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Working Paper Series

'Dutch versus First-Price Auctions with Dynamic Expectations-Based Reference-Dependent Preferences'

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No. 05 / 2020 **June 2020**

Dutch versus First-Price Auctions with Dynamic Expectations-Based Reference-Dependent Preferences*

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June 7, 2020

Abstract

We study the behavior of expectations-based loss-averse bidders in Dutch and first-price auctions with independent private values. With loss-averse preferences, the strategic equivalence between these formats no longer holds. Intuitively, as the Dutch auction unfolds, a bidder becomes more optimistic about her chances of winning; this stronger "attachment" effect pushes her to bid more aggressively than in the first-price auction. Thus, Dutch auctions raise more revenue than first-price ones. Indeed, we show that the Dutch auction raises the most revenue among standard auction formats. Our results imply that with expectations-based reference-dependent preferences sequential mechanisms might outperform static ones.

JEL classification: D44, D81, D82.

Keywords: Reference-Dependent Preferences; Loss Aversion; Dutch Auctions; Revenue Equivalence; Personal Equilibrium.

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*We thank Jeff Ely, Botond Kőszegi, Marco Pagnozzi and audiences at the 2019 Sydney Workshop on Experimental Economics and Theory, the 2019 Organizational Economics Summer Symposium at the University of Sydney, QUT, UTS, UQ, the 2019 Australasian Economic Theory Workshop in Adelaide and the Berlin Theory Colloquium for helpful comments and suggestions. Antonio Rosato gratefully acknowledges financial support from the Australian Research Council (ARC) through the ARC Discovery Early Career Researcher Award DE180100190. The first two authors also thank the 'healthy' Café in Manly for its great hospitality while some of the work on this paper was completed.

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1 Introduction

The static first-price auction (FPA) and its dynamic counterpart, the Dutch auction, are among the most prominent auction formats. A central result in auction theory is that these two formats are strategically equivalent. The crucial insight, due originally to Vickrey (1961), is that the information bidders obtain during the Dutch auction does not affect their optimal strategies; therefore, bidders choose their bids solely based on their prior information. Indeed, the equivalence between these two formats holds in many different environments (e.g., independent or correlated private values, pure common values, interdependent values with affiliated signals, etc...), even under risk aversion. The strategic equivalence further implies that the two auction formats generate the same expected revenue. However, evidence from both laboratory and field experiments shows that revenue equivalence may fail. For instance, Lucking-Reiley (1999) conducts a field experiment by selling *Magic* game cards via Internet auctions and reports that the Dutch auction produces 30-percent higher revenues than the FPA. Katok and Kwasnica (2008) obtain similar results in a laboratory experiment when the price in the Dutch auction drops slowly. These studies suggest that the Dutch auction tends to generate more revenue than the FPA, especially if the price clock of the Dutch auction is relatively slow.[1]

In this paper, we provide a novel explanation for the strategic (and hence, revenue) non-equivalence between the FPA and the Dutch auction based on reference-dependent preferences and loss aversion. We analyze both auction formats in a symmetric environment where bidders have independent private values (IPV) and are expectations-based loss averse à la Kőszegi and Rabin (2006, 2007, 2009).[2] We show that loss-averse bidders bid more aggressively in the Dutch auction than in the FPA. Intuitively, the larger the probability with which a loss-averse bidder expects to win the prize, the stronger her incentives to raise her bid in order to avoid experiencing disappointment from losing the auction. This is what Kőszegi and Rabin (2006) call "attachment effect". We argue that, although the two auction formats select the same winner, they create different levels of attachment for the bidders. Consider, for instance, a bidder with a fairly low value. When submitting her bid in the FPA, she knows it is quite likely that one of her opponents has a higher value. Thus, she is rather pessimistic about her chances of winning the auction and not very attached to the prize; therefore, she does not have a strong incentive to bid high. In contrast, consider the same bidder participating in a Dutch auction and imagine the clock is only slightly above the price at which she had originally planned to buy. By now she has updated her beliefs about her strongest opponent's value and is very optimistic that it is below hers – after all,

[1] However, earlier experiments by Coppinger *et al.* (1980) and Cox *et al.* (1982) report higher revenues for FPA than for Dutch; Cox *et al.* (1983) attribute this finding to probability miscalculations in the Dutch auction.

[2] For experimental evidence on Kőszegi and Rabin's model see Abeler *et al.* (2011), Ericson and Fuster (2011), Gill and Prowse (2012), Banerji and Gupta (2014), Heffetz and List (2014), Karle *et al.* (2015), Sprenger (2015), Zimmermann (2015), Gneezy *et al.* (2017), Smith (2019), Cerulli-Harms *et al.* (2019), and Rosato and Tymula (2019). While most of the evidence indicates that expectations play an important role in shaping reference points, a few studies have also documented some violations of the model's directional predictions.

if (one of) her opponents had a much larger valuation than hers, they would have already stopped the clock. Thus, she is very much attached to the prize. In this case, the bidder has a strong incentive to raise her bid and stop the clock at an earlier price in order to reduce the chances of experiencing a loss if another bidder stops the clock before her. In other words, the bidding strategy of a loss-averse bidder in the FPA is shaped by the attachment effect arising from her *initial* beliefs about how likely she is to win the auction. In the Dutch auction, in contrast, she becomes increasingly more optimistic about her chances of winning as the auction unfolds; this creates a stronger attachment effect which induces her to bid more aggressively than in the FPA.

As the theoretical equivalence between the Dutch auction and the FPA holds for many different environments, some authors have suggested that its empirical breakdown might be caused by non-standard risk preferences. Karni (1988) is the first to point out that these two formats are equivalent if and only if bidders are expected-utility maximizers. Nakajima (2011) considers bidders whose preferences exhibiting the Allais paradox (Allais, 1953) and shows that the Dutch auction systematically yields more revenue than the FPA.[3] Auster and Kellner (2019) obtain the same result for ambiguity-averse bidders. Another strand of literature, however, attributes the breakdown of the FPA-Dutch equivalence to bidders' time preferences. In fact, in those studies where the Dutch generates a higher revenue than the FPA, the clock of the Dutch auction is rather slow. Katok and Kwasnica (2008) and Carare and Rothkopf (2005) explain this observation by appealing to bidders' impatience or their opportunity cost of the time spent in the Dutch auction. While our model belongs to the realm of non-expected-utility preferences, it also relates to this second explanation as the speed of the Dutch auction's clock likely affects the adjustment of the reference point. Indeed, as the adjustment of the reference point might require some time, a slower clock should provide bidders with enough time to update their beliefs.

Section 2 describes the auction environment, bidders' preferences, and solution concept. We consider a standard symmetric environment with independent private values where bidders have expectations-based reference-dependent preferences as in Kőszegi and Rabin (2009). Hence, in addition to classical material utility, a bidder experiences "gain-loss utility" from comparing her material outcomes to a reference point equal to her (rational) beliefs about these outcomes, as well as "news utility" from updating her reference point from old to new beliefs; both gain-loss and news utility attach a higher weight to losses than to equal-size gains. We focus on symmetric equilibria in increasing strategies; thus, the bidder with the highest value wins the auction.

In Section 3 we begin our analysis by characterizing the equilibrium strategy of loss-averse bidders in the FPA and illustrating how the attachment effect operates. We show that the attachment effect pushes loss-averse bidders to bid more aggressively than their risk-neutral counterparts. Indeed, because bidders keep their reference point fixed when choosing their strategy, they are willing to bid more in order to reduce their chances of losing the auction.

[3] Weber (1982) shows that the FPA yields a higher revenue than the Dutch auction when bidders' preferences exhibits the *counter* Allais paradox.

Next, we turn to the Dutch auction. Here, the main intricacy in characterizing the equilibrium is a form of beliefs-based time inconsistency that arises even though bidders' preferences are time consistent.[4] That is, the price at which a bidder stops the clock in equilibrium need not be – and in general it is not – the price at which the bidder would have preferred to stop the clock from the outset. This happens because, as the auction unfolds and the reference point adjusts, the bidder is tempted to "surprise" herself by stopping the clock earlier or later than originally planned. In equilibrium, however, the bidder's plan must be consistent with her expectations so that she stops the clock exactly at the price at which she had planned to do so. We show that there can be multiple consistent bidding plans and identify the one that provides bidders with the highest utility from an ex-ante perspective.

Finally, we compare the equilibrium strategies of the two formats and show that loss-averse bidders bid more aggressively in the Dutch auction than in the FPA. An immediate corollary is that the Dutch auction raises more revenue than the FPA. More generally, we argue that managing buyers' expectations is crucial for the performance of a selling mechanism, and show that the expected revenue of the four standard auctions ranks as follows: Dutch > FPA = second-price auction (SPA) > English. Indeed, buyers' beliefs about their likelihood of winning at the time of bidding, and hence their attachment, coincide in the static FPA and SPA. By contrast, attachment is the weakest in the English auction as bidders become increasingly more pessimistic about their likelihood of winning as the auction unfolds.

Section 4 concludes the paper by discussing the implications of our results for the existing literature on reference dependence and mechanism design. In particular, we highlight that with expectations-based reference-dependent preferences, two mechanisms that allocate the prize to the same bidder might still result in different payoffs for both the bidders and the seller, depending on how the allocation is implemented. Hence, the "Revelation Principle" might fail when bidders have expectations-based reference-dependent preferences.

2 The Model

In this section, we describe the environment, bidders' preferences, and solution concept.

2.1 Environment

An indivisible item is auctioned off to $N \geq 2$ bidders. Each bidder $i \in \{1, \ldots, N\}$ has a private value $\theta_i \in \Theta := \left[\underline{\theta}, \overline{\theta}\right] \subseteq \mathbb{R}_+$. Values are independently and identically distributed across bidders according to a CDF $F : \Theta \to [0, 1]$ admitting a continuous PDF f. Let F_1 and f_1 respectively denote the CDF and PDF of the highest order statistic among $N - 1$; similarly, let $F_1(\cdot|x)$ and $f_1(\cdot|x)$ respectively denote its CDF and PDF conditional on being lower than x.

[4] See Kőszegi and Rabin (2009) and Pagel (2016, 2017) for further elaborations.

In the FPA, bidders simultaneously submit sealed bids; the highest bidder wins the auction and pays her bid. Regarding the Dutch auction, we assume that the clock starts at some sufficiently high price (e.g., higher than $\overline{\theta}$) and then drops in small steps of size $\varepsilon > 0$. The first bidder who stops the clock wins the auction and pays the price displayed on the clock.

2.2 Preferences and Solution Concept

Throughout the paper, we restrict attention to symmetric pure-strategy equilibria with strictly increasing, differentiable bidding functions, $\beta : \Theta \to \mathbb{R}_+$. Consider a type-$\theta$ bidder bidding (in either the FPA or the Dutch auction) as if her type were $\tilde{\theta} \neq \theta$. If she wins the auction, she obtains an item she values θ and pays the price $\beta(\tilde{\theta})$; denote this outcome by $(\theta, \beta(\tilde{\theta}))$. If she loses the auction, she gets nothing and pays nothing; denote this outcome by $(0, 0)$. Hence, the set of material outcomes is $\tilde{\mathcal{O}} = \{(\theta, \beta(\tilde{\theta})), (0, 0)\}$ and the bidder's possible material payoffs are $\theta - \beta(\tilde{\theta})$ and 0, respectively. Following Kőszegi and Rabin (2006, 2007, 2009) we assume that, in addition to classical material utility, the bidder also derives psychological gain-loss utility from comparing her material outcomes to a reference outcome given by her recent expectations (probabilistic beliefs). If the bidder *plans* to bid $\beta(\theta)$, her reference outcomes are $\mathcal{O} = \{(\theta, \beta(\theta)), (0, 0)\}$ and her reference point at any point in time is a distribution over the set of reference outcomes $\mathcal{O}$. We first elaborate on the reference point of a type-θ bidder *at the beginning* of the auction as induced by the bidding strategy β. Since, in a symmetric equilibrium, the bidder wins with probability $F_1(\theta)$ if she plans to bid $\beta(\theta)$, her reference point is given by

$$r = \begin{cases} (\theta, \beta(\theta)) & \text{with probability} \quad F_1(\theta) \\ (0, 0) & \text{with probability} \quad 1 - F_1(\theta) \end{cases}.$$

Moreover, the bidder updates her reference point based on the arrival of new information about her material outcomes. In the FPA, updating only takes place at the end of the auction when the bidder learns whether or not she won and her beliefs become degenerate. In the Dutch auction, instead, at each price drop the bidder observes whether an opponent stopped the clock and updates her beliefs about the opponents' types (and hence her likelihood of winning) accordingly. If at price $\beta(\theta') > \beta(\theta)$ the auction is still running, a type-θ bidder updates her likelihood of winning — given her plan to stop the clock at price $\beta(\theta)$ — to $F_1(\theta|\theta')$. Such updating of the reference point induces itself psychological gains and/or losses. In particular, following Kőszegi and Rabin (2009), we assume that the bidder makes an "ordered comparison" percentile-by-percentile between her previous beliefs and her new ones.[5] Formally, for any $p \in (0, 1)$ let $c_r(p)$ and $c_{\tilde{r}}(p)$ denote the consumption levels at percentile p under two reference point's distributions r and $\tilde{r}$, respectively. The gain-loss utility arising from updating the reference point from $\tilde{r}$ to r in dimension $k \in \{g, m\}$

[5] Kőszegi and Rabin (2009) call this "news" utility or "prospective" gain-loss utility; we assume that bidders place the same weight on prospective and contemporaneous gain-loss utility. For a slightly different definition of prospective gain-loss utility see Pagel (2019).

is defined as follows:

$$N(r,\tilde{r}) = \sum_{k\in\{g,m\}} \int_0^1 \mu^k(c_r(p) - c_{\tilde{r}}(p))dp.$$

Following most of the literature, we assume that the gain-loss function μ^k is piecewise linear:

$$\mu^k(x) = \begin{cases} \eta^k x & \text{if } x \geq 0 \\ \eta^k \lambda^k x & \text{if } x < 0 \end{cases}$$

with $\eta^k > 0$ and $\lambda^k > 1$ for $k \in \{g,m\}$.

For instance, consider the gain-loss utility of a type-θ bidder when the clock of the Dutch auction drops from price $\beta(\theta')$ to price $\beta(\theta'')$. If no opponent buys in this time interval, then the probability which the bidder expects to win increases by $F_1(\theta|\theta'') - F_1(\theta|\theta')$. Hence, the bidder experiences a gain in the item dimension and a loss in the money dimension equal to:

$$N = \eta^g \left[F_1(\theta|\theta'') - F_1(\theta|\theta')\right]\theta - \eta^m\lambda^m \left[F_1(\theta|\theta'') - F_1(\theta|\theta')\right]\beta(\theta).$$

Let $U(\tilde{\theta}|\theta,\theta')$ denote a type-θ bidder's total expected utility when the current clock price is $\beta(\theta')$ and the bidder — who had planned to stop the clock at price $\beta(\theta) < \beta(\theta')$ — is considering to deviate by stopping the clock at price $\beta(\tilde{\theta}) < \beta(\theta')$. We impose the restriction that a bidder's strategy is credible given the reference point it generates:

Definition 1. *A strategy $\beta(\theta)$ is a personal equilibrium (PE) for a bidder with type θ if, taking as given the distribution of bids induced by $\beta(\theta)$, for all $\theta' \geq \theta$ it holds that*

$$U(\theta|\theta,\theta') \geq U(\tilde{\theta}|\theta,\theta'),$$

for any credible deviation $\tilde{\theta} < \theta'$.

The restriction to credible strategies (and deviations) is important. Indeed, notice that at price $\beta(\theta')$ a bidder might be tempted to deviate from her equilibrium strategy of stopping the clock at price $\beta(\theta)$ to an alternative strategy — such as, for instance, stopping the clock at some other price $\beta\left(\widehat{\theta}\right)$ — even though she would not carry this plan through when it is time to execute it. The reason is that the bidder might enjoy additional psychological gain-loss utility from the change in the reference point caused by non-credible deviations; once the reference point has adjusted to the new plan, however, the bidder might want to deviate again (and again...). The restriction to credible strategies implies that a bidder will only entertain a plan that she is willing to follow through given the reference point implied by the plan.

We can now define our solution concept for the auction game:

Definition 2. *A bidding function β constitutes a symmetric personal equilibrium if for each type θ, given the knowledge that opponents bid according to β, the strategy $\beta(\theta)$ is a personal equilibrium.*

In the FPA, where bidders submit sealed bids at the beginning of the auction, updating of the reference point happens only when the auction is over. Hence, in the FPA a bidding strategy $\beta(\theta)$ is a personal equilibrium if and only if at the beginning of the auction $U(\theta|\theta) \geq U(\tilde{\theta}|\theta)$ for all $\tilde{\theta}$. Finally, as there can be multiple symmetric personal equilibria, we assume bidders collectively select the one yielding the highest expected utility — the "preferred personal equilibrium" (PPE).

3 Analysis

In this section, we derive the equilibrium bidding strategies in the FPA and Dutch auction and highlight how the attachment effect affects the incentives of loss-averse bidders. In particular, the magnitude of the attachment effect depends on how optimistic a bidder is at the time of submitting her bid; this, in turn, will imply that the Dutch auction raises more revenue than the FPA.

3.1 Equilibrium Bidding in the FPA

Consider a type-θ bidder who has planned to bid $\beta_I(\theta)$ but deviates by mimicking a bidder with type $\tilde{\theta} \geq \theta$.[6] In this case, her expected payoff is:

$$\begin{aligned} U(\tilde{\theta}|\theta) &= F_1(\tilde{\theta})\left[\theta - \beta_I(\tilde{\theta})\right] - \eta^g\lambda^g\left[1-F_1(\tilde{\theta})\right]F_1(\theta)\theta + \eta^g F_1(\tilde{\theta})\left[1-F_1(\theta)\right]\theta \\ &+\eta^m\left[1-F_1(\tilde{\theta})\right]F_1(\theta)\beta_I(\theta) - \eta^m\lambda^m F_1(\tilde{\theta})\left[1-F_1(\theta)\right]\beta_I(\tilde{\theta}) - \eta^m\lambda^m F_1(\tilde{\theta})F_1(\theta)\left[\beta_I(\tilde{\theta})-\beta_I(\theta)\right]. \end{aligned} \tag{1}$$

The first term in (1) represents the standard expected material payoff. The other terms capture expected gain-loss utility and are derived as follows. The second term captures the loss in the item dimension for a bidder who expected to win the auction with probability $F_1(\theta)$ but ends up losing it — an event happening with probability $1-F_1(\tilde{\theta})$ — and thus experiences a loss equal to $\eta^g\lambda^g F_1(\theta)\theta$. Similarly, the third term captures the gain in the item dimension for a bidder who expected to lose with probability $1-F_1(\theta)$ but ends up winning — an event happening with probability $F_1(\tilde{\theta})$ — and thus experiences a gain equal to $\eta^g\left[1-F_1(\theta)\right]\theta$. The fourth and fifth terms capture the corresponding expected gains and losses in the money dimension. The final term captures the loss in the money dimension when winning at a price higher than expected. Differentiating (1) with respect to $\tilde{\theta}$ and evaluating the resulting first-order condition at $\tilde{\theta}=\theta$ yields a differential equation whose solution provides us with the equilibrium bidding strategy:

Proposition 1. *The symmetric PPE bidding strategy in the FPA is given by*

$$\beta_I(\theta) = \int_{\underline{\theta}}^{\theta} \frac{1+\eta^g\lambda^g F_1(x) + \eta^g\left[1-F_1(x)\right]}{F_1(\theta)\left(1+\eta^m\lambda^m\right)} e^{\frac{\eta^m(\lambda^m-1)[F_1(\theta)-F_1(x)]}{1+\eta^m\lambda^m}} x f_1(x)dx. \tag{2}$$

[6] Balzer and Rosato (2019) show that upward deviations are the most relevant ones.

Balzer and Rosato (2019) derived the symmetric PPE bidding function for an environment with interdependent values and independent signals; as the IPV model is a special case of theirs, applying their characterization result to our environment yields expression (2).

Next, we compare the loss-averse bidding strategy with the risk-neutral benchmark. Let $\eta^m = \lambda^m = 0$.[7] Then, the equilibrium bidding function becomes

$$\beta_I(\theta) = \mathbb{E}_{F_1}[v(x)|x \leq \theta], \tag{3}$$

where

$$v(x) = \{1 + \eta^g \lambda^g F_1(x) + \eta^g [1 - F_1(x)]\} x.$$

Since $v(x) > x$ for any $\eta^g > 0$, every type overbids compared to the risk-neutral benchmark. The term $v(x)$ can be interpreted as the "opportunity value "of winning the auction for a bidder with type x. In addition to classical utility the bidder experiences a gain of $\eta^g [1 - F_1(x)] x$ and escapes the counterfactual disappointment of losing and receiving a loss of $\eta^g \lambda^g F_1(x)x$. Hence, as in the risk-neutral framework, bidders in equilibrium bid the expectation of their strongest opponent's valuation conditional on winning, where, however, "valuation" must be understood as the belief-dependent opportunity value of winning. Importantly, note that the opportunity value is taken with respect to the ex-ante beliefs; this will be fundamentally different in the Dutch auction.

3.2 Equilibrium Bidding in the Dutch Auction

Differently from the FPA, the Dutch auction is a dynamic format where a bidder's beliefs (and hence her reference point) evolve throughout the auction. Moreover, when she submits her bid in the FPA, the bidder is unsure about whether she will win; in the Dutch auction, instead, when she submits her bid by stopping the clock, the bidder is sure to win.

In a symmetric equilibrium, a type-θ bidder stops the clock at price $\beta_D(\theta)$. In particular, the bidder prefers executing this plan over switching to any other credible plan at any point in time. Suppose the current clock price is $\beta_D(\theta') > \beta_D(\theta)$ and a type-θ bidder considers deviating to another plan, $\beta_D(\tilde{\theta}) > \beta_D(\theta)$.[8] In this case, her expected payoff is:

$$\begin{aligned} U(\tilde{\theta}|\theta, \theta') &= F_1(\tilde{\theta}|\theta')[\theta - \beta_D(\tilde{\theta})] + \eta^g \theta \left[F_1(\tilde{\theta}|\theta') - F_1(\theta|\theta')\right] \\ &\quad - \eta^m \lambda^m [F_1(\tilde{\theta}|\theta')\beta_D(\tilde{\theta}) - F_1(\theta|\theta')\beta_D(\theta)] + \mathbb{E}\left[N(\tilde{\theta}|\theta, \theta')\right] \end{aligned} \tag{4}$$

where $\mathbb{E}\left[N(\tilde{\theta}|\theta, \theta')\right]$ is the expected news utility of a type-θ bidder generated between price $\beta_D(\theta')$ and price $\beta_D(\tilde{\theta})$ given the new plan to buy at price $\beta_D(\tilde{\theta})$.

[7] Throughout the paper, when describing the intuition behind our results, we sometimes abstract from loss aversion over money in order to simplify the exposition; yet, the same intuition applies more generally.

[8] As for the FPA, upward deviations are the most relevant ones.

The first term on the right-hand side of (4) is the standard expected material payoff. The other terms capture expected gain-loss utility. By deviating from her plan to buy at price $\beta_D(\theta)$ to the new plan of buying at price $\beta_D(\tilde{\theta})$, the bidder's probability of winning increases from $F_1(\theta|\theta')$ to $F_1(\tilde{\theta}|\theta')$. Hence, by deviating she experiences a gain in the item dimension equal to $\eta^g\theta[F_1(\tilde{\theta}|\theta') - F_1(\theta|\theta')]$. At the same time, however, the bidder also increases her expected payment from $F_1(\theta|\theta')\beta_D(\theta)$ to $F_1(\tilde{\theta}|\theta')\beta_D(\tilde{\theta})$, thereby experiencing a loss in the money dimension equal to $\eta^m\lambda^m[F_1(\tilde{\theta}|\theta')\beta_D(\tilde{\theta}) - F_1(\theta|\theta')\beta_D(\theta)]$. The last term on the right-hand side of (4) captures news utility; that is, the expected gain-loss utility stemming from changes in beliefs and the resulting updating of the reference point as the auction unfolds. The next result allows to re-write this expression in terms of the model's primitives.

Lemma 1. *Let the current clock price be $\beta_D(\theta')$ and consider a bidder of type θ planning to stop the clock at price $\beta_D(\tilde{\theta}) < \beta_D(\theta')$. For $\varepsilon \to 0$, the following equality holds:*

$$\mathbb{E}\left[N(\tilde{\theta}|\theta,\theta')\right] = -\left[\eta^g(\lambda^g - 1)\theta + \eta^m(\lambda^m - 1)\beta_D(\tilde{\theta})\right]\int_{\tilde{\theta}}^{\theta'} f_1(x|\theta')F_1(\tilde{\theta}|x)dx. \tag{5}$$

The term on the right-hand side of (5) is a natural generalization of the static expected gain-loss utility to a dynamic environment. We discuss it by focusing on the risk in the item dimension, but a similar intuition applies for the money dimension. From the perspective of a bidder who is active at price $\beta_D(\theta')$, at any future price $\beta_D(x) \in \left[\beta_D(\tilde{\theta}), \beta_D(\theta')\right]$ only one of the following events can realize. Either the auction continues and the bidder learns that her strongest opponent's type is below x which, given the current price is $\beta_D(\theta')$, happens with probability $F_1(x|\theta')$; in this case, the bidder updates her beliefs and her probability of winning increases by $-\frac{\partial}{\partial x}F_1(\tilde{\theta}|x) = f_1(x|x)F_1(\tilde{\theta}|x)$, generating a gain equal to $\eta^g f_1(x|\theta')F_1(\tilde{\theta}|x)$. Alternatively, the auction ends and the bidder learns that her strongest opponent's type is exactly x which, given the current price is $\beta_D(\theta')$, happens with (marginal) probability $f_1(x|\theta')$; in this case, she learns that she lost and her beliefs about winning drop from $F_1(\tilde{\theta}|x)$ to zero, generating a loss equal to $\eta^g\lambda^g f_1(x|\theta')F_1(\tilde{\theta}|x)$.

An equilibrium bid is a credible plan of when to stop the clock such that, at any point during the auction, the bidder prefers executing it over switching to another credible plan. Verifying that an equilibrium bid is indeed a credible plan is technically tedious and so we relegate it to Appendix A. Yet, equilibrium behavior is rather intuitive: at any price $\beta_D(\theta') > \beta_D(\theta)$ a type-θ bidder prefers to stay in the auction instead of buying immediately; hence, $U(\theta|\theta,\theta') \geq U(\theta'|\theta,\theta')$. In Appendix A we show that letting $\theta' \to \theta$ yields a lower bound on the derivative of the bidding function. Making this lower bound bind and solving the resulting differential equation provides us with the equilibrium bidding strategy:

Proposition 2. *The symmetric PPE bidding strategy in the Dutch auction is given by*

$$\beta_D(\theta) = \int_{\underline{\theta}}^{\theta} \frac{1+\eta^g\lambda^g}{F_1(\theta)\left(1+\eta^m\lambda^m\right)}\left[\frac{F_1(\theta)}{F_1(x)}\right]^{\frac{\eta^m(\lambda^m-1)}{1+\eta^m\lambda^m}} x f_1(x)dx. \tag{6}$$

Again, let $\eta^m = \lambda^m = 0$. Then, the equilibrium bidding function simplifies to

$$\beta_D(\theta) = (1 + \eta^g \lambda^g)\mathbb{E}_{F_1}[x | x \leq \theta]. \tag{7}$$

Compared to the risk-neutral benchmark every bidder overbids by a factor $1 + \eta^g \lambda^g$. Again, bidders bid the expectation of their strongest opponent's opportunity value, where this value now is given by

$$v(x) = (1 + \eta^g \lambda^g)x.$$

Indeed, in the Dutch auction, when stopping the clock at an earlier price, the bidder is certain to win. This magnifies the attachment effect compared to the FPA, where by deviating to a higher bid, the bidder is "only" marginally increasing her chances of winning.

3.3 Revenue Comparison

We now show that, by creating a stronger attachment effect, the Dutch auction raises more revenue than the FPA.

Suppose $\eta^m = \lambda^m = 0$ and consider the incentive constraint of a type-θ bidder who contemplates mimicking type $\theta' > \theta$. In equilibrium, $U(\theta|\theta) \geq U(\theta'|\theta)$; hence, by condition (1), the following must hold:

$$F_1(\theta')\beta_I(\theta') - F_1(\theta)\beta_I(\theta) \geq [F_1(\theta') - F_1(\theta)]\,\theta + \eta^g \theta [F_1(\theta') - F_1(\theta)]\,[1 + (\lambda^g - 1)\,F_1(\theta)]\,. \tag{8}$$

Similarly, in the Dutch auction for any price $\beta_D(\theta') > \beta_D(\theta)$ it holds in equilibrium that $U(\theta|\theta,\theta') \geq U(\theta'|\theta,\theta')$; multiplying both sides in (4) by $F_1(\theta')$ and re-arranging yields:

$$\begin{aligned} F_1(\theta')\beta_D(\theta') - F_1(\theta)\beta_D(\theta) \;\; &\geq \;\; [F_1(\theta') - F_1(\theta)]\,\theta \\ &+\eta^g \theta [F_1(\theta') - F_1(\theta)] + \eta^g(\lambda^g - 1)\theta F_1(\theta') \int_\theta^{\theta'} F_1(\theta|x) f_1(x|\theta')dx. \end{aligned} \tag{9}$$

The term on the left-hand side and the first term on the right-hand side of (8) and (9) represent the respective familiar material costs and benefits associated with mimicking a bidder with a higher type — trading off a higher probability of winning against paying a higher price. The additional terms on the right-hand sides capture the impact of beliefs on the bidding incentives of a loss-averse bidder. This impact, however, is stronger in the Dutch auction because

$$F_1(\theta') \int_\theta^{\theta'} F_1(\theta|x) f_1(x|\theta')dx > \int_\theta^{\theta'} F_1(\theta) f_1(x)dx = [F_1(\theta') - F_1(\theta)]F_1(\theta).$$

Hence, in the Dutch auction loss-averse bidders have a stronger incentive to bid aggressively since by doing so they can reduce the expected losses caused by the fluctuations in their beliefs due to updating of the reference point. Indeed, we have the following result:

Proposition 3. *$\beta_D(\theta) \geq \beta_I(\theta)$ and this inequality is strict for all $\theta > \underline{\theta}$.*

Thus, loss-averse bidders bid more in the Dutch auction than in the FPA. The following result then is an immediate corollary of Proposition 3:

Corollary 1. *With loss-averse bidders the Dutch auction yields a higher revenue than FPA.*

The attachment effect in the FPA depends on a bidder's ex-ante likelihood of winning. In the Dutch auction, instead, the attachment effect grows over time since, as the price at which a bidder had planned to stop the clock approaches, her beliefs about her chances of winning — and hence her willingness to pay — become higher. This form of dynamic inconsistency pushes a bidder to (plan to) stop the clock at a higher price than the one she would bid in the FPA. Moreover, combining the previous corollary with results obtained by Balzer and Rosato (2019) and von Wangenheim (2018), we obtain that the Dutch auction raises the most revenue among the four main auction formats:

Proposition 4. *With loss averse bidders, in terms of expected revenue, the four main auction formats can be ranked as follows:*

$$\textit{Dutch} > \textit{FPA} = \textit{SPA} > \textit{English}.$$

Intuitively, the FPA and SPA are revenue equivalent as, since they are both static formats where a bidder's reference point depends on her ex-ante likelihood of winning, they induce the same level of attachment. The English auction raises the lowest revenue since a bidder becomes less optimistic about her chances of winning as the auction unfolds; this in turn lowers the bidder's reference point, inducing her to bid less aggressively than in the SPA. In other words, while in a Dutch auction a bidder's initial attachment grows as the auction evolves, in an English auction a bidder becomes less attached to the item as the auction continues. Hence, by creating the strongest attachment for bidders, the Dutch auction raises the largest revenue among standard formats.

Finally, notice that the results in this section hold also under the alternative solution concept of *choice-acclimating personal equilibrium* (CPE) whereby, when contemplating whether to deviate from her equilibrium bid, a bidder immediately adjusts her reference point accordingly (see Kőszegi and Rabin, 2007). Indeed, under CPE, since a bidder's expected gain-loss utility is U-shaped in her likelihood of winning, bidding more aggressively increases the bidder's expected gain-loss utility only if this likelihood is at least 50%; moreover, the larger is her likelihood of winning, the more the bidder is tempted to raise her bid. Thus, bidders still bid more aggressively in the Dutch auction than in any other format.[9]

[9]The only different prediction of CPE is that the FPA revenue dominates the SPA as the latter exposes bidders to additional risk in the money dimension, thereby pushing their bids down; see Lange and Ratan (2010).

4 Conclusion

We have shown that, in both the Dutch auction and the FPA, the incentives of expectations-based loss-averse bidders are driven by the attachment effect: the higher the probability with which a bidder expects to win the auction, the larger her disappointment if she loses and hence her willingness to pay. In the Dutch auction, bidders become more optimistic about their chances of winning as the auction unfolds; in the FPA, instead, the strategy of a loss-averse bidder depends on her ex-ante likelihood of winning. Hence, the Dutch auction induces a stronger attachment than the FPA.

The key insight emerging from our analysis is that when bidders are expectations-based loss averse, managing their level of attachment is crucial for the performance of a selling mechanism. Indeed, using this general insight we were able to rank the four main standard auction formats: the Dutch auction raises more revenue than the FPA which is revenue equivalent to the SPA; and the latter two formats yield a higher revenue than the English auction. The evidence from both the lab and the field seems broadly consistent with this ranking. Indeed, Lucking-Reiley (1999) and Katok and Kwasnica (2008) find that the Dutch auction raises more revenue than the FPA. Moreover, several studies show that with private values the SPA tends to raise more revenue than the English auction; see Kagel *et al.* (1987) and Harstad (2000). Finally, Cheema *et al.* (2012) find that the Dutch auction yields higher revenue than the English auction, and even more so when the clocks of the two auctions are relatively slow.

More generally, with expectations-based reference-dependent preferences, two mechanisms that allocate the prize to the same bidder might still result in different payoffs for both the bidders and the seller, depending on how the allocation is implemented. In other words, the "Revelation Principle" might fail if bidders have expectations-based reference-dependent preferences. These preferences have been fruitfully applied in many different areas of economics.[10] Yet, contributions in the area of mechanism design have mainly focused on static mechanisms. Our results, however, imply that focusing on static mechanisms is not always without loss of generality.

[10] For applications to firms' pricing and advertising strategies, see Heidhues and Kőszegi (2008, 2014), Herweg and Mierendorff (2013), Karle and Peitz (2014, 2017), Rosato (2016), and Karle and Schumacher (2017). For applications to contracy theory, market design and mechanism design, see Herweg *et al.* (2010, 2018), Lange and Ratan (2010), Daido and Murooka (2016), Macera (2018), Eisenhuth (2018), Ehrhart and Ott (2017), von Wangenheim (2018), Balzer and Rosato (2019), Rosato (2019), Dreyfuss *et al.* (2020) and Meisner and von Wangenheim (2020).

A Proofs

Proof of Proposition 1: See Balzer and Rosato (2019). ■

Proof of Lemma 1: Consider a bidder planning to stay in the auction until $\beta_D(\tilde{\theta})$. At price $\beta_D(x) > \beta_D(\tilde{\theta})$ she expects to win with probability $F_1(\tilde{\theta}|x)$. Consider a decremental price drop to $\beta_D(x-\Delta) = \beta_D(x) - \varepsilon$. If an opponent stops the clock at price $\beta_D(x-\Delta)$ — an event happening with conditional probability $1 - F_1(x-\Delta|x)$ — the bidder loses the auction in which case her gain-loss utility is

$$-\eta^g \lambda^g F_1(\tilde{\theta}|x)\theta + \eta^m F_1(\tilde{\theta}|x)\beta_D(\tilde{\theta}).$$

With probability $F_1(x-\Delta|x)$ no opponent buys and the probability with which the bidder expects to win increases by $F_1(\tilde{\theta}|x) - F_1(\tilde{\theta}|x-\Delta)$. In this case, her gain-loss utility is

$$\eta^g \left[F_1(\tilde{\theta}|x) - F_1(\tilde{\theta}|x-\Delta)\right]\theta - \eta^m \lambda^m \left[F_1(\tilde{\theta}|x) - F_1(\tilde{\theta}|x-\Delta)\right]\beta_D(\tilde{\theta}).$$

Hence, her expected news utility of a price drop from $\beta_D(x)$ to $\beta_D(x-\Delta)$ is

$$\begin{aligned}
&\mathbb{E}\left[N(x-\Delta|\theta,x)\right] \\
&= \left[1 - F_1(x-\Delta|x)\right] F_1(\tilde{\theta}|x)\left[-\eta^g\lambda^g\theta + \eta^m\beta_D(\tilde{\theta})\right] + F_1(x-\Delta|x)\left[F_1(\tilde{\theta}|x) - F_1(\tilde{\theta}|x-\Delta)\right]\left[\eta^g\theta - \eta^m\lambda^m\beta_D(\tilde{\theta})\right] \\
&= \left[1 - F_1(x-\Delta|x)\right] F_1(\tilde{\theta}|x)\left[-\eta^g\lambda^g\theta + \eta^m\beta_D(\tilde{\theta})\right] + \left[F_1(x-\Delta|x)F_1(\tilde{\theta}|x) - F_1(\tilde{\theta}|x)\right]\left[\eta^g\theta - \eta^m\lambda^m\beta_D(\tilde{\theta})\right] \\
&= \left[1 - F_1(x-\Delta|x)\right] F_1(\tilde{\theta}|x)\left[-\eta^g\lambda^g\theta + \eta^m\beta_D(\tilde{\theta})\right] + F_1(\tilde{\theta}|x)\left[F_1(x-\Delta|x) - 1\right]\left[\eta^g\theta - \eta^m\lambda^m\beta_D(\tilde{\theta})\right] \\
&= -\left[1 - F_1(x-\Delta|x)\right] F_1(\tilde{\theta}|x)[\eta^g(\lambda^g-1)\theta + \eta^m(\lambda^m-1)\beta_D(\tilde{\theta})].
\end{aligned}$$

When the current clock price is $\beta_D(\theta')$, for $x \in [\tilde{\theta}, \theta']$ price $\beta_D(x)$ is reached with probability $F_1(x|\theta')$. Hence, from the perspective of price $\beta_D(\theta')$, the expected news utility associated with a price drop from $\beta_D(x)$ to $\beta_D(x-\Delta)$ is given by:

$$F_1(x|\theta')\mathbb{E}\left[N(x-\Delta|\theta,x)\right] = -F_1(x|\theta')\left[\frac{F_1(x) - F_1(x-\Delta)}{F_1(x)}\right] F_1(\tilde{\theta}|x)[\eta^g(\lambda^g-1)\theta + \eta^m(\lambda^m-1)\beta_D(\tilde{\theta})]. \quad (10)$$

Total expected news utility is the sum of all these incremental expected gain-loss utility terms for all prices from $\beta_D(\theta')$ to $\beta_D(\tilde{\theta})$. Notice that, since β is continuously increasing, as $\varepsilon \to 0$ we have $\Delta \to 0$ and $\frac{F_1(x)-F_1(x-\Delta)}{\Delta F_1(\theta')} \to f_1(x|\theta')$. Hence, the expected news utility, (10), approaches $-\left[\eta^g(\lambda^g-1)\theta + \eta^m(\lambda^m-1)\beta_D(\tilde{\theta})\right]\int_{\tilde{\theta}}^{\theta'} f_1(x|\theta')F_1(\tilde{\theta}|x)dx$. ■

Proof of Proposition 2: We prove Proposition 2 in three steps. First, using only necessary conditions, we derive a lower bound on the equilibrium bid. Then, we focus on sufficient conditions and show that the lower bound is indeed attainable and thus constitutes a PE. Finally, we show that the PPE is the PE that involves the lowest bid.

Step 1. In a symmetric equilibrium a type-θ bidder prefers executing her plan of buying at price $\beta_D(\theta)$ over buying at price $\beta_D(\tilde{\theta})$ at any clock price $\beta_D(\theta') > \beta_D(\theta)$ if and only if $\Delta U(\tilde{\theta}|\theta,\theta') := F_1(\theta')[U(\tilde{\theta}|\theta,\theta') - U(\theta|\theta,\theta')] \leq 0$ for all $\theta' \geq \theta$ and all credible deviations $\tilde{\theta} \leq \theta'$. For any upward deviation $\tilde{\theta} \geq \theta$ we have

$$\Delta U(\tilde{\theta}|\theta,\theta') = (1+\eta^g)\left[F_1(\tilde{\theta}) - F_1(\theta)\right]\theta + \eta^g(\lambda^g-1)\theta\left(\int_\theta^{\theta'} F_1(\theta|x)f_1(x)dx - \int_{\tilde{\theta}}^{\theta'} F_1(\tilde{\theta}|x)f_1(x)dx\right)$$
$$-(1+\eta^m\lambda^m)\left[F_1(\tilde{\theta})\underline{\beta_D}(\tilde{\theta}) - F_1(\theta)\underline{\beta_D}(\theta)\right]$$
$$+\eta^m(\lambda^m-1)\left(\underline{\beta_D}(\theta)\int_\theta^{\theta'} F_1(\theta|x)f_1(x)dx - \underline{\beta_D}(\tilde{\theta})\int_{\tilde{\theta}}^{\theta'} F_1(\tilde{\theta}|x)f_1(x)dx\right). \tag{11}$$

Differentiation yields

$$\frac{\partial \Delta U(\tilde{\theta}|\theta,\theta')}{\partial\tilde{\theta}} = (1+\eta^g\lambda^g)\theta f_1(\tilde{\theta}) - (1+\eta^m)\underline{\beta_D}(\tilde{\theta})f_1(\tilde{\theta}) - (1+\eta^m\lambda^m)F_1(\tilde{\theta})\underline{\beta}'_D(\tilde{\theta})$$
$$-\eta^g(\lambda^g-1)\theta\int_{\tilde{\theta}}^{\theta'} f_1(\tilde{\theta}|x)f_1(x)dx - \eta^m(\lambda^m-1)\underline{\beta_D}(\tilde{\theta})\int_{\tilde{\theta}}^{\theta'} f_1(\tilde{\theta}|x)f_1(x)dx$$
$$-\eta^m(\lambda^m-1)\underline{\beta}'_D(\tilde{\theta})\int_{\tilde{\theta}}^{\theta'} F_1(\tilde{\theta}|x)f_1(x)dx \tag{12}$$

In equilibrium the bidder does not want to deviate upwards locally, i.e. $\lim_{\theta'\searrow\theta}\frac{\partial\Delta U(\tilde{\theta}|\theta,\theta')}{\partial\tilde{\theta}} \leq 0$ for $\tilde{\theta} = \theta'$, which leads to the necessary condition

$$(1+\eta^m\lambda^m)\,F_1(\theta)\beta'_D(\theta) + (1+\eta^m)\,\beta_D(\theta)f_1(\theta) \geq (1+\eta^g\lambda^g)\,f_1(\theta)\theta. \tag{13}$$

Imposing that (13) holds with equality and solving the resulting differential equation yields a lower bound on any PE bid; call this lower bound $\underline{\beta_D}$. This is expression (2) in the main text.

Step 2. We show that $\underline{\beta_D}$ satisfies the sufficient conditions for a PE. For upward deviations, note that $\frac{\partial^2\Delta U(\tilde{\theta}|\theta,\theta')}{\partial\tilde{\theta}\partial\theta'} < 0$. Hence, a deviation to $\tilde{\theta} > \theta$ is profitable at price $\underline{\beta_D}(\theta') > \underline{\beta_D}(\tilde{\theta})$ if and only if it is profitable at price $\underline{\beta_D}(\theta') = \underline{\beta_D}(\tilde{\theta})$. But for any $\tilde{\theta} = \theta' > \theta$, we have from (12) that the (right-)derivative is

$$\left.\frac{\partial\Delta U(\tilde{\theta}|\theta,\theta')}{\partial\tilde{\theta}}\right|_{\tilde{\theta}=\theta'} = (1+\eta^g\lambda^g)\theta f_1(\tilde{\theta}) - (1+\eta^m\lambda^m)\,F_1(\tilde{\theta})\underline{\beta}'_D(\tilde{\theta}) - (1+\eta^m)\,\underline{\beta_D}(\tilde{\theta})f_1(\tilde{\theta}) = (1+\eta^g\lambda^g)(\theta-\tilde{\theta})f_1(\tilde{\theta}) < 0$$

where the second equality follows since (13) holds with equality for type $\tilde{\theta}$.

Next, we show that the bidding function $\underline{\beta_D}$ is immune to downward deviations. Fix $\tilde{\theta} < \theta < \theta'$ and suppose that when the clock price is $\underline{\beta_D}(\theta')$ a type-θ bidder deviates to the plan of *buying at price* $\underline{\beta_D}(\tilde{\theta}) < \underline{\beta_D}(\theta)$. Such a deviation is only a concern if it is a credible plan; that is, if the bidder actually carries it through. This, however, is not the case. Indeed, since (13) hodls with equality for a type-$\tilde{\theta}$ bidder, such bidder would be indifferent towards a local upward deviation around price $\beta_D(\tilde{\theta})$. As the right-hand side of (12) is strictly increasing in θ, and $\theta > \tilde{\theta}$, a type-θ bidder strictly benefits from such a local upward deviation at $\underline{\beta_D}(\tilde{\theta})$.

Step 3. In Step 1 we showed that $\underline{\beta_D}$ is the lowest PE bid. Moreover, notice that all other strictly increasing PE bidding functions that arise in a symmetric equilibrium lead to the same allocation of the good. In equilibrium, no bidder deviates from her strategy and therefore it is easy to see, by using (4), that a bidder's equilibrium payoff decreases in her bid. Thus, $\underline{\beta_D}$ is every bidder type's preferred symmetric PE bidding function and hence the PPE. ■

Proof of Proposition 3: Observe that the bid in the FPA, (2), is bounded above by $\int_{\underline{\theta}}^{\theta} \frac{1+\eta^g \lambda^g}{F_1(\theta)[1+\eta^m \lambda^m]} e^{\frac{\eta^m(\lambda^m-1)}{1+\eta^m\lambda^m}[F_1(\theta)-F_1(s)]} f_1(s) s ds$. Thus, it is sufficient to show that

$$\left[\frac{F_1(\theta)}{F_1(s)}\right]^{\frac{\eta^m(\lambda^m-1)}{1+\lambda^m\eta^m}} \geq e^{\frac{\eta^m(\lambda^m-1)}{1+\eta^m\lambda^m}[F_1(\theta)-F_1(s)]}$$
$$\Leftrightarrow \ln(F_1(\theta)) - \ln(F_1(s)) \geq F_1(\theta) - F_1(s)$$
$$\Leftrightarrow \ln(F_1(\theta)) - F_1(\theta) \geq \ln(F_1(s)) - F_1(s). \quad (14)$$

As $\theta \geq s$, (14) holds if $\ln(F_1(x)) - F_1(x)$ is increasing in x. This is the case since $\frac{f_1(x)}{F_1(x)} - f_1(x) = f_1(x)\frac{1-F_1(x)}{F_1(x)} \geq 0$. ■

Proof of Proposition 4: The inequality *Dutch* > *FPA* follows from Corollary 1. The equality *FPA* = *SPA* follows from the results in Balzer and Rosato (2019). Finally, the inequality *SPA* > *English* follows from von Wangenheim (2018). ■

References

[1] Abeler, J., A. Falk, L. Goette and D. Huffman (2011), "Reference Points and Effort Provision" *American Economic Review*, 101(2), 470-492.

[2] Allais, M. (1953), "Le comportement de l'homme rationnel devant le risque: Critique des postulats et axiomes de l'école Américaine" *Econometrica*, 21(4), 503–546.

[3] Auster, S. and C. Kellner (2019), "Robust Bidding and Revenue in Descending Price Auctions" forthcoming in *Journal of Economic Theory*.

[4] Balzer, B. and A. Rosato (2019), "Expectations-Based Loss Aversion in Auctions with Interdependent Values: Extensive vs. Intensive Risk" forthcoming in *Management Science*.

[5] Banerji, A. and N. Gupta (2014), "Detection, Identification and Estimation of Loss Aversion: Evidence from an Auction Experiment" *American Economic Journal: Microeconomics*, 6(1), 91-133.

[6] Carare, O. and M. Rothkopf (2005), "Slow Dutch Auctions" *Management Science*, 51(3), 365-373.

[7] Cerulli-Harms, A., L. Goette and C. Sprenger (2019), "Randomizing Endowments: An Experimental Study of Rational Expectations and Reference-Dependent Preferences" *American Economic Journal: Microeconomics*, 11(1), 185–207.

[8] Cheema, A., D. Chakravarti and A. Sinha (2012), "Bidding Behavior in Descending and Ascending Auctions" *Marketing Science*, 31(5), 779–800.

[9] Coppinger, V., V. Smith and J. Titus (1980), "Incentives and Behavior in English, Dutch and Sealed-bid Auctions" *Economic Inquiry*, 18(1), 1–22.

[10] Cox, J., B. Robertson and V. Smith (1982), "Theory and behavior of single object auctions" in Research in Experimental Economics (Vernon L. Smith, ed.), 1–43, JAI Press, Greenwich, Connecticut.

[11] Cox, J., V. Smith and J. Walker (1983), "A Test that Discriminates between Two Models of the Dutch-First Auction Non-Isomorphism" *Journal of Economic Behavior & Organization*, 4(2-3), 205–219.

[12] Daido, K. and T. Murooka (2016), "Team Incentives and Reference-Dependent Preferences" *Journal of Economics & Management Strategy*, 25(4), 958-989.

[13] Dreyfuss, B., O. Heffetz and M. Rabin (2020), "Expectations-Based Loss Aversion May Help Explain Seemingly Dominated Choices in Strategy-Proof Mechanisms" *Working Paper*.

[14] Ehrhart, K.M. and M. Ott (2017), "Reference-Dependent Bidding in Dynamic Auctions" *Working Paper*.

[15] Eisenhuth, R. (2018), "Reference Dependent Mechanism Design" forthcoming in *Economic Theory Bulletin*.

[16] Ericson Marzilli, K.M. and A. Fuster (2011), "Expectations as Endowments: Evidence on Reference-Dependent Preferences from Exchange and Valuation Experiments" *Quarterly Journal of Economics*, 126(4), 1879-1907.

[17] Gill, D. and V. Prowse (2012), "A Structural Analysis of Disappointment Aversion in a Real Effort Competition" *American Economic Review*, 102(1), 469-503.

[18] Gneezy, U., L. Goette, C. Sprenger and F. Zimmermann (2017), "The Limits of Expectations-Based Reference Dependence" *Journal of the European Economic Association,* 15(4), 861-876.

[19] Harstad, R. (2000), "Dominant Strategy Adoption and Bidders' Experience with Pricing Rules" *Experimental Economics*, 3(3), 261–280.

[20] Heffetz, O. and J. List (2008), "Is the Endowment Effect an Expectations Effect?" *Journal of the European Economic Association*, 12(5), 1396–1422.

[21] Heidhues, P. and B. Kőszegi (2008), "Competition and Price Variation when Consumers are Loss Averse" *American Economic Review*, 98(4), 1245-1268.

[22] Heidhues, P. and B. Kőszegi (2014) "Regular Prices and Sales" *Theoretical Economics*, 9(1), 217–251.

[23] Herweg, F., H. Karle and D. Müller (2018), "Incomplete Contracting, Renegotiation, and Expectation-Based Loss Aversion" *Journal of Economic Behavior & Organization,* 145, 176-201.

[24] Herweg, F. and K. Mierendorff (2013), "Uncertain Demand, Consumer Loss Aversion, and Flat-Rate Tariffs" *Journal of the European Economic Association,* 11(2), 399-432.

[25] Herweg, F., D. Müller and P. Weinschenk (2010), "Binary Payment Schemes: Moral Hazard and Loss Aversion" *American Economic Review*, 100(5), 2451-2477.

[26] Kagel, J., R. Harstad and D. Levin (1987), "Information Impact and Allocation Rules in Auctions with Affiliated Private Values: A Laboratory Study" *Econometrica*, 55(6), 1275–1304.

[27] Karle, H., G. Kirchsteiger and M. Peitz (2015), "Loss Aversion and Consumption Choice: Theory and Experimental Evidence" *American Economic Journal: Microeconomics,* 7(2), 101-120.

[28] Karle, H. and M. Peitz (2014), "Competition under Consumer Loss Aversion" *RAND Journal of Economics,* 45(1), 1-31.

[29] Karle, H. and M. Peitz (2017), "De-Targeting: Advertising an Assortment of Products to Loss-Averse Consumers" *European Economic Review,* 95, 103-124.

[30] Karle, H. and H. Schumacher (2017), "Advertising and Attachment: Exploiting Loss Aversion through Pre-Purchase Information" *RAND Journal of Economics,* 48(4), 927-948.

[31] Karni E. (1988) "On the Equivalence between Descending Bid Auctions and First Price Sealed Bid Auctions" *Theory and Decision*, 25(3), 211-217.

[32] Katoc, E. and A. Kwasnica (2008), "Time is Money: The Effect of Clock Speed on Seller's Revenue in Dutch Auctions" *Experimental Economics,* 11(4), 344-357.

[33] Kőszegi, B. and M. Rabin (2006), "A Model of Reference-Dependent Preferences" *Quarterly Journal of Economics*, 121(4), 1133-1165.

[34] Kőszegi, B. and M. Rabin (2007), "Reference-Dependent Risk Attitudes" *American Economic Review*, 97(4), 1047-1073.

[35] Kőszegi, B. and M. Rabin (2009), "Reference-Dependent Consumption Plans" *American Economic Review*, 99(3), 909-936.

[36] Lange, A. and A. Ratan (2010), "Multi-dimensional Reference-dependent Preferences in Sealed-bid auctions: How (most) Laboratory Experiments Differ from the Field" *Games and Economic Behavior*, 68(2), 634-645.

[37] Lucking-Reiley, D. (1999), "Using Field Experiments to Test Equivalence between Auction Formats: Magic on the Internet" *American Economic Review*, 89(5), 1063-1080.

[38] Macera, R. (2018) "Intertemporal Incentives Under Loss Aversion" *Journal of Economic Theory,* 178, 551-594.

[39] Meisner, V. and J. von Wangenheim (2020), "School Choice and Loss Aversion" *Working Paper.*

[40] Nakajima, D. (2011), "First-price Auctions, Dutch Auctions, and Buy-it-now Prices with Allais Paradox Bidders" *Theoretical Economics*, 6(3), 473-498.

[41] Pagel, M. (2016), "Expectations-Based Reference-Dependent Preferences and Asset Pricing" *Journal of the European Economic Association*, 14(2), 468–514.

[42] Pagel, M. (2017), "Expectations-Based Reference-Dependent Life-Cycle Consumption" *Review of Economic Studies*, 84(2), 885–934.

[43] Pagel, M. (2019), "Prospective gain-loss utility: Ordered versus separated comparison" *Journal of Economic Behavior and Organization*, 168, 62–75.

[44] Rosato, A. (2016), "Selling Substitute Goods to Loss-Averse Consumers: Limited Availability, Bargains and Rip-offs" *RAND Journal of Economics*, 47(3), 709-733.

[45] Rosato, A. (2019), "Loss Aversion in Sequential Auctions: Endogenous Interdependence, Informational Externalities and the Afternoon Effect" *Working Paper.*

[46] Rosato, A. and A. Tymula (2019), "Loss Aversion and Competition in Vickrey Auctions: Money Ain't No Good" *Games and Economic Behavior,* 115, 188-208.

[47] Smith, A. (2019), "Lagged Beliefs and Reference-Dependent Utility" *Journal of Economic Behavior & Organization*, 167, 331-340.

[48] von Wangenheim, J. (2018), "English vs. Vickrey Auctions with Loss Averse Bidders" *Working Paper.*

[49] Weber, R. (1982), "The Allais paradox, Dutch auctions, and alpha-utility theory" *Working Paper.*

[50] Zimmermann, F. (2015), "Clumped or Piecewise? - Evidence on Preferences for Information" *Management Science*, 61(4), 740–753.