{\underline{t}}^{\tilde{t}} \int_{\underline{t}}^{t_1} \left[2\Phi_2(\tilde{D}_2|t_1, y_1) - 1\right] \left[\beta_{II}^{BB}(t_1) - \beta_{II}^{BB}(y_1)\right] f_1(y_1) dy_1 f_1(t_1) dt_1. \quad (25) \end{aligned}$$

As for the FPA, whenever winning might be bad news — that is whenever $\tilde{S} \neq 0$ — loss-averse bidders who bracket broadly experience an additional disutility that leads them to bid less aggressively compared to the case where winning is always good news. Moreover, like in the FPA, this reduction in bids leads to a larger indirect equilibrium utility compared to the case where winning is always goods news and compared to the case of narrow bracketing. To formally see this, in Appendix A we evaluate the first-order condition of the direct utility at $t = \tilde{t}$, and show that the corresponding differential equation has a unique solution. Once established the existence of an equilibrium, the envelope theorem reveals that the indirect equilibrium utility reads

$$EU_{BB}(t) = \int_{\underline{t}}^{t} \left\{ \gamma^{BB}(s)/N - 2\Lambda F_1(s) \left[1 - F_1(s)\right] \int_{\underline{t}}^{s} \tilde{S}_1(N\beta_{II}^{BB}(t_1) - t_1 - t, t_1) f_1(t_1) dt_1 \right\} ds, \quad (26)$$

where $\tilde{S}_1(N\beta_{II}^{BB}(t_1) - t_1 - t, t_1) = \tilde{F}_2(N\beta_{II}^{BB}(t_1) - t_1 - t|t_1)/N$ if $N\beta_{II}^{BB}(t_1) - t_1 - t > (N-2)\underline{t}$ and zero otherwise. Thus, type t enjoys an additional information rent if and only if there is a lower type t' for whom winning is not unambiguously good news. That is, there is a type $t' \leq t$ such that $N\beta_{II}^{BB}(t_1) - t_1 - t' > (N-2)\underline{t}$ for some type $t_1 \leq t'$. The intuition for this result is that, similar to the FPA, whenever winning might be bad news for a type t', this type will reduce his bid. Higher types like t, in turn, receive a larger information rent because by mimicking type t', they not only reduce their expected payments but are also less likely than type t' to suffer a loss when winning with bid $\beta_{II}^{BB}(t')$.

4.3 FPA vs. SPA under Broad Bracketing

In the SPA, the winning price and the good's value conditional on winning are positively correlated. Thus, under narrow bracketing the intensive risk is lower in the FPA than in the SPA, and, as a result, the FPA yields a higher revenue than the SPA. The next proposition shows that this is not necessarily the case under broad bracketing.

Proposition 11. *Suppose winning is always good news. The auction format that gives rise to less intensive risk fetches a higher revenue. Moreover, we have payoff equivalence between the FPA and the SPA.*

Proposition 11 states that revenue equivalence between FPA and SPA fails when bidders are loss averse and bracket broadly. In particular, while under narrow bracketing the FPA always yields a higher expected revenue than the SPA, the opposite might happen under broad bracketing. Indeed, under narrow bracketing the SPA is a riskier format for loss-averse bidders who, compared to the FPA, are exposed to additional uncertainty with respect to their payment, even conditional on winning the auction. Yet, under broad bracketing the SPA can sometimes be a less risky format than the FPA because the uncertainty over the payment embedded in the SPA counterbalances the uncertainty over the value of the prize and reduces the uncertainty over the bidder's net surplus conditional on winning. Hence, under broad bracketing the SPA can expose bidders to less intensive risk than the FPA. In either case, as long as winning is unambiguously good news, the seller bears the entire cost of the additional intensive risk and both auction formats are payoff equivalent for the bidders. Finally notice that the result that under broad bracketing the SPA might yield a higher expected revenue than the FPA is a direct by-product of the common-value framework and stands in stark contrast with the result obtained by Lange and Ratan (2010) according to which, in a private-value environment, the FPA always yields a higher expected revenue than the SPA under narrow as well as under broad bracketing.

5 Loss Aversion vs. Risk Aversion

Bidders in common-value auctions are exposed to two kinds of risk: *extensive risk* and *intensive risk*. We showed that both under narrow bracketing as well as under broad bracketing the intensive risk always induces loss-averse bidders to shade their bids down. However, our analysis also revealed some important differences between how loss-averse bidders behave under narrow bracketing and under broad bracketing. These differences are related to (i) whether winning the auction can lead to experiencing a loss ex post and (ii) how bidders are affected by the extensive risk. Below we elaborate on both of these points and compare the behavior of loss-averse bidders with that of risk-averse bidders.

With respect to point (i), recall that under narrow bracketing winning the auction always leads to a gain in the consumption dimension. Under broad bracketing, however, winning the auction might sometimes generate a feeling of loss for the winner. In this case, in both the FPA and the SPA, bidders shade their bids and obtain a larger equilibrium utility than under narrow bracketing. Thus, the behavior of loss-averse bidders in this case resembles that of DARA bidders who enjoy a similar effect as shown by Eső and White (2004). This effect arises because, under DARA, the degree of risk aversion decreases in a bidder's wealth, which intrinsically depends on the bidder's privately known type. Thus, the bidder gains an additional information rent. By contrast, when winning is always good news, loss-averse bidders who bracket broadly are similar to CARA bidders in that they obtain the same payoff in the SPA and FPA; moreover, for such bidders common-value auctions are payoff equivalent to private-value auctions (up to a scaling factor).

Next, let us turn to point (ii) and focus on the case when, under broad bracketing, winning the auction is always good news. In this case, the extensive risk is the same under narrow bracketing and under broad bracketing. However, how bidders are affected by this risk depends on whether they bracket broadly or narrowly. Under broad bracketing, every bidder's type increases his bid to reduce the magnitude of potential losses. By contrast, under narrow bracketing winning the auction entails a gain in the good dimension and a loss in the monetary dimension. Therefore, hedging is not possible. Instead, bidder types increase their bids if and only if mimicking a bidder with a lower type leads to a larger extensive risk.

Finally, we compare the behavior of loss-averse bidders who bracket broadly (and when winning is always good news) with that of CARA bidders. With CARA preferences, bidders are exposed to the same extensive risk in the FPA and the SPA. Thus, also under CARA it is the intensive risk that determines the performance of a given selling mechanism. Indeed, the next proposition mirrors Proposition 11.

Proposition 12. *With CARA bidders the auction format that induces the smaller variance in the bidders' payoff conditional on winning raises the highest revenue. Moreover, CARA bidders behave less aggressively than risk-neutral bidders.*

The first part of Proposition 12 highlights that also for CARA bidders it is the intensive risk

that determines the performance of a selling mechanism. Indeed, both auction formats implement the same allocation regarding the good and thereby generate the same extensive risk. Yet, under CARA preferences the SPA might entail a smaller intensive risk than the FPA. In this case, the winner's expected payment is larger in the SPA than in the FPA and a revenue-maximizing auctioneer facing CARA bidders will prefer the SPA over the FPA. The same result might also arise with loss-averse bidders under broad bracketing (and when winning is always good news), while the opposite result holds under narrow bracketing.

Even though the ranking in terms of revenue between the SPA and the FPA under CARA is the same as under broad-bracketing loss aversion, notice that the implications for bidders' behavior are different. Indeed, the second part of Proposition 12 states that in both auction formats CARA bidders bid less than risk-neutral bidders. In contrast, as shown in the previous section, loss-averse bidders who bracket broadly might bid more than risk-neutral bidders. Therefore, while under CARA the negative effect of the intensive risk dominates the positive effect of the extensive risk, pushing bidders to bid less aggressively and thereby lowering the seller's expected revenue compared to the risk-neutral benchmark, the opposite might happen when bidders are loss averse.

6 Conclusion

Ample evidence, gathered from both the field and the lab, indicates that people evaluate outcomes not (only) in absolute terms but (also) relative to a reference point, and that losses (relative to this reference point) loom larger than equal-size gains; see, for instance, Kahneman *et al.* (1990) on the endowment effect in laboratory trade experiments, Odean (1998) and Genesove and Mayer (2001) on the disposition effect in the stock and housing market respectively, and Crawford and Meng (2011) on cabdrivers' labor supply decisions. In particular, as shown by Lange and Ratan (2010), Banerj and Gupta (2014), Eisenhuth (2018), Rosato (2017b), Rosato and Tymula (2018) and von Wangenheim (2017), expectations-based loss aversion has important implications for auction design.

While previous contributions have focused solely on auctions with private values, our paper is the first to study the role of expectations-based loss aversion in common-value auctions. We have provided a full characterization of the behavior of loss-averse bidders in first-price and second-price common-value auctions, contrasting it with the behavior of risk-neutral bidders as well as that of risk-averse bidders. Our analysis highlights how the behavior of loss-averse bidders and the performance of different auction formats depend on how bidders react to the extensive and the intensive risk, and on whether they bracket narrowly or broadly. Moreover, we have shown that with loss-averse bidders revenue equivalence fails even if bidders have independent signals about the common value; indeed, under narrow bracketing the FPA always yields a higher revenue than the SPA whereas the opposite might hold under broad bracketing. In particular, we find that the performance of a selling mechanism crucially depends on the relative magnitude of intensive risk

and extensive risk as the former generates a precautionary bidding effect that pushes bidders to be less aggressive whereas the latter can induce bidders to be more aggressive. These findings have important implications for the design of optimal selling mechanisms. For instance, consider a private-value setting where bidders are only partially informed about the value of the good, e.g. $V_i = t_i + X$, where X is a stochastic component that can be realized either before or after the selling mechanism is implemented. Our results imply that a revenue-maximizing seller prefers bidders to learn the realization of X before the mechanism takes place, as in this way the intensive risk disappears. More generally, the extensive risk stems from strategic uncertainty — that is, the uncertainty with respect to the behavior of a bidder's opponents; by contrast, the intensive risk is conditional on winning and independent of the behavior of a bidder's opponents. Our results imply that sellers should favor mechanisms that give rise to as little non-strategic uncertainty as possible. The case of strategic uncertainty is more intricate. If bidders bracket broadly, the performance of a selling mechanism increases in the extensive risk, while the opposite holds true under narrow bracketing. Indeed, the extensive risk can be directly influenced through the design of the selling mechanisms. For example, buyers are exposed to more extensive risk in a sealed-bid auction than in an Dutch auction or in bilateral negotiations.

There are several directions left for future research. One that strikes us as particularly interesting is to analyze common-value English auctions with loss-averse bidders using the dynamic model of reference-dependent preferences introduced in Kőszegi and Rabin (2009). von Wangenheim (2017) uses this model to analyze English auctions with private values and shows that loss-averse bidders bid less aggressively in the English auction than in the second-price auction. The intuition is that, as the English auction unfolds, a loss-averse bidder becomes more pessimistic about his chances of eventually winning the auction — because the fact that the auction is not over yet implies that his opponents' values are relatively high — and this pushes him to bid less aggressively. However, in a common-value environment there is an additional effect going in the opposite direction because as the auction unfolds a bidder becomes more optimistic with respect to the prize's value. Without a thorough investigation of loss aversion in common-value English auctions, it is not possible to say which of these two effects dominates.

Another interesting direction for future research would be to investigate the relationship between expectations-based loss aversion and the winner's curse. As we have shown, the winner's curse can be a by-product of equilibrium bidding for expectations-based loss-averse bidders. In particular, under narrow bracketing, bidders with high signals overbid relative to the risk-neutral Bayesian Nash equilibrium, while those with low signals underbid. Hence, expectations-based loss aversion makes the opposite prediction of "cursedness" à la Eyster and Rabin (2005). Furthermore, as shown by Crawford and Iriberri (2007), "level-k" thinking can also provide a non-equilibrium explanation for the winner's curse in common-value auctions, often yielding similar predictions as cursedness. Hence, future theoretical and experimental work could attempt to further tease out and test the different implications of these models.

A Proofs

Proof of Lemma 1: Comparing (2) and (3), the terms $\gamma\left(\tilde{t}\right)$, $q\left(\tilde{t}\right)$ and $\mathcal{T}_I(\tilde{t})$ are readily identified. To identify the term $\Omega\left(\tilde{t}\right)$ in (2), observe that

$$\begin{aligned}
& \eta^g \int_{\underline{t}(N-1)}^{\tilde{t}(N-1)} \int_{\underline{t}(N-1)}^{x} \left(\frac{x-y}{N}\right) \tilde{f}(y|\tilde{t})dy\tilde{f}(x|\tilde{t})dx + \eta^g\lambda^g \int_{\underline{t}(N-1)}^{\tilde{t}(N-1)} \int_{x}^{\tilde{t}(N-1)} \left(\frac{x-y}{N}\right) \tilde{f}(y|\tilde{t})dy\tilde{f}(x|\tilde{t})dx \\
= & -\underbrace{\eta^g\left(\lambda^g-1\right)}_{\equiv\Lambda^g} \int_{\underline{t}(N-1)}^{\tilde{t}(N-1)} \int_{\underline{t}(N-1)}^{x} \left(\frac{x-y}{N}\right) \tilde{f}(y|\tilde{t})dy\tilde{f}(x|\tilde{t})dx \\
= & -\frac{\Lambda^g}{N}\underbrace{\tilde{F}((N-1)\tilde{t}|\tilde{t}) \int_{\underline{t}(N-1)}^{\tilde{t}(N-1)} \tilde{F}(x|\tilde{t})dx - \int_{\underline{t}(N-1)}^{\tilde{t}(N-1)} \tilde{F}(x|\tilde{t})^2dx}_{=\Omega(\tilde{t})/F_1(\tilde{t})^2},
\end{aligned}$$

where the first equality follows by changing the order of integration and the second equality follows by partial integration. ■

Proof of Proposition 1: First, we need to show that $\beta_I^*(t)$ is increasing in t. Define $F^*(b) = \Pr\left[\max_{n\neq i}\left\{\beta_I^*\right\}_n \leq b\right]$. Moreover, define $\tilde{\gamma}(b) := F^*(b)\left\{1-\Lambda^g\left[1-F^*(b)\right]\right\}$. With this notation, it is easy to see that $\beta_I^*(t)$ satisfies

$$\beta_I^*(t) \in \arg\max_b \tilde{\gamma}(b)\left[t+\tilde{Q}(b)\right]/N - T(b)$$

where $\tilde{Q}$ and T are some functions. Observe that

$$\begin{aligned}
\frac{\partial^2\left\{\tilde{\gamma}(b)\left[t+\tilde{Q}(b)\right]/N - T(b)\right\}}{\partial t\partial b} &= \frac{\partial\tilde{\gamma}(b)}{\partial b}\frac{1}{N} \\
&= \frac{dF^*(b)}{db}\left\{1-\Lambda^g\left[1-2F^*(b)\right]\right\}/N > 0,
\end{aligned}$$

because $\frac{dF^*(b)}{db} > 0$ on the support of $F^*(b)$ and $\Lambda^g \leq 1$. Therefore, the objective function satisfies the strict Spence–Mirrlees condition as defined in Milgrom and Shannon (1994). Because $b \in \mathbb{R}$, the objective function is also quasi-supermodular in b. Thus, Theorem 4 in Milgrom and Shannon (1994) implies that every maximizer of the objective must be increasing. As a result the objective satisfies single crossing and thus the maximizer, for every non-constant $\beta_I^*(t)$, must be increasing.

Next, we establish sufficiency by using standard mechanism design arguments. Fix t and $t' \neq t$. Then,

$$\begin{aligned}
EU(t',t) &= \gamma(t')\left[t+q(t')\right]/N - \mathcal{T}_I(t') - \Lambda^g\Omega(t')/N \\
&= EU(t',t') + \gamma(t')(t-t')/N \\
&\Leftrightarrow EU(t',t) - EU(t,t) + EU(t,t) - EU(t',t') = \gamma(t')(t-t')/N \\
&\Leftrightarrow EU(t',t) - EU(t,t) \\
&= -\mathbb{I}[t>t']\int_{t'}^{t}\gamma(s)/Nds + \mathbb{I}[t<t']\int_{t}^{t'}\gamma(s)/Nds + \gamma(t')(t-t')/N. \qquad (27)
\end{aligned}$$

Observe that the RHS is of (27) is negative (independently whether $t > t'$ or $t < t'$) because $\gamma(t)$ is increasing by Assumption 1. ■

Proof of Proposition 2: We proceed in several steps. First, we use the envelope theorem to derive an inequality that determines the sign of the difference $\beta_I^{RN}(t) - \beta_I^*(t)$. In step 2 we derive a necessary condition for when this inequality is satisfied for the highest type and prove claim (ii). In step 3 we use the condition to prove claim (i).

Step 1: Expression for the Difference in Bids. Recall that the envelope theorem implies

$$\gamma(t)t + \gamma(t)q(t) - \Lambda^g\Omega(t) - N\mathcal{T}_I(t) = \int_{\underline{t}}^{t} \gamma(s)ds$$

$$\Leftrightarrow \quad (1-\Lambda^g)\left\{F_1(t)[t+q(t)] - \int_{\underline{t}}^{t} F_1(v)dv\right\} - N\mathcal{T}_I(t) = \Lambda^g\left\{\int_{\underline{t}}^{t} F_1^2(v)dv - F_1^2(t)[t+q(t)] + \Omega(t)\right\}.$$

By the envelope theorem we also know that in the risk-neutral case, $F_1(t)[t+q(t)] - \int_{\underline{t}}^{t} F_1(v)dv = NF_1(t)\beta_I^{RN}(t)$. Thus,

$$\begin{aligned} F_1(t)\left[\beta_I^{RN}(t) - \beta_I^*(t)\right] &= \Lambda^g\left\{F_1(t)[t+q(t)] - \int_{\underline{t}}^{t} F_1(v)dv + \int_{\underline{t}}^{t} F_1^2(v)dv - F_1^2(t)[t+q(t)] + \Omega(t)\right\}/N \\ &\quad + \Lambda^m F_1(t)\left[1-F_1(t)\right]\beta_I^*(t). \end{aligned}$$

Dividing both sides of the above expression by $F_1(t)$ and re-arranging yields

$$\beta_I^{RN}(t) - \beta_I^*(t) = \Lambda^g\left\{\left[1-F_1(t)\right][t+q(t)] - \frac{\int_{\underline{t}}^{t} F_1(v)\left[1-F_1(v)\right]dv - \Omega(t)}{F_1(t)}\right\}/N + \Lambda^m\left[1-F_1(t)\right]\beta_I^*(t).$$

Substituting $\beta_I^*(t) = \frac{\gamma(t)[t+q(t)] - \Lambda^g\Omega(t) - \int_{\underline{t}}^{t}\gamma(s)ds}{NF_1(t)\{1+\Lambda^m[1-F_1(t)]\}}$ into this expression leads

$$\begin{aligned} \beta_I^{RN}(t) - \beta_I^*(t) &= \Lambda^g\left\{\left[1-F_1(t)\right][t+q(t)] - \frac{\int_{\underline{t}}^{t} F_1(v)\left[1-F_1(v)\right]dv - \Omega(t)}{F_1(t)}\right\}/N \\ &\quad + \frac{\gamma(t)[t+q(t)] - \Lambda^g\Omega(t) - \int_{\underline{t}}^{t}\gamma(s)ds}{NF_1(t)}\frac{\Lambda^m\left[1-F_1(t)\right]}{1+\Lambda^m\left[1-F_1(t)\right]}. \end{aligned} \tag{28}$$

The RHS of (28) is an expression of the primitives alone. First, notice that $\gamma(t)\left[t+q(t)\right] - \Lambda^g\Omega(t) - \int_{\underline{t}}^{t}\gamma(s)ds$ is positive, as otherwise equilibrium payoffs and bids were negative. Hence, the RHS increases in Λ^m. Moreover, $\lim_{\Lambda^m\to\infty}\frac{\Lambda^m[1-F_1(t)]}{1+\Lambda^m[1-F_1(t)]} = 1$ for all $t \in [0,\bar{t})$ whereas $\frac{\Lambda^m[1-F_1(t)]}{1+\Lambda^m[1-F_1(t)]} = 0$ for $t = \bar{t}$.

Step 2: Proof of claim (ii)**.** Focus on (28). For the highest type the sign of the term in brackets after Λ^g is determined by whether $\Omega(t) \leq \int_{\underline{t}}^{t} F_1(v) - F_1(v)^2 dv$ or not, that is, (8).

Finally observe that whenever $\beta_I^*(\bar{t}) > \beta^{RN}(\bar{t})$ then continuity of the bid difference in t implies that this assertion holds true for an interval of types at the top.

Step 3: Prof of claim (i)**.** Now, let us focus on a bidder whose type is below t^m, with $F_1(t^m) = 0.5$. We show that (28) is positive. Thus, w.l.o.g. let $\Lambda^m = 0$. We establish that the term in brackets after Λ^g is strictly positive. That is, $F_1(t)\left[1-F_1(t)\right][t+q(t)] - \int_{\underline{t}}^{t} F_1(v)(1-F_1(v))dv +$

$\Omega(t) > 0$. A sufficient condition for this is $F_1(t)\left[1 - F_1(t)\right] t - \int_{\underline{t}}^{t} F_1(v)(1 - F_1(v))dv > 0$. Because $F_1(t)\left[1 - F_1(t)\right]$ is increasing in t on $[0, t^m]$, this last inequality is satisfied. ■

Proof of Proposition 3: Recall that the objective reads $EU(\tilde{t}, t) = \gamma(\tilde{t})\left[t + q(\tilde{t})\right]/N - \Lambda^g \Omega(\tilde{t})/N - \mathcal{T}_{II}(\tilde{t})$. The first-order condition yields:

$$\left(\gamma(t)q(t)\right)'/N - \Lambda^g \Omega'(t)/N + \gamma'(t)t/N = \mathcal{T}'_{II}(t), \tag{29}$$

where $\mathcal{T}'_{II}(t) = \left[\beta^*_{II}(t)[1 + \Lambda^m] - 2\Lambda^m \int_{\underline{t}}^{t} \beta^*_{II}(v) f_1(v) dv\right] f_1(t)$. Define $\mathcal{Q}(t) := \frac{(\gamma(t)q(t))' - \Lambda^g \Omega'(t)}{N f_1(t)}$ and $\mathcal{G}(t) := \frac{\gamma'(t)}{N f_1(t)}$. Multiplying both sides of (29) by $\frac{1}{f_1(t)(1+\Lambda^m)}$ and differentiating yields

$$\frac{1}{1 + \Lambda^m}\left[\mathcal{Q}'(t) + \mathcal{G}(t) + \mathcal{G}'(t)t\right] = \beta^{*\prime}_{II}(t) - \frac{2\Lambda^m f_1(t)}{1 + \Lambda^m}\beta^*_{II}(t).$$

Solving this first-order linear differential equation gives:

$$\beta^*_{II}(t) = \frac{e^{\frac{2\Lambda^m F_1(t)}{1+\Lambda^m}}}{1 + \Lambda^m} \int_{\underline{t}}^{t} e^{-\frac{2\Lambda^m F_1(v)}{1+\Lambda^m}} \left[\mathcal{Q}'(v) + \mathcal{G}(v) + \mathcal{G}'(v)v\right] dv. \tag{30}$$

Next, we simplify (30) by applying partial integration. Observe that

$$\begin{aligned} e^{\frac{2\Lambda^m F_1(t)}{1+\Lambda^m}} \int_{\underline{t}}^{t} e^{-\frac{2\Lambda^m F_1(v)}{1+\Lambda^m}} \mathcal{Q}'(v) dv &= \mathcal{Q}(t) - \mathcal{Q}(0) e^{\frac{2\Lambda^m F_1(t)}{1+\Lambda^m}} + \frac{2\Lambda^m}{1 + \Lambda^m} \int_{\underline{t}}^{t} \mathcal{Q}(v) e^{-\frac{2\Lambda^m [F_1(v) - F_1(t)]}{1+\Lambda^m}} f_1(v) dv \\ &= \frac{(\gamma(t)q(t))' - \Lambda^g \Omega'(t)}{N f_1(t)} + \frac{2\Lambda^m}{1 + \Lambda^m} \int_{\underline{t}}^{t} \left[\frac{(\gamma(v)q(v))' - \Lambda^g \Omega'(v)}{N}\right] e^{\frac{-2\Lambda^m [F_1(v) - F_1(t)]}{1+\Lambda^m}} dv. \end{aligned}$$

Similarly, we have

$$\begin{aligned} e^{\frac{2\Lambda^m F_1(t)}{1+\Lambda^m}} \int_{\underline{t}}^{t} e^{-\frac{2\Lambda^m F_1(v)}{1+\Lambda^m}} \left[\mathcal{G}(v) + \mathcal{G}'(v)v\right] dv &= \mathcal{G}(t)t + \frac{2\Lambda^m}{1 + \Lambda^m} \int_{\underline{t}}^{t} \mathcal{G}(v) v e^{-\frac{2\Lambda^m [F_1(v) - F_1(t)]}{1+\Lambda^m}} f_1(v) dv \\ &= \frac{\gamma'(t)t}{N f_1(t)} + \frac{2\Lambda^m}{1 + \Lambda^m} \int_{\underline{t}}^{t} \frac{\gamma'(v)v}{N} e^{-\frac{2\Lambda^m [F_1(v) - F_1(t)]}{1+\Lambda^m}} dv. \end{aligned}$$

Substituting into (30) leads to (10) from the main text. Sufficiency and monotonicity of β^*_{II} follows from similar arguments as those employed in the proof of Proposition 1. ■

Proof of Proposition 4: We begin by noticing that integration by parts implies that

$$\int_{\underline{t}}^{t} \left(\int_{\underline{t}}^{x} (\beta^*_{II}(x) - \beta^*_{II}(y)) f_1(y)\, dy\right) f_1(x)\, dx = \int_{\underline{t}}^{t} F_1(x)\, \beta^*_{II}(x) f_1(x)\, dx - \int_{\underline{t}}^{t} \left(\int_{\underline{t}}^{x} \beta^*_{II}(y) f_1(y)\, dy\right) f_1(x)\, dx$$

and

$$\int_{\underline{t}}^{t} \left(\int_{\underline{t}}^{x} \beta^*_{II}(y) f_1(y)\, dy\right) f_1(x)\, dx = F_1(t) \int_{\underline{t}}^{t} \beta^*_{II}(x) f_1(x)\, dx - \int_{\underline{t}}^{t} F_1(x)\, \beta^*_{II}(x) f_1(x)\, dx.$$

Hence, type-t's dis-utility from the expected payment reads

$$\mathcal{T}_{II}(t) = \int_{\underline{t}}^{t} \beta_{II}^{*}(x) f_1(x)\, dx + \Lambda^m \underbrace{\left\{ [1 - 2F_1(t)] \int_{\underline{t}}^{t} \beta_{II}^{*}(x) f_1(x)\, dx + 2\int_{\underline{t}}^{t} F_1(x)\, \beta_{II}^{*}(x) f_1(x)\, dx \right\}}_{:=C(t)}. \quad (31)$$

Moreover, the envelope theorem implies that

$$\begin{aligned} N\mathcal{T}_{II}(t) &= \gamma(t)[t + q(t)] - \int_{\underline{t}}^{t} \gamma(s) ds - \Lambda^g \Omega(t) \quad (32) \\ &= (1 - \Lambda^g)\left\{ F_1(t)[t + q(t)] - \int_{\underline{t}}^{t} F_1(s)\, ds \right\} + \Lambda^g \left\{ F_1^2(t)[t + q(t)] - \int_{\underline{t}}^{t} F_1^2(s)\, ds \right\} - \Lambda^g \Omega(t). \end{aligned}$$

Similarly, for the risk-neutral case the envelope theorem implies that

$$N \int_{\underline{t}}^{t} \beta_{II}^{RN}(x) f_1(x)\, dx = F_1(t)[t + q(t)] - \int_{\underline{t}}^{t} F_1(s)\, ds. \quad (33)$$

Combining (31), (32) and (33) we obtain

$$N \int_{\underline{t}}^{t} \beta_{II}^{*}(x) f_1(x)\, dx = N(1 - \Lambda^g) \int_{\underline{t}}^{t} \beta_{II}^{RN}(x) f_1(x)\, dx + \Lambda^g \left\{ F_1^2(t)[t + q(t)] - \int_{\underline{t}}^{t} F_1^2(s)\, ds \right\} - \Lambda^g \Omega(t) - \Lambda^m NC(t).$$

Therefore, we have that

$$\begin{aligned} N\int_{\underline{t}}^{t} \left[\beta_{II}^{*}(x) - \beta_{II}^{RN}(x)\right] f_1(x)\, dx &= \Lambda^g \left\{ F_1^2(t)[t + q(t)] - N\int_{\underline{t}}^{t} \beta_{II}^{RN}(x) f_1(x)\, dx - \int_{\underline{t}}^{t} F_1^2(s)\, ds \right\} - \Lambda^g \Omega(t) - \Lambda^m NC(t) \\ &= \Lambda^g \left\{ \int_{\underline{t}}^{t} F_1(s)[1 - F_1(s)]\, ds - F_1(t)[1 - F_1(t)][t + q(t)] \right\} - \Lambda^g \Omega(t) - \Lambda^m NC(t), \end{aligned}$$

where the last step follow from (33). Observe further that $\frac{d\left(\int_{\underline{t}}^{t} \beta_{II}^{*}(x) f_1(x) dx - \int_{\underline{t}}^{t} \beta_{II}^{RN}(x) f_1(x) dx\right)}{dt} = f_1(t)\left(\beta_{II}^{*}(t) - \beta_{II}^{RN}(t)\right)$. Therefore, $\beta_{II}^{*}(t) > \beta_{II}^{RN}(t)$ if and only if

$$\Lambda^g \left\{ \int_{\underline{t}}^{t} F_1(s)[1 - F_1(s)]\, ds - F_1(t)[1 - F_1(t)][t + q(t)] \right\}' - \Lambda^g \Omega'(t) - \Lambda^m NC'(t) > 0. \quad (34)$$

To prove the first claim notice that the sign of the expression on the LHS of (34) depends on the sign of the following expression

$$\Lambda^g f_1(t)[2F_1(t) - 1][t + q(t)] - F_1(t)[1 - F_1(t)]\, q'(t) - \Lambda^g \Omega'(t) - \Lambda^m NC'(t). \quad (35)$$

Fix now a t such that $F_1(t) \leq \frac{1}{2}$. As $q'(t) > 0$ and $-\Omega'(t) < 0$, a sufficient condition for

expression (35) to be negative is that $C'(t) \geq 0$. We have

$$\begin{aligned} C'(t) &= -2f_1(t)\int_{\underline{t}}^{t} \beta_{II}^*(x) f_1(x)\,dx + \beta_{II}^*(t) f_1(t) \geq 0 \\ &\Leftrightarrow \beta_{II}^*(t) \geq 2\int_{\underline{t}}^{t} \beta_{II}^*(x) f_1(x)\,dx \\ &\Leftrightarrow \beta_{II}^*(t) - 2\left[F_1(t)\,\beta_{II}^*(t) - \int_{\underline{t}}^{t} \beta_{II}^{*\prime}(x) F_1(x)\,dx\right] > 0 \\ &\Leftrightarrow \beta_{II}^*(t)\left[1 - 2F_1(t)\right] + \int_{\underline{t}}^{t} \beta_{II}^{*\prime}(x) F_1(x)\,dx > 0. \end{aligned}$$

As $\beta_{II}^*(t)$ is increasing, it follows that $C'(t) \geq 0$ for any $t \leq t^m$. Hence, for any such type expression (35) is negative, which proves the first claim.

Finally, to prove the second claim, let $\Lambda^m = 0$. Then, it is easy to see that for $t = \bar{t}$ equation (35) is satisfied if and only if condition (13) is satisfied. Hence, since (35) is continuous in t and Λ^m, there exists a positive measure of types at the top of the signal distribution's support for which $\beta_{II}^*(t) > \beta_{II}^{RN}(t)$ if Λ^m is sufficiently small. ■

Proof of Proposition 5: Recall that the envelope theorem implies that in both auction formats $EU(t,t) = \int_{\underline{t}}^{t} \gamma(s)/N ds$, which proves the first statement.

Moreover, this indirect utility implies

$$N\mathcal{T}_k(t) = \gamma(t)\left[t + q(t)\right] - \Lambda^g \Omega(t) - \int_{\underline{t}}^{t} \gamma(s)\,ds,$$

where $k \in \{I,\ II\}$. Therefore, $\mathcal{T}_I = \mathcal{T}_{II}$. Moreover, notice that

$$N\mathcal{T}_I(t) = \{1 + \Lambda^m\left[1 - F_1(t)\right]\} \times \underbrace{F_1(t)\,\beta_I^*(t)}_{\text{Expected payment to the seller from type } t},$$

and

$$N\mathcal{T}_{II}(t) = \{1 + \Lambda^m\left[1 - F_1(t)\right]\} \times \underbrace{\int_{\underline{t}}^{t} \beta_{II}^*(s) f_1(s)\,ds}_{\text{Expected payment to the seller from type } t} + \Lambda^m I(t),$$

where $I(t) = \int_{\underline{t}}^{t}\left(\int_{\underline{t}}^{x}(\beta_{II}^*(x) - \beta_{II}^*(v)) f_1(v)\,dv\right) f_1(x)\,dx$. Thus, $F_1(t)\,\beta_I^*(t) = \int_{\underline{t}}^{t} \beta_{II}^*(s) f_1(s)\,ds$ for $\Lambda^m = 0$. Next, suppose $\Lambda^m > 0$. Because $tI(t) > 0$ and $\mathcal{T}_I(t) = \mathcal{T}_{II}(t)$, it follows that $t\left[F_1(t)\,\beta_I^*(t) - \int_{\underline{t}}^{t} \beta_{II}^*(s) f_1(s)\,ds\right] > 0$. ■

Proof of Proposition 6: First, assume that winning is always good news, that is, $N\widetilde{\beta}_I^{BB}(t) < t + (N-1)\underline{t}$ for all t.

A straightforward application of the envelope theorem yields

$$\widetilde{\beta}_I^{BB}(t) = \frac{\gamma^{BB}(t)\left[t + q(t)\right] - \int_{\underline{t}}^{t} \gamma^{BB}(s)\,ds - \Lambda\Omega_I(t)}{N\gamma^{BB}(t)}.$$

Substituting and re-arranging yields the expression in the main text. Sufficiency and monotonicity of $\widetilde{\beta}_I^{BB}$ follow from a similar argument as in the proof of Proposition 1.

Next, assume without loss of generality that $N\tilde{\beta}_I^{BB}(t) > t + (N-1)\underline{t}$ for some t.

Existence and Uniqueness. Define c as the infimum of the set $\{t \in [\underline{t}, \bar{t}] | N\tilde{\beta}_I^{BB}(t) > t + (N-1)\underline{t}\}$. For any $t \in [\underline{t}, c)$ it is easy to see the equilibrium strategy is $\tilde{\beta}_I^{BB}$. For any $t \in (c, \bar{t}]$, we take the first-order condition of (18), evaluate it at $\tilde{t} = t$, rewrite it in the standard form of an ordinary differential equation and obtain

$$\beta_I^{BB}(t)' = \frac{\left(\gamma^{BB}(\tilde{t})(\frac{t+q(\tilde{t})}{N})\right)'|_{\tilde{t}=t} - \Lambda\Omega_I'(t) - (2\Lambda F_1(t)\left[1 - F_1(t)\right])' S\left(N\beta_I^{BB}(t) - t\right) - \gamma^{BB}(t)'\beta_I^{BB}(t)}{\gamma^{BB}(t) + 2\Lambda F_1(t)(1 - F_1(t))NS'\left(N\beta_I^{BB}(t) - t\right)} \tag{36}$$

Call the term on the RHS $M(t, \beta_I^{BB})$. First, note that $M(t, b)$ is continuos in both in t and b. Moreover, $M(t, b)$ is continuously differentiable in b and therefore Lipschitz continuous. Thus, we know that the differential equation has exactly one solution on $(c, \bar{t}]$ given the boundary condition $\beta_I^{BB}(c) = \tilde{\beta}_I^{BB}(c)$.

Monotonicity. For any $t \in [\underline{t}, c]$ it follows that $\tilde{\beta}_I^{BB}$ is increasing from a similar argument as provided in the proof of Proposition 1. Thus, β_I^{BB} is increasing on that interval. Now fix $t \in (c, \bar{t}]$ and note that

$$\frac{\partial EU(\tilde{t}, t)}{\partial t} = \gamma^{BB}(\tilde{t})/N + 2\Lambda F_1(\tilde{t})\left[1 - F_1(\tilde{t})\right] S'\left(N\beta_I^{BB}(\tilde{t}) - t\right) \tag{37}$$

Note that (37) is differentiable, because S is. Suppose first that $S'\left(N\beta_I^{BB}(\tilde{t}) - t\right) = 0$. Then,

$$\frac{\partial^2 EU(\tilde{t}, t)}{\partial t \partial \tilde{t}} = \left(\gamma^{BB}(\tilde{t})\right)' \geq 0$$

Next, if $S'\left(N\beta_I^{BB}(\tilde{t}) - t\right) = \tilde{F}(N\beta_I^{BB}(\tilde{t}) - t|\tilde{t})$ rewrite that CDF in terms of first-order statistics, that is, $F_1(\tilde{t})\tilde{F}(N\beta_I^{BB}(\tilde{t}) - t|\tilde{t}) = \int_{\underline{t}}^{\tilde{t}} \int_{(N-2)\underline{t}}^{N\beta_I^{BB}(\tilde{t})-t-t_1} \tilde{f}_2(x|t_1)dx f_1(t_1)dt_1$, where $\tilde{f}_2(\cdot|t_1)$ is the pdf of $\sum_{i\geq 2} t_i$ conditional on being lower than $(N-2)t_1$ and observe that (37) becomes

$$\gamma^{BB}(\tilde{t}) + 2\Lambda(1 - F_1(\tilde{t})) \int_{\underline{t}}^{\tilde{t}} \int_{(N-2)\underline{t}}^{N\beta_I^{BB}(\tilde{t})-t-t_1} \tilde{f}_2(x|t_1)dx f_1(t_1)dt_1 \tag{38}$$

We differentiate the second term of (38) w.r.t. $\tilde{t}$:

$$\begin{aligned} &f_1(\tilde{t})2\Lambda\left[-\int_{\underline{t}}^{\tilde{t}} \int_{(N-2)\underline{t}}^{N\beta_I^{BB}(\tilde{t})-t-t_1} \tilde{f}_2(x|t_1)dx f_1(t_1)dt_1 + \int_{(N-2)\underline{t}}^{N\beta_I^{BB}(\tilde{t})-t-\tilde{t}} \tilde{f}_2(x|t_1)dx\right] \\ &+2\Lambda\left[1 - F_1(\tilde{t})\right]\left(\beta_I^{BB}(\tilde{t})\right)' \int_{\underline{t}}^{\tilde{t}} \tilde{f}_2(N\beta_I^{BB}(\tilde{t}) - t - t_1|t_1)f_1(t_1)dt_1 \geq -2\Lambda F_1(\tilde{t})f_1(\tilde{t}) \end{aligned}$$

The inequality follows because the first term in the first square bracket is smaller than $F_1(\tilde{t})$. This holds because the density in the integrand, f_1, is weighed down by some number lower than 1, $\tilde{f}_2(\cdot|t_1)$. All other terms are positive. Therefore, the derivative of (38) satisfies

$$\frac{\partial^2 EU(\tilde{t}, t)}{\partial t \partial \tilde{t}} \geq f_1(\tilde{t})\left[(1 - \Lambda) + 2\Lambda F_1(\tilde{t})\right] - 2\Lambda f_1(\tilde{t})F_1(\tilde{t}) = f_1(\tilde{t})(1 - \Lambda) > 0.$$

We therefore conclude that $EU(\tilde{t},t)$ satisfies the strict Spence-Mirrlees conditions. Using the same argument as in the proof of Proposition 1 establishes that β_I^{BB} is also increasing on $(c,\bar{t}]$.

Sufficiency. Fix t and $\tilde{t} \neq t$. Then, using (18)

$$\begin{aligned} EU(\tilde{t},t) &= \gamma^{BB}(\tilde{t})(\frac{t+q(\tilde{t})}{N} - \beta_I^{BB}(\tilde{t})) - \Lambda\Omega_I(\tilde{t}) - 2\Lambda F_1(\tilde{t})(1-F_1(\tilde{t}))S\left(N\beta_I^{BB}(\tilde{t}) - t\right) \\ &= EU(\tilde{t},\tilde{t}) + \gamma^{BB}(\tilde{t})(t-\tilde{t})/N - 2\Lambda F_1(\tilde{t})\left[1-F_1(\tilde{t})\right]\left[S(N\beta_I^{BB}(\tilde{t})-t) - S(N\beta_I^{BB}(\tilde{t})-\tilde{t})\right] \\ &\Leftrightarrow EU(\tilde{t},t) - EU(t,t) + EU(t,t) - EU(\tilde{t},\tilde{t}) \\ &= \gamma^{BB}(\tilde{t})(t-\tilde{t})/N - 2\Lambda F_1(\tilde{t})\left[1-F_1(\tilde{t})\right]\left[S(N\beta_I^{BB}(\tilde{t})-t) - S(N\beta_I^{BB}(\tilde{t})-\tilde{t})\right] \end{aligned} \tag{39}$$

Substitute the representation (19) into (39) to get

$$\begin{aligned} EU(\tilde{t},t) - EU(t,t) &= \\ & \gamma^{BB}(\tilde{t})(t-\tilde{t})/N + 2\Lambda F_1(\tilde{t})\left[1-F_1(\tilde{t})\right]\left[S(N\beta_I^{BB}(\tilde{t})-\tilde{t}) - S(N\beta_I^{BB}(\tilde{t})-t)\right] \\ -\mathbb{I}[t &> \tilde{t}]\int_{\tilde{t}}^{t}\frac{\partial EU(s,s)}{\partial t}ds + \mathbb{I}[t<\tilde{t}]\int_{t}^{\tilde{t}}\frac{\partial EU(s,s)}{\partial t}ds \\ &\leq \gamma^{BB}(\tilde{t})(t-\tilde{t})/N + 2\Lambda F_1(\tilde{t})\left[1-F_1(\tilde{t})\right]\left[S(N\beta_I^{BB}(\tilde{t})-\tilde{t}) - S(N\beta_I^{BB}(\tilde{t})-t)\right] \\ -\mathbb{I}[t &> \tilde{t}]\int_{\tilde{t}}^{t}\frac{\partial EU(s,\tilde{t})}{\partial t}ds + \mathbb{I}[t<\tilde{t}]\int_{t}^{\tilde{t}}\frac{\partial EU(s,\tilde{t})}{\partial t}ds, \end{aligned} \tag{40}$$

where the inequality follows from the fact that $\frac{\partial EU(t,\tilde{t})}{\partial t} \geq 0$. Finally, evaluating the integrals reveals that the upper bound on (40) is zero.

Proof of Proposition 7: Recall that $\beta_I^{BB}(t) = \frac{\gamma^{BB}(t)[t+q(t)] - \int_{\underline{t}}^{t}\gamma^{BB}(s)ds - \Lambda\Omega_I(t)}{N\gamma^{BB}(t)}$ and $\beta_I^{RN}(t) = \frac{t+q(t)}{N} - \frac{\int_{\underline{t}}^{t}F_1(s)ds}{NF_1(t)}$. Therefore, it follows that

$$\beta_I^{BB}(t) - \beta_I^{RN}(t) = \frac{\Lambda}{N\gamma^{BB}(t)}\left[F_1(t)\int_{\underline{t}}^{t}F_1(s)\,ds - \int_{\underline{t}}^{t}F_1(s)^2\,ds - \Omega_I(t)\right].$$

Finally, observe that

$$\frac{F_1(t)\int_{\underline{t}}^{t}F_1(s)\,ds - \int_{\underline{t}}^{t}F_1(s)^2\,ds}{F_1(t)^2} = \int_{\underline{t}}^{t}F_1(x|t)[1-F_1(x|t)]dx.$$

Proof of Proposition 8: We apply a guess and verify approach. First, we guess that winning is always good news and that $\beta_{II}^{BB}(t_1) - t_1$ is increasing. Under these hypothesis, we have that

$$\begin{aligned} \Omega_{II}(\tilde{t}) &= \underbrace{-\int_{\underline{t}}^{\tilde{t}}\int_{\underline{t}}^{t_1} t_1 - y_1 f_1(y_1)dy_1 f_1(t_1)dt_1}_{=\Omega_I} + 2\int_{\underline{t}}^{\tilde{t}}\int_{\underline{t}}^{t_1}\left[\beta_{II}^{BB}(t_1) - \beta_{II}^{BB}(y_1)\right]f_1(y_1)dy_1 f_1(t_1)dt_1 \\ &= \Omega_I(\tilde{t}) - 2\left[2\int_{\underline{t}}^{\tilde{t}}F_1(x)\beta_{II}^{BB}(x)f_1(x)dx - F_1(\tilde{t})\int_{\underline{t}}^{\tilde{t}}\beta_{II}^{BB}(x)f_1(x)dx\right]. \end{aligned} \tag{41}$$

The direct utility becomes

$$\gamma^{BB}(\tilde{t})[t+q(\tilde{t})]/2+\Lambda\Omega_I(\tilde{t})/2-2\Lambda\int_{\underline{t}}^{\tilde{t}}F_1(x)\beta_{II}^{BB}(x)f_1(x)dx-(1-\Lambda)\int_{\underline{t}}^{\tilde{t}}\beta_{II}^{BB}(x)f_1(x)dx. \quad (42)$$

Differentiating (42) and evaluating the derivative at $\tilde{t}=t$ reveals

$$\beta_{II}^{BB}(t)=\frac{(\gamma^{BB}(t))'[t+q(t)]/2+\gamma^{BB}(t)q'(t)/2+\Lambda\Omega_I'(t)/2}{((1-\Lambda)+2\Lambda F_1(t))f_1(t)}.$$

Straightforward substitution shows that $\beta_{II}^{BB}(t)=t$ which verifies the initial guess. ■

Proof of Proposition 9: For $N=2$ the bid in the SPA is equal to the risk-neutral bid. We now show that the same is true in the FPA. Recall that $\beta_I^{BB}(t)=(t+q(t))/N-(\int_{\underline{t}}^t\gamma^{BB}(s)ds+\Lambda\Omega_I(t))/(\gamma^{BB}(t)N)$ and note that

$$\begin{aligned}
\beta_I^{BB}(t) &= \beta_I^{RN}(t)=(t+q(t))/N-\int_{\underline{t}}^t F_1(s)ds/(F_1(t)N)\\
&\Leftrightarrow (\int_{\underline{t}}^t\gamma^{BB}(s)ds+\Lambda\Omega_I(t))/(\gamma^{BB}(t))=\int_{\underline{t}}^t F_1(s)ds/(F_1(t))\\
&\Leftrightarrow F_1(t)(\int_{\underline{t}}^t\gamma^{BB}(s)ds+\Lambda\Omega_I(t))=\gamma^{BB}(t)\int_{\underline{t}}^t F_1(s)ds\\
&\Leftrightarrow \Lambda\int_{\underline{t}}^t F_1(s)^2ds+\Lambda\Omega_I(t)=\Lambda F_1(t)\int_{\underline{t}}^t F_1(s)ds\\
&\Leftrightarrow \Omega_I(t)=F_1(t)\int_{\underline{t}}^t F_1(s)ds-\int_{\underline{t}}^t F_1(s)^2ds,
\end{aligned}$$

which is true for $N=2$. ■

Proof of Proposition 10: Differentiating the direct utility, (25), and evaluating the first-order condition at $\tilde{t}=t$ yields

$$\begin{aligned}
\beta_{II}^{BB}(t)=\Big\{\Big(\gamma^{BB}(\tilde{t})(t+q(\tilde{t}))\Big)'|_{\tilde{t}=t}/N+\Lambda\tilde{\Omega}_{II}'(t)/N+2\Lambda f_1(t)\left[2F_1(t)-1\right]\int_{\underline{t}}^t\tfrac{\tilde{S}(\beta_{II}^{BB}(t_1)-t_1-t,t_1)}{N}f_1(t_1)dt_1\\
-2\Lambda F_1(t)\left[1-F_1(t)\right]\tfrac{\tilde{S}(\beta_{II}^{BB}(t)-2t,t)}{N}f_1(t)-\Lambda f_1(t)\int_{\underline{t}}^t\beta_{II}^{BB}(x)f_1(x)dx\\
-\Lambda\int_{\underline{t}}^t(2\Phi_2(\tilde{D}_2|t,y_1)-1)\left[\beta_{II}^{BB}(t)-\beta_{II}^{BB}(y_1)\right]f_1(y_1)dy_1f_1(t)\Big\}/\Big\{\left[1-\Lambda+\Lambda F_1(t)\right]f_1(t)\Big\}.
\end{aligned}$$

Simplifying and re-arranging the above equation, we obtain

$$\begin{aligned}
\Big\{\Lambda\int_{\underline{t}}^t(2\Phi_2(\tilde{D}_2|t,y_1)-1)f_1(y_1)dy_1+\left[1-\Lambda+\Lambda F_1(t)\right]\Big\}\beta_{II}^{BB}(t)=\Big\{\frac{\Big(\gamma^{BB}(\tilde{t})(t+q(\tilde{t}))\Big)'|_{\tilde{t}=t}+\Lambda\tilde{\Omega}_{II}'(t)}{f_1(t)N}\\
+2\Lambda\left[2F_1(t)-1\right]\int_{\underline{t}}^t\tfrac{\tilde{S}(\beta_{II}^{BB}(t_1)-t_1-t,t_1)}{N}f_1(t_1)dt_1-2\Lambda F_1(t)(1-F_1(t))\tfrac{\tilde{S}(\beta_{II}^{BB}(t)-2t,t)}{N}f_1(t)\\
-\Lambda\int_{\underline{t}}^t\beta_{II}^{BB}(x)f_1(x)dx+\Lambda\int_{\underline{t}}^t(2\Phi_2(\tilde{D}_2|t,y_1)-1)\beta_{II}^{BB}(y_1))f_1(y_1)dy_1\Big\}
\end{aligned} \quad (43)$$

Differentiate (43) w.r.t. t and re-arrange the FOC such that only terms containing $(\tilde{\beta}_{II}^{BB}(t))'$ are on the RHS. Let us denote the terms on the LHS of the so constructed differential equation by $M(t, \beta_{II}^{BB}(t))$ which is is continuous in both arguments. Moreover, $M(t, b)$ is continuously differentiable in b (and thus Lipschitz continuous) because $[1 - \Lambda + \Lambda F_1(t)] > 0$. Hence, the differential equation has a unique solution on $(\underline{t}, \bar{t}]$. Sufficiency and monotonicity follows from similar steps than those in the FPA.

Then, the first statement in the Proposition is a direct consequence of the definition of $\tilde{S}$. The second statement is a direct implication of the indirect utility, (26), presented in the main text. Finally, the third statement follows from the envelope theorem.■

Proof of Lemma 2: Fix bidder i and two realizations of his toughest competitor, t_1, and y_1 with $t_1 \geq y_1$. Recall that $\tilde{D}_2 = y_1 - t_1 + N\left[\beta_{II}^{BB}(t_1) - \beta_{II}^{BB}(y_1)\right]$. For given y_1 and t_1, whenever $D_2 \equiv \sum_{i \geq 2}(t_i - y_i)$ falls short this threshold, then $t_1 - \beta_{II}^{BB}(t_1) - y_1 + \beta_{II}^{BB}(y_1) + D_2 < 0$ and else positive. Let $\phi_2(\cdot|t_1, y_1)$ be the density of D_2 conditional on the realizations t_1 and y_1. Moreover, assume for the moment that $\tilde{D}_2 > -(N-2)(y_1 - \underline{t})$. Then, $\Omega_{II}(\tilde{t})$ reads

$$\int_{\underline{t}}^{\tilde{t}} \int_{\underline{t}}^{t_1} \int_{\tilde{D}_2}^{(N-2)(t_1-\underline{t})} \left\{(t_1 - y_1 + x) - N\left[\beta_{II}^{BB}(t_1) - \beta_{II}^{BB}(y_1)\right]\right\} \phi_2(x|t_1, y_1) dx f_1(y_1) dy_1 f_1(t_1) dt_1$$
$$+ \int_{\underline{t}}^{\tilde{t}} \int_{\underline{t}}^{t_1} \int_{-(N-2)(y_1-\underline{t})}^{\tilde{D}_2} \left\{-(t_1 - y_1 + x) + N\left[\beta_{II}^{BB}(t_1) - \beta_{II}^{BB}(y_1)\right]\right\} \phi_2(x|t_1, y_1) dx f_1(y_1) dy_1 f_1(t_1) dt_1$$

Now use that

$$\int_{\tilde{D}_2}^{(N-2)(t_1-\underline{t})} (\cdot)\phi_2(x|t_1, y_1) dx = \int_{-(N-2)(y_1-\underline{t})}^{(N-2)(t_1-\underline{t})} (\cdot)\phi_2(x|t_1, y_1) dx - \int_{-(N-2)(y_1-\underline{t})}^{\tilde{D}_2} (\cdot)\phi_2(x|t_1, y_1) dx.$$

The intensive risk therefore reads

$$\Omega_{II}(\tilde{t}) = \int_{\underline{t}}^{\tilde{t}} \int_{\underline{t}}^{t_1} \left[2\tilde{\Phi}_2(\tilde{D}_2|t_1, y_1) - 1\right] \left[\beta_{II}^{BB}(t_1) - \beta_{II}^{BB}(y_1)\right] f_1(y_1) dy_1 f_1(t_1) dt_1 + \hat{\Omega}(\tilde{t}),$$

where

$$\hat{\Omega}(\tilde{t}) = \Omega_I(\tilde{t}) - 2\int_{\underline{t}}^{\tilde{t}} \int_{\underline{t}}^{t_1} \int_{-(N-2)(y_1-\underline{t})}^{\tilde{D}_2} (t_1 - y_1 + D_2)\phi_2(x|t_1, y_1) dx f_1(y_1) dy_1 f_1(t_1) dt_1.$$

Finally, we account for the possibility that $\tilde{D}_2 < -(N-2)(y_1 - \underline{t})$. Define the non-negative, increasing and differentiable function $W(\cdot|t_1, y_1)$ the following way: $W(\tilde{D}_2|t_1, y_1) = \int_{(N-2)(y_1-\underline{t})}^{\tilde{D}_2} (t_1 - y_1 + D_2)\phi_2(x|t_1, y_1) dx$ if $\tilde{D}_2 \geq -(N-2)(y_1 - \underline{t})$ and else $W(x|t_1, y_1) = 0$. Then, it is easy to see that

$$\hat{\Omega}(\tilde{t}) = \Omega_I(\tilde{t}) - 2\int_{\underline{t}}^{\tilde{t}} \int_{\underline{t}}^{t_1} W(\tilde{D}_2|t_1, y_1) f_1(y_1) dy_1 f_1(t_1) dt_1$$

as stated in the text. ■

Proof of Lemma 3: Assume for the moment that there exists a t such that $N\beta_{II}^{BB}(t_1) - t_1 - t >$

$(N-2)\underline{t}$ for all $t_1 \leq t$. Let $\tilde{t}$ be in a neighborhood of t. Then we have that

$$\begin{aligned}
& \int_{\underline{t}}^{\tilde{t}} ... \int_{\underline{t}}^{t_{N-2}} \left| \frac{t+\sum t_i}{N} - \beta_{II}^{BB}(t_1) \right| f_{N-1}(t_{N-1}|t_{N-2}) dt_{N-1} ... f_1(t_1|\tilde{t}) dt_1 \\
= & \int_{\underline{t}}^{\tilde{t}} \int_{N\beta_{II}(t_1)-t_1-t}^{(N-2)t_1} \left(\frac{t+x+t_1}{N} - \beta_{II}^{BB}(t_1) \right) \tilde{f}_2(x|t_1) dt_1 f_1(t_1|\tilde{t}) dt_1 \\
& -2 \int_{\underline{t}}^{\tilde{t}} \int_{(N-2)\underline{t}}^{N\beta_{II}(t_1)-t_1-t} \left(\frac{t+x+t_1}{N} - \beta_{II}^{BB}(t_1) \right) \tilde{f}_2(x|t_1) dt_1 f_1(t_1|\tilde{t}) dt_1 \\
= & \ t + q(\tilde{t}) - \left[1 - F_1(\tilde{t})\right] \int_{\underline{t}}^{\tilde{t}} \beta_{II}^{BB}(t_1) f_1(t_1) dt_1 + 2 \int_{\underline{t}}^{\tilde{t}} \int_{(N-2)\underline{t}}^{N\beta_{II}^{BB}(t_1)-t_1-t} \frac{\tilde{F}_2(x|t_1)}{N} dx f_1(t_1) dt_1,
\end{aligned}$$

where $\tilde{F}_2(x|t_1)$ is the CDF of $\sum_{i=2} t_i$ conditional on t_1, with density $\tilde{f}_2(x|t_1)$. Note that the last equality follows by partial integration. Next, assume that there is a t such that $N\beta_{II}^{BB}(t_1) - t_1 - t < (N-2)\underline{t}$ for all t_1. We have that

$$\begin{aligned}
& \int_{\underline{t}}^{\tilde{t}} ... \int_{\underline{t}}^{t_{N-1}} \left| \frac{t+\sum t_i}{N} - \beta_{II}^{BB}(t_1) \right| f_{N-1}(t_{N-1}|t_{N-2}) dt_{N-1} ... f_1(t_1|\tilde{t}) dt_1 \\
= & \int_{\underline{t}}^{\bar{t}} \int_{(N-2)\underline{t}}^{(N-2)t_1} \left[\frac{t+x+t_1}{N} - \beta_{II}^{BB}(t_1) \right] \tilde{f}_2(x|t_1) dx f_1(t_1|\tilde{t}) dt_1.
\end{aligned}$$

Thus, let $\tilde{S}(\beta_{II}^{BB}(t_1) - t_1 - t, t_1) = 0$ if $N\beta_{II}^{BB}(t_1) - t_1 - t < (N-2)\underline{t}$ and else equal to $\int_{(N-2)\underline{t}}^{N\beta_{II}^{BB}(t_1)-t_1-t} \frac{\tilde{F}_2(x|t_1)}{N} dx$. Then the formula for the direct utility presented in the main text easily follows. ■

Proof of Proposition 11: Observe that the envelope theorem implies that both in the FPA and in the SPA a bidder's indirect utility satisfies $\int_{\underline{t}}^{t} \gamma^{BB}(s) ds$, which proves the last statement. Moreover, equating the direct utility with the indirect utility yields the following condition:

$$\begin{aligned}
\int_{\underline{t}}^{t} \gamma^{BB}(s)\, ds &= \gamma^{BB}(t)\,[t + q(t)] - \Lambda\Omega_k(t) - N\mathcal{T}_k^{BB}(t) \Leftrightarrow \\
\Lambda\Omega_k(t) + N\mathcal{T}_k^{BB}(t) &= \gamma^{BB}(t)\,[t + q(t)] - \int_{\underline{t}}^{t} \gamma^{BB}(s)\, ds, \qquad (44)
\end{aligned}$$

where $\mathcal{T}_{II}^{BB}(t) := \{1 - \Lambda\,[1 - F_1(t)]\} \int_{\underline{t}}^{t} \beta_{II}^{BB}(x) f_1(x)\, dx$, $\mathcal{T}_I^{BB}(t) := \beta_I^{BB}(t) F_1(t) \{1 - \Lambda\,[1 - F_1(t)]\}$, $k \in \{I,\ II\}$ and $\Omega_I(t) = \Omega(t)$.

Notice that equation (44) implies that

$$\Lambda\Omega_I(t)/N + \mathcal{T}_I^{BB}(t) = \Lambda\Omega_{II}(t)/N + \mathcal{T}_{II}^{BB}(t). \qquad (45)$$

Substituting and re-arranging, it is straightforward to see that (45) is equivalent to

$$\{1 - \Lambda\,[1 - F_1(t)]\} \left[F_1(t)\, \beta_I^{BB}(t) - \int_{\underline{t}}^{t} \beta_{II}^{BB}(x) f_1(x)\, dx \right] = \Lambda\,\{\Omega_{II}(t) - \Omega_I(t)\}/N. \quad ■$$

Proof of Proposition 12: We begin by proving the first statement. With a slight abuse of

notation we denote by $EU(t)$ type-t bidder's indirect equilibrium utility. We know that

$$EU_{II}^{CARA,CV}(t) = EU_{II}^{CARA,PV}(t) = EU_{I}^{CARA,PV}(t) = EU_{I}^{CARA,CV}(t)$$

where the first and last equalities follow from Eső and White (2004) and the middle one follows from Matthews (1983). Hence,

$$\begin{aligned} EU_{II}^{CARA,CV}(t) &= EU_{I}^{CARA,CV}(t) \quad \Leftrightarrow \\ U(CE_{II}(t)) &= U(CE_{I}(t)) \quad \Leftrightarrow \\ CE_{II}(t) &= CE_{I}(t) \end{aligned}$$

where CE is the certainty equivalent. Now observe that in equilibrium, that is, for a fixed bidding strategy β_k^{CARA}, $k \in \{I, II\}$, a bidder of type t_i faces a lottery over basic outcomes. Let $X_k^{t_i}$ denote the random variable associated with this lottery in the FPA and the SPA, respectively. For the FPA, we have that $X_I^{t_i} = t_i + \sum_{j \neq i} t_j - \beta_I^{CARA}(t_i)$ if t_i is larger than the signal of i's strongest opponent, that is, if $t_i > t_1$, and $X_I^{t_i} = 0$ otherwise. Similarly, in the SPA, we have that $X_{II}^{t_i} = t_i + \sum_{j \neq i} t_j - \beta_{II}^{CARA}(t_1)$ if $t_i > t_1$ and $X_{II}^{t_i} = 0$ otherwise. The FPA gives rise to more intensive risk than the SPA iff $Var\left(X_I^{t_i} | X_I^{t_i} > 0\right) \geq Var\left(X_{II}^{t_i} | X_{II}^{t_i} > 0\right)$. Both auction formats give rise to the same extensive risk. That is, the probability that outcome 0 realizes is the same in both auction formats. Therefore, $Var\left(X_I^{t_i} | X_I^{t_i} > 0\right) \geq Var\left(X_{II}^{t_i} | X_{II}^{t_i} > 0\right)$ iff $Var\left(X_I^{t_i}\right) \geq Var\left(X_{II}^{t_i}\right)$. As the bidder's utility function is concave and $CE_{II}(t) = CE_I(t)$, it must be that

$$\begin{aligned} Var\left(X_I^{t_i} | X_I^{t_i} > 0\right) &\geq Var\left(X_{II}^{t_i} | X_{II}^{t_i} > 0\right) \\ \Leftrightarrow Var\left(X_I^{t_i}\right) &\geq Var\left(X_{II}^{t_i}\right) \\ \Leftrightarrow \mathbb{E}\left[X_I^{t_i}\right] &\geq \mathbb{E}\left[X_{II}^{t_i}\right] \\ \Rightarrow \mathbb{E}\left[\beta_{II}^{CARA}(t_1)\right] &\geq \mathbb{E}\left[\beta_I^{CARA}(t)\right]. \end{aligned} \tag{46}$$

Finally, as a bidder's expected payment is higher in the auction that leads to fewer intensive risk for any type, it follows that the seller's expected revenue is also higher in that auction.

Next, we turn to the last statement. For the SPA, the result follows from Milgrom and Weber (1982). For the FPA, we have

$$F_1(t)\,\beta_I^{RN}(t) = \int_{\underline{t}}^{t} \beta_{II}^{RN}(s)\, f_1(s)\, ds > \int_{\underline{t}}^{t} \beta_{II}^{CARA}(s)\, f_1(s)\, ds > F_1(t)\,\beta_I^{CARA}(t)$$

where the first inequality follows from Milgrom and Weber (1982) and the second one follows from (46). ■

B Relaxing Assumption 1

In this section we briefly show how to amend the analysis of the FPA and SPA if Assumption 1 does not hold.[19] Recall that Assumption 1 ensures that the weight a bidder places on expected

[19] We do this for the narrow bracketing specification, but similar results apply under broad bracketing.

gain-loss utility does not (strictly) exceed the weight he puts on consumption utility. When this assumption is violated, a loss-averse agent dislikes uncertainty in his consumption outcomes so much that he might select first-order stochastically-dominated options.[20] That is, holding everything else fixed, the expected utility of a bidder that expects to win the auction with a rather low probability decreases in the probability of winning the good. As a consequence, single-crossing fails for low-type bidders. Thus, the equilibrium involves pooling at the bottom. Let t^* denote the unique interior solution to the following equation:

$$\gamma(t)\left[t+q(t)\right]-\Lambda^g\Omega(t)=0. \tag{47}$$

The following proposition describes the equilibrium strategy in the FPA and SPA when Assumption 1 does not hold.

Proposition 13. *Assume $\Lambda^g>1$. Let $\gamma^*(t):=1\left[t\geq t^*\right]\gamma(t)$, $q^*(t):=1\left[t\geq t^*\right]q(t)$ and $\Omega^*(t):=1\left[t\geq t^*\right]\Omega(t)$. Then, in both the FPA and the SPA all types below t^* submit a bid equal to zero. In the FPA all types (weakly) above t^* bid according to the following strategy*

$$\tilde{\beta}_I^*(t)=\frac{\gamma^*(t)\left[t+q^*(t)\right]-\Lambda^g\Omega^*(t)-\int_{\underline{t}}^{t}\gamma^*(s)\,ds}{NF_1(t)\left\{1+\Lambda^m\left[1-F_1(t)\right]\right\}}.$$

Similarly, in the SPA all types (weakly) above t^* bid according to the strategy for the case with $\Lambda^g\leq 1$ when replacing $\gamma(t)$, $q(t)$ and $\Omega(t)$ with $\gamma^*(t)$, $q^*(t)$ and $\Omega^*(t)$, respectively and defining $\gamma^{*\prime}(t)$, $q^*(t)'$ and $\Omega^{*\prime}(t)$ as right-hand-side derivatives.

Proof of Proposition 13: Recall that in both the FPA and SPA the following holds

$$EU\left(\tilde{t},t\right)=\gamma^*\left(\tilde{t}\right)\left[t+q^*\left(\tilde{t}\right)\right]/N-\Omega^*\left(\tilde{t}\right)/N-\mathcal{T}_k^*\left(\tilde{t}\right),$$

where $k\in\{I,\ II\}$, and $\mathcal{T}_k^*\left(\tilde{t}\right)$ is defined by the posited bidding strategies.

First, notice that bidding according to his signal is optimal for type t^*. Indeed, mimicking a bidder with a lower signal would yield a payoff of zero just like reporting truthfully would. Suppose then that the bidder mimics a bidder with a higher signal than his. As shown in the proof of Proposition 1 such a deviation is not profitable whenever $\Lambda^g\left[1-2F_1\left(\tilde{t}\right)\right]<1$. Furthermore, we know that $\gamma(t^*)\left[t^*+q(t^*)\right]-\Omega(t^*)=0$. As $\Omega(t^*)$ is strictly positive, it follows that $\gamma(t^*)$ must be strictly positive as well and this implies that $\Lambda^g\left[1-2F_1(t^*)\right]<1$.

Next, consider a bidder with a signal lower than t^*. Such a bidder prefers to lose the auction with certainty rather than winning with probability (weakly) less than $F_1(t^*)$. Moreover, such a type, say t', never wants to deviate globally and mimic type $t''>t^*$ because

$$EU\left(t'',t'\right)\leq EU\left(t'',t^*\right)\leq EU\left(t^*,t^*\right)=0=EU\left(t',t'\right).$$

Now consider a bidder with a signal weakly higher than t^*. The posited equilibrium bidding functions satisfy the first-order conditions by construction. The second-order conditions are satisfied by much the same argument as for type t^*.

[20] To see why, consider a loss-averse agent who is deciding whether to accept a 50-50 gamble between winning \$100 or \$0. Under CPE the agent would reject the gamble whenever $\eta(\lambda-1)>2$.

Next, we show that the bidding function is increasing. Applying an argument similar to the one in the proof of Proposition 1, we see that the bidding function is increasing for $t \in [t^*, \overline{t}]$ if $\Lambda^g [1 - 2F_1(t)] < 1$. As F_1 is increasing, the result follows.

Finally, it remains to show that the solution to (47) is unique. By contradiction, suppose there is more than one solution. Let t^* be the smallest solution and t' be the largest one. Then, it must be that

$$\gamma(t')[t' + q(t')] - \Lambda^g \Omega(t') = 0 = \gamma(t^*)[t^* + q(t^*)] - \Lambda^g \Omega(t^*)$$

However, observe that $EU(t^*, t') = \gamma(t^*)(t' - t^*) + EU(t^*, t^*) > 0$. Moreover, by incentive compatibility, $EU(t', t') \geq EU(t^*, t') > 0$. Because $\tilde{\beta}_k^*(t)$ is increasing and $\tilde{\beta}_k^*(t^*) = 0$, $EU(t', t') > 0$ implies that $\gamma(t')[t' + q(t')] - \Lambda^g \Omega(t') > 0$, yielding a contradiction. ■

As in the private-value case, if Assumption 1 fails, the equilibrium strategy calls for partial pooling whereby some bidders with strictly positive types choose not to participate in the auction and bid zero instead (see Lange and Ratan, 2010). Therefore, only bidders with a sufficiently large probability of winning the auction place positive bids. The intuition for this is that bidders with relatively low signals do not expect to win with a high probability and hence prefer to eliminate all the uncertainty in their consumption outcomes by not participating in the auction. These are those bidders for whom the left-hand-side of (47) is negative: their expected gain-loss disutility is so large that they would attain a negative total payoff even if they were to obtain the good for free. Moreover, notice that with common values the threshold type t^* is larger compared to the case of private values. The reason is that in a common-value environment bidders face risk not only with respect to whether or not they get the good for sale, but also with respect to what the good's value is. Hence, when Assumption 1 fails more types choose not to participate in the auction in an environment with common values than in one with private values.

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