UTS Business School –
Department of Economics
Dr Chau Chak Wing Building
14 – 28 Ultimo Road, Ultimo NSW 2007

T: +61 2 9514 7713
economics@uts.edu.au

Department of Economics
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'Signaling by Bayesian Persuasion and Pricing Strategy. Short Title: Disclosure and Price Signaling'

Yanlin Chen [1]
Jun Zhang [2]

[1] Nanjing Audit University, China
[2] University of Technology Sydney, Australia

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Signaling by Bayesian Persuasion and Pricing Strategy
Short Title: Disclosure and Price Signaling

Yanlin Chen [*] Jun Zhang [†]

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Abstract

This paper investigates how a privately informed seller could signal her type through Bayesian persuasion and pricing strategy. We find that it is generally impossible to achieve separation through one channel alone. Furthermore, the outcome that survives the intuitive criterion always exists and is unique. This outcome is separating, for which a closed-form solution is provided. The signaling concern forces the high-type seller to disclose inefficiently more information and charge a higher price, resulting in fewer sales and lower profit. Finally, we show that a regulation on minimal quality could potentially hurt social welfare, and private information hurts the seller.

Keywords: Bayesian persuasion, signaling, information disclosure, informed principal.

JEL Classifications: D82, D83, L12

1 Introduction

In many, if not most, real-life sales situations, buyers do not have accurate information about the value of a product at the beginning due to their heterogeneous preferences in matching with the

[*]Institute for Social and Economic Research, Nanjing Audit University, China, Email: yanlin.chen2019@outlook.com.

[†]Corresponding author. Economics Discipline Group, School of Business, University of Technology Sydney; Tel: +61 2 95143203, Fax: +61 2 95147722, E-mail: zhangjunqueens@gmail.com. Jun Zhang gratefully acknowledges financial support from ARC, Australia (DE160100104).We are grateful to the editor, Gilat Levy, and three anonymous referees for insightful comments and suggestions. We thank Yeon-Koo Che, Anton Kolotilin, Hongyi Li, Jingfeng Lu, Junjie Zhou as well as seminar participants at Chinese University of Hongkong-Shengzhen, Shanghai University of Finance and Economics, East China Normal University, Jiangxi University of Finance and Economics, National University of Singapore, Nanyang University of Technology, 2017 Australasian Economic Theory Workshop, 2017 Nanjing International Conference on Game Theory and the Fourth Microeconomic workshop for valuable comments. The usual disclaimers apply.

features of the product. Sellers can often decide to what extent buyers will be allowed to access further information that can be used to refine their estimates. Consider the following situations. When a new video game is developed, whether players will like it is uncertain. One thing the company can do is to first launch a trial version of the video game and allow players to learn more information before formally launching it. The company can fully design the trial version and control how much information to reveal to influence players' estimations of the matching value. When an automobile manufacturer releases a new vehicle model, it can either allow customers to have a test drive so that they can get more familiar with the functions of the vehicle or simply sell it as it is. An innovator with new cost-reduction technology can decide whether to offer a demonstration of the technology so that potential purchasing firms can see how well the technology would fit with their current technology. A federal government's outer continental shelf (OCS) can choose the number of oil-field tests that can be carried out by potential bidders to influence the accuracy of the estimate. In the financial market for acquisition of assets, the selling company can choose how much proprietary information to reveal so that potential buyers can evaluate the synergy with their current assets.

Since Kamenica and Gentzkow's pioneering work, Bayesian persuasion has been widely used to model how information can be disclosed in a wide range of environments, and provides a systematic way to examine all possible information disclosure rules under full commitment. One key assumption necessary to make the model work is that the sender (seller here) can commit to the adopted statistical experiment, i.e., the realization of the signal from this statistical experiment must be truthfully transmitted to the receiver (buyer here). This assumption trivially holds in selling problems, since most of the time it is actually the buyer who directly observes the realized signal instead of the seller. For example, whether players like the game after the trial, whether a buyer is comfortable with the features of a car, whether new technology fits well with current technology, results of oil-field tests and the synergies among assets are all buyers' ex-post private information. Therefore, Bayesian persuasion serves as an appropriate tool for studying information disclosure in selling problems.

Another common feature in selling problems is that sellers usually have an unverifiable informational advantage over the buyers since they may observe informative signals about buyers' valuations–i.e., the lemon problem. For example, game companies know the specific features of the game they develop. Innovators know the intrinsic characteristics of their own technology. Automobile makers know the quality of their new models. The OCS may have already performed its own tests. Companies selling financial assets hold superior information about their own assets. As such, the signaling issue arises naturally. How can a high-type seller distinguish herself from a low-type seller in this situation? Will the signaling consideration urge the seller to disclose more or less information? Who will benefit from the signaling? Will the signaling effect enhance social efficiency? These are the questions we aim to answer in this paper.

In this paper, a seller has one unit of indivisible product to sell to a representative buyer. At the beginning, the value of the product to the buyer is uncertain, and the seller can control how much information to allow the buyer to access. Meanwhile, the seller possesses binary unverifiable private information that is informative about the buyer's value. To sell the product, the seller's strategy has two parts: information disclosure and pricing strategy. For information disclosure, the seller can design a statistical experiment whose realization can be conditional on the true value of the buyer, i.e., Bayesian persuasion. For pricing strategy, we assume that the seller commits to an ex ante price at which the buyer can buy the product after he learns the signal realization from the statistical experiment. Since both the choice of Bayesian persuasion and pricing strategy are functions of the seller's private type, they both could signal the seller's private information.

We first characterize some properties of perfect Bayesian equilibria. We show that the seller cannot signal her type by either information disclosure or pricing strategy alone. As is common in the literature, due to the multiplicity of equilibria, we use the intuitive criterion proposed by Cho and Kreps (1987) to refine the perfect Bayesian equilibrium (intuitive equilibrium). We find that the outcome that survives the intuitive criterion always exists and is unique. This equilibrium outcome is separating, and must be in pure strategy. In this equilibrium outcome, the low-type seller reveals an efficient amount of information and sets the selling price such that the buyer is indifferent between buying and not buying. This is her optimal strategy if her type is known by the buyer. The high-type seller adopts a *monotone binary partition* disclosure rule, in which the buyer only learns whether his value is higher or lower than a cutoff.[1] This cutoff is determined such that it is just enough to deter the low-type seller from mimicking the high type. The high-type seller sets the selling price at the buyer's expected value of the product conditional on being higher than the cutoff. Compared to the case in which the higher-type seller's quality is known by the buyer, the signaling effect urges the high-type seller to disclose inefficiently more information and sets a higher selling price, which excludes more buyers from trading. As a result, the high-type seller is worse off. In a sense, the Akerlof effect still arises: signaling through persuasion and pricing can mitigate adverse selection problem but does not get rid of it.

Based on the characteristics of the unique intuitive outcome, we derive some results on comparative statistics. When we fix the high type and increase the low type in terms of likelihood ratio dominance, the high type's cutoff is not monotone. Notably, the high type could set up a higher cutoff and earn a lower profit. Furthermore, while the low type earns a higher profit, the social welfare could have been decreasing. This has important policy implications. Minimal quality

[1] Anderson and Renault (2006) provide nice interpretations of how to implement monotone binary partition disclosure policies.

standards are widely adopted in many countries. Our result thus implies that imposing a minimal quality standard could result in a loss in social welfare. Furthermore, we show that private information hurts the seller as in Alonso and Câmara (2018).

The rest of the paper is organized as follows. In Section 2, we conduct a literature review. In Section 3, we describe the model. In Section 4, we simplify the problem. In section 5, we derive the properties of perfect Bayesian equilibria. In Section 6, we impose the intuitive criterion to select the equilibrium. In Section 7, we conduct some comparative statistics. In Section 8, we discuss how our findings are related to the literature. Section 9 concludes and an appendix contains some technical proofs.

2 Literature review

In their pioneering work on Bayesian persuasion, Kamenica and Gentzkow (2011) consider the environment with a single Sender and a single Receiver. The Sender can commit to an informative signal device about the state of nature which is initially unknown to everyone. The Receiver takes an action after updating his belief about the state of nature upon observing the signal realization. Kamenica and Gentzkow's theory is then extended in several directions. For example, Gentzkow and Kamenica (2016) and Au and Kawai (2017) allow multiple senders and investigate whether competition among senders will cause more information to be revealed. Kolotilin *et al.* (2017) allow a privately informed receiver. The general theory is also applied to study information disclosure in specific settings, such as voting in Wang (2012) and contests in Zhang and Zhou (2015) .

Our paper is more related to the literature on Bayesian persuasion with an informed sender.[2] Perez-Richet (2014) provides the first step in analyzing interim Bayesian persuasion by a sender. He assumes that the sender's payoff is constant in the receiver's belief except for a single discontinuity, and the sender is perfectly informed about the state of nature. He shows that it is without loss of generality to focus on pooling equilibria in his setting. Alonso and Câmara (2018) demonstrate that private information is not beneficial to the sender, compared to the case without information advantage in a perfect Bayesian equilibrium. Since their main concern is comparison of the sender's equilibrium payoff, it is not necessary to characterize the equilibrium strategies and refine them. Hedlund (2017) considers a model in which a privately informed Sender can use Bayesian persuasion to influence the receiver's belief about the state of nature. He characterizes the properties of equilibria selected by the D1 criterion and shows that there are multiple equilibria that are either separating or fully disclosing, which provides a framework in which the result of Alonso and Câmara (2018) does not apply. All of these papers assume that the sender can signal

[2] See Hedlund (2017) for an excellent review of the literature.

her private information only through Bayesian persuasion. In selling problems, however, pricing strategies are always feasible for signaling. We show that the availability of a pricing strategy in addition to Bayesian persuasion yields a dramatically different conclusion. First, our model always yields a unique refined equilibrium outcome. Although the analysis becomes more involved due to the multidimensional strategy space of the seller, a larger strategy space also makes it possible to block more unreasonable equilibria. This suggests that allowing the seller to have more signaling tools may not necessarily be a curse. Second, all of the above works focus on the binary state of nature. While the case of the binary state of nature is more tractable, as Hedlund (2017) points out, "it is also one of the more important limitations from a theoretical point of view." In this paper, we allow the state of nature to be continuous.[3]

Hedlund (2017) notes that a possible interpretation of his model is that 'the sender provides free trials of a product of uncertain quality and extracts a price that is increasing in the receiver's ultimate belief that the quality is high.' (Hedlund, 2017, p.33) Thus, we can interpret his paper as a seller/buyer model in which after the signal realization is observed, the price is automatically adjusted to be equal to the receiver's ex post valuation. For completeness, we show that if such an ex post price is indeed available, the high-type seller always discloses full information, which is consistent with his finding. In reality, however, the buyer usually observes the signal realization privately, and this makes such an ex post price infeasible. We show that with our ex ante price full disclosure is usually not optimal for the high-type seller, which demonstrates the significant difference between ex ante and ex post prices.

Our paper is also related to Alonso and Câmara (2016a), especially their online appendix. Although their paper is mainly about how politicians can utilize Bayesian persuasion to influence voters, their model can be interpreted as a seller/buyer model with exogenous price. The seller wants to maximize the probability that the buyer's ex post valuation will be above the exogenously fixed price. Their Proposition 2 shows that the optimal disclosure takes the form of a cutoff rule, where the receiver learns whether his valuation is above a certain cutoff or not. In their online appendix, they describe an extension of their model in which the sender privately observes the true state before choosing the signal. In this case, they show that the best equilibrium for the sender is one in which all informed sender types pool on the same signal as the uninformed sender. In contrast, our paper shows that if the seller also strategically chooses the ex ante price, then we need to look at the separating equilibrium.

This paper is also related to the literature in which an informed sender can signal her private information through information disclosure other than Bayesian persuasion. Gill and Sgroi (2012) investigate the firm's decision on the toughness of the test. Chung and Esõ (2013) explore the

[3]We could also allow the seller's type to be continuous; see our discussion in the conclusion.

trade-off between persuasion and learning in a signaling game and observe countersignaling. Li and Li (2013) investigate a costly signaling game in which a politician decides on the accuracy of the signal on her own qualifications and her opponent's. Miyamoto (2012) studies independent multidimensional information and argues that the degree of one dimension's noise is a signal for that of the other dimension. In our paper, the two dimensions of uncertainty–the seller's type and the buyer's value of the product–are correlated with each other. Degan and Li (2015) explore the persuasive signaling game in which a perfectly informed sender can chooses the precision of information. In all these papers, the sender may incur some cost to choose among a constrained set of feasible signals with various accuracy. In contrast, our paper assumes costless information disclosure and we do not impose restrictions on the statistical experiment following Kamenica and Gentzkow (2011). Skreta (2011) considers a seller who observes a vector of signals correlated with buyers' valuations. Before proposing a selling mechanism, the seller chooses a disclosure policy to disclose her observed information to buyers about their competitors. In contrast, our paper discusses a seller who design information disclosure about the buyer's value and selling price simultaneously.[4]

Finally, this paper is related to the literature on signaling product quality in the marketplace. The seller may communicate the quality of her product to buyers through a variety of channels, such as prices in Bagwell and Riordan (1991) and Desai and Srinivasan (1995); uninformative advertising in Milgrom and Roberts (1986); performance warranties in Spence (1977); umbrella branding in Wernerfelt (1988); the reputation of the retailer in Chu and Chu (1994); refund policies in Moorthy and Srinivasan (1995) and Shieh (1996); and the selection of the selling mechanism itself in Kremer and Skrzypacz (2004). In this strand of literature, the only uncertainty in the buyer's valuation is the seller's private information. In our model, however, the seller's information is not necessarily perfectly correlated with the buyer's value, which raises the issue of signaling through Bayesian persuasion.

3 The model

A risk-neutral seller (she) has one unit of indivisible product for sale to a risk-neutral buyer (he). The seller has private and unverifiable information θ that affects the distribution of the buyer's value of the product. This private information is characterized by a binary distribution on $\{L, H\}$, with probability μ_H^0 and $\mu_L^0 = 1 - \mu_H^0$, respectively. To focus on the informed seller problem, we assume that the buyer does not have private information at the beginning. The buyer's value of the product v depends on how well his idiosyncratic preference matches with the product's

[4]Eso and Szentes (2007) and Li and Shi (2017) consider information disclosure in mechanism design but there is no signaling issue.

characteristics and the seller's type. At the beginning, no one observes v directly. When the seller's type is θ, the buyer's value v follows an atomless distribution with $c.d.f\ F_\theta(v)$ and $p.d.f\ f_\theta(v)$ on the common support $[0,\overline{v}]$. We assume that $F_\theta(v)$ satisfies the monotone likelihood ratio property: $\frac{f_H(v)}{f_L(v)} > \frac{f_H(v')}{f_L(v')}, \forall v > v'$. This implies that the buyer is willing to pay more if the seller is of type H; thus, we can refer to type θ as the quality of the seller. Bayes' rule implies that the buyer's prior belief about the value of the product is $f(v) = \mu_H^0 f_H(v) + \mu_L^0 f_L(v)$. The seller's value of the product (or, equivalently, the production cost) is known as $c \in (0,\overline{v})$ regardless of θ.[5]

The seller's strategy

To sell the product, the seller chooses a strategy that consists of a disclosure rule and a pricing strategy. We model the disclosure rule as Bayesian persuasion, following Kamenica and Gentzkow (2011) and Hedlund (2017). A disclosure rule is a costless statistical experiment π, which is a family of distributions $\pi(s|v)$ over a finite set of realization space S such that $\sum_{s\in S}\pi(s|v) = 1, \forall v$. Regarding the pricing strategy, we assume that the seller commits to an ex ante price $p \in \mathbb{R}_+$, at which the buyer can buy the product after he learns the signal realization from the statistical experiment. Note that sellers with different types can choose different statistical experiment, as well as using different pricing strategies. Therefore, the seller can transmit her private information through both channels. A (mixed) strategy of the seller $\sigma : \Theta \to \Delta(\Pi \times \mathbb{R}_+)$ is a mapping from her type space to her strategy space.

The buyer's belief and strategy

In the continuation game following the seller's strategy (π, p), let $\mu_H(\pi, p)$ denote the buyer's belief about the seller's type being H. Then the density of the buyer's interim belief (before learning the signal realization s) about the value of the product is

$$f(v|\mu_H(\pi,p)) = \mu_H(\pi,p) f_H(v) + [1-\mu_H(\pi,p)] f_L(v).$$

After the buyer learns the signal realization s, the density of the buyer's final belief about the value of the product is

$$f(v|\mu_H(\pi,p),\pi,s) = \frac{\pi(s|v) f(v|\mu_H(\pi,p))}{\int_0^{\overline{v}} \pi(s|v) f(v|\mu_H(\pi,p)) dv}. \tag{1}$$

Given the buyer's final belief, he decides whether to buy the product at price p. Assume that the buyer buys the product if he is indifferent. Thus, it is without loss of generality to focus on pure strategies for the buyer. Denote the buyer's strategy as $\rho(\pi, p, s) \in \{0, 1\}$, where $\rho = 1$ if and

[5]The same result can be obtained if the seller's value of the product depends on θ. The main reason for this assumption is to demonstrate that the separation is not due to the seller's heterogeneous values. When $c = 0$, all results go through except the uniqueness of the low-type seller's strategy. However, they all lead to the same outcome as the one we characterized in Theorem 1.

only if the buyer buys the product.

Timing

To summarize, the timing of the game is as follows.

1. The nature draws a private type θ for the seller.
2. The seller chooses a signal π and a selling price p.
3. The buyer observes the seller's strategy and a signal realization s that is generated according to the statistical experiment π, then decides whether to buy the product at price p or not.

Payoffs

If the buyer buys the product worthy of v at price p, his payoff is $v-p$; otherwise, his payoff is normalized to zero. Therefore, the expected payoff of the buyer is

$$u(\pi,p,s,\rho)=\left[\int_0^{\bar{v}}(v-p)f(v|\mu_H(\pi,p),\pi,s)dv\right]\rho(\pi,p,s).$$

Denote $\tau_\theta(s|\pi)=\int_0^{\bar{v}}\pi(s|v)f_\theta(v)dv$ as the expected probability of generating signal s from type-θ seller's point of view. When the buyer buys the product at price p, the seller obtains a profit $p-c$; otherwise, the seller's profit is zero. Therefore, the expected profit of the type θ seller is

$$r_\theta(\pi,p,\rho)=(p-c)\sum_{s\in S}\tau_\theta(s|\pi)\rho(\pi,p,s).$$

The equilibrium concept

The solution concept is perfect Bayesian equilibrium (PBE) selected by the intuitive criterion.

LEMMA 1 *A PBE of the game is* $(\sigma,\rho,\mu_H(\pi,p))$ *such that*

(1) *Given* $\mu_H(\pi,p)$*, the buyer buys the product if and only if* $\frac{\int_0^{\bar{v}}vf(v|\mu_H(\pi,p),\pi,s)dv}{\int_0^{\bar{v}}f(v|\mu_H(\pi,p),\pi,s)dv}\geq p$.

(2) *Given* ρ *and* $\mu_H(\pi,p)$*, a strategy* (π,p) *is adopted by seller* θ *only if* $(\pi,p)\in\arg\max r_\theta(\pi,p,\rho)$.

(3) *If* $\mu_H^0\sigma_H(\pi,p)+\mu_L^0\sigma_L(\pi,p)>0$*, then* $\mu_H(\pi,p)=\frac{\mu_H^0\sigma_H(\pi,p)}{\mu_H^0\sigma_H(\pi,p)+\mu_L^0\sigma_L(\pi,p)}$ *and* $\mu_L(\pi,p)=1-\mu_H(\pi,p)$.

(1) is the buyer's sequential rationality constraint, and means that the buyer buys the product if and only if the final expected value of the product is higher than the price. (2) is the seller's

sequential rationality constraint, and means that the seller puts a positive probability on a strategy only if it maximizes her expected profit. (3) is the consistency constraint that the buyer's belief about the seller's type is Bayes rational whenever possible.

Due to the multiplicity of PBE, we will adopt the intuitive criterion proposed by Cho and Kreps (1987) for refinement, which puts a restriction on the receiver's off-equilibrium-path belief based on strategy dominance. A reasonable off-equilibrium-path belief assigns zero probability to those types who are strictly worse off than their equilibrium profit. Any PBE in which there exists some type θ that has the incentive to deviate, given that the buyer has a reasonable belief, will be eliminated. PBEs that survive the intuitive criterion are called intuitive equilibria. In our model, the intuitive criterion can be stated in a more intuitive way.

LEMMA 2 *A PBE with equilibrium payoff r_L and r_H for the low and high-type seller, respectively, violates the intuitive criterion, if there exists a blocking strategy (π', p') such that*

(a) $r_L(\pi', p', \mu_H(\pi', p')) < r_L$, $\forall\ 0 \leq \mu_H(\pi', p') \leq 1$, and (b) $r_H(\pi', p', 1) > r_H$.

Condition (a) means that (π', p') yields strictly lower profit for the low-type seller, regardless of the belief upon seeing (π', p'). Condition (b) means that (π', p') yields strictly higher profit for the high-type seller if the buyer believes that the seller is of the high type for sure upon seeing (π', p'). By the intuitive criterion, Condition (a) ensures that upon seeing (π', p'), the buyer should assign zero probability for being low; Condition (b) then implies that the high-type seller will have incentive to deviate to (π', p'). Therefore, such a PBE violates the intuitive equilibrium.

4 Simplifying the seller's problem

Potentially, the signal space S is large. In this section, we aim to simplify the seller's problem by restricting the signal space.

DEFINITION 1 *Two equilibria are outcome equivalent if they yield the same profit for each type of the seller, buyer's payoff for each possible valuation, and ex post allocation of the object.*

The definition of outcome equivalency is in a strong notion since it must hold for each type of the seller and buyer. The following lemma shows that if we fix the buyer's belief, we can always replace a general signal space with a binary signal space.

LEMMA 3 *Given the buyer's belief, for any statistical experiment with a general signal space, there exists an outcome equivalent statistical experiment with binary signal space.*

This lemma is an application of the revelation principle similar to Kamenica and Gentzkow (2011). In our model, the buyer's action space is binary: to buy or not to buy. Thus, the ex post allocation of the object is either selling or not selling. Furthermore, the ex post allocation of the object uniquely determines the profit for each type of the seller and the buyer's payoff for each possible valuation. The idea is to combine any signals that lead to the same outcome into a unique signal while maintaining the same price. With the same belief and price, the incentive constraints of the seller and the buyer are not affected. Since there are only two possible outcomes, we need at most two signals.

In contract to Kamenica and Gentzkow (2011), the above lemma does not immediately imply that it is without loss of generality to focus on binary signal space due to the signaling effect. This is because it is assumed that (1) the buyer's interim belief remains constant after the seller changes the statistical experiment, and (2) changing the statistical experiment is not affecting the incentives of the seller. Our approach is to assume binary signal space first and then show later that this is indeed without loss of generality by utilizing the above lemma.

From now, we let $S = \{s_1, s_2\}$. Clearly, we have

$$\pi(s_1|v) + \pi(s_2|v) = 1 \text{ and } 0 \leq \pi(s_1|v) \leq 1.$$

Since we can always exchange the role of the two signals, we will refer to s_1 (s_2) as the signal that leads to a higher (lower) expected value of the product. In addition, we assume that two statistical experiments that differ in zero measure will be perceived as the same statistical experiment by the buyer.[6] Denote Π as the set of all possible statistical experiments. We assume that Π is the same for both types of seller. The signal realization is only observable to the buyer.

A statistical experiment $\pi(s|v)$ is a *monotone binary partition* if

$$\pi(s_1|v) = \begin{cases} 0 & if\ v < y \\ 1 & if\ v \geq y \end{cases},$$

for some cutoff $y \in (0, \bar{v})$. The monotone binary partition statistical experiment only informs the buyer whether his value is above the threshold y or not, and plays an important role in our analysis.

[6]This is mainly for expositional convenience. Otherwise, for any equilibrium, there exists infinite many other equilibria that differ in zero measure.

5 Properties of PBE

Since the multiplicity of PBE in signaling games is common, instead of solving for all possible PBE, we aim to characterize some useful properties of PBE and refine the PBE with the intuitive criterion in the next section. We work backward and start with the buyer's strategy. Denote the buyer's expected final value of the product as

$$\mathbf{E}\{V|\mu_H(\pi,p),\pi,s\} \equiv \frac{\int_0^{\overline{v}} v\pi(s|v)f(v|\mu_H(\pi,p))dv}{\int_0^{\overline{v}} \pi(s|v)f(v|\mu_H(\pi,p))dv}.$$

Simply rewriting Condition (1) in Lemma 1 yields the equilibrium conditions for the buyer:

LEMMA 4 *Given the seller's strategy (π, p) and a belief system $\mu_H(\pi,p)$, the buyer buys the product when signal s is realized if and only if $p \leq \mathbf{E}\{V|\mu_H(\pi,p),\pi,s\}$. Denote the set of signals leading to sale as $B(\mu_H(\pi,p))$.*

Now we study the seller's strategy. Given the buyer's equilibrium strategy described above and the belief system $\mu_H(\pi,p)$, the type θ seller's profit by choosing strategy (π,p) is

$$r_\theta(\pi,p,\mu_H(\pi,p)) = (p-c)\int_0^{\overline{v}} \left[\sum\nolimits_{s\in B(\mu_H(\pi,p))} \pi(s|v)\right] f_\theta(v)dv.$$

Note that with slight abuse of notation, we replaced the buyer's strategy in the r_θ function with the belief, since the buyer's strategy only depends on the belief. The following lemma shows how the expected final value and the seller's profit changes with the buyer's belief.

LEMMA 5 *For π and s leading to a non-degenerated belief of v, $E\{V|\mu_H(\pi,p),\pi,s\}$ is strictly increasing in $\mu_H(\pi,p)$; otherwise, $E\{V|\mu_H(\pi,p),\pi,s\}$ is constant with respect to $\mu_H(\pi,p)$. And $r_\theta(\pi,p,\mu_H(\pi,p))$ is weakly increasing in $\mu_H(\pi,p)$ when $p \geq c$.*

This lemma is intuitive. The first part means that when the product is more likely to be from a high-type seller, the consumer values it more. The second part means that when the price is higher than the cost, the seller's profit is higher when the buyer thinks the product is more likely from a high-type seller. This implies that regardless of the seller's type, the worst thing is believed to be the low type for sure.

To derive PBEs, we often need to know a seller's optimal strategy given a certain belief $\mu_H(\pi,p)$. A useful benchmark is when the seller's type is known to the buyer: $\mu_H(\pi,p) = 1$ for high-type seller and $\mu_H(\pi, p) = 0$ for low-type seller. The optimal strategy in this case is characterized in the following lemma.

LEMMA 6 *If the seller's type θ is observable to the buyer, her unique profit-maximizing strategy is*

$$\pi_\theta(s_1|v) = \begin{cases} 1, v \geq c, \\ 0, \ otherwise, \end{cases}$$

$$p_\theta = \frac{\int_c^{\overline{v}} v f_\theta(v) dv}{\int_c^{\overline{v}} f_\theta(v) dv}$$

and the corresponding profit is $\int_c^{\overline{v}} (v - c) f_\theta(v) dv$.

Thus, when types are observable, the unique optimal strategy for each type is to adopt a monotone binary partition with a cutoff equal to the cost and set the price at the expected value conditional on being higher than the cost. This is optimal since it induces full efficiency and the seller extracts all the surplus.

In what follows, we characterize some properties that are satisfied by any PBE. Note that here we do not restrict ourselves to pure strategy equilibrium. We begin by establishing a lower bound for the equilibrium profit.

PROPOSITION 1 *In any PBE, a type θ seller makes a profit weakly higher than $\underline{r}_\theta$, where*

$$\underline{r}_L \equiv \int_c^{\overline{v}} (v - c) f_L(v) dv > 0,$$

$$\underline{r}_H \equiv \frac{1 - F_H(c)}{1 - F_L(c)} \int_c^{\overline{v}} (v - c) f_L(v) dv > \underline{r}_L.$$

Note that $\underline{r}_\theta > 0$. This proposition states that in any PBE, there exists a strictly positive lower bound profit for both types of seller. If a seller charges a price $p \leq c$, the most she can obtain is zero profit. Furthermore, if a seller charges a price $p > \mathbf{E}\{V|1, \pi, s_1\}$, the buyer will not buy the product for sure. Therefore, the following property of the PBE follows directly.

PROPOSITION 2 *In any PBE, a type θ seller charges a price p_θ such that $c < p_\theta \leq \mathbf{E}\{V|1, \pi, s_1\}$.*

While we have assumed binary signal space, the following proposition shows that in terms of PBE, this is indeed without loss of generality.

PROPOSITION 3 *For any PBE in the game with general signal space, there exists an outcome equivalent PBE in the game with binary signal space, and vice versa.*

If a general signal space is allowed, the set of strategy becomes much larger. However, it turns out that (1) while more PBE can be identified, the set of outcome does not change, and (2) while more off-equilibrium-path deviations have to be considered, it is sufficient to consider the ones with binary signal.

5.1 Properties of separating equilibria

While the above properties hold for any PBE, in what follows we demonstrate some properties that are specific to separating equilibria, which allow for mixed strategies. Separating equilibria are important, since the type of the seller will be fully inferred by their strategies–i.e., signaling happens. Besides being useful on their own, some of the properties are important for establishing the intuitive equilibrium. It turns out that the strategy for the low-type seller in separating equilibria is unique.

PROPOSITION 4 *In separating equilibria, the low-type seller's equilibrium strategy is unique:*

$$\pi_L^*(s_1|v) = \begin{cases} 1, v \geq c, \\ 0, \ otherwise, \end{cases}$$

$$p_L^* = \frac{\int_c^{\overline{v}} v f_L(v)dv}{\int_c^{\overline{v}} f_L(v)dv}.$$

and she obtains an expected profit $r_L^* = \underline{r}_L$.

In this unique strategy, the low-type seller does not randomize. She discloses only whether the value is above the cost or not, and posts a price that is equal to the expected value of the product conditional on being higher than the cost. Note that this is also the optimal strategy for the low-type seller if her type is known by the buyer. For the high-type seller, we have

PROPOSITION 5 *In any separating equilibria, the high-type seller's equilibrium strategies satisfy the following properties:*

(A) the disclosure rule is such that both signal s_1 *and* s_2 *will realize with strictly positive probability, and* $\mathbf{E}\{1, \pi, s_2\} < \mathbf{E}\{1, \pi, s_1\}$.

(B) the price is such that $\mathbf{E}\{1, \pi, s_2\} < p \leq \mathbf{E}\{1, \pi, s_1\}$.

(C) neither the disclosure rule nor the price is the same as that for the low-type seller.

In a separating equilibrium, the high-type seller should trade with probability in $(0, 1)$, otherwise one type will have incentive to mimic the other. As a result, Part (A) and (B) follow directly. Both signals should be utilized by the high-type seller. The buyer values the product strictly higher when the signal realization is s_1 than s_2. The price should be set such that trade occurs if and only if s_1 is realized.

Part (C) means that any separating equilibria must have the two types setting different disclosure rules as well as different prices. In a sense, separations cannot be achieved via price or disclosure rule alone. For instance, if the two types adopt the same information disclosure but different price, the one with lower price would mimic the other. However, this does not mean that no separating equilibrium could arise when one channel is shut down. For example, if the price is fixed at zero for both types, information disclosure will not matter as the buyer buys the product for sure. As a result, different types of seller can adopt different information disclosure in PBE. Despite these properties, it is worthy to note that the high-type seller's equilibrium strategy in (separating) PBE is still quite flexible. For instance, all of the following features could be supported: the disclosure rule could be a non-monotone binary partition–and could even be randomizing over the two signals for some values, and s_1 could be sent even if the value is lower than the cost. As such, refinements are necessary for credible prediction.

6 The intuitive equilibrium

As is common in the literature, there are many PBEs even for binary types. In this section, we impose the intuitive criterion to refine the equilibria. In our analysis, the main step in deriving the intuitive equilibrium is to construct the blocking strategy to exclude PBEs that do not satisfy the intuitive criterion. The following lemma is important for us to construct a blocking strategy.

LEMMA 7 *Consider a strategy* (π, p) *with associated belief* $\mu_H(\pi, p)$ *such that* $r_H(\pi, p, \mu_H(\pi, p)) > 0$.

(i) π *is not a monotone binary partition with cutoff* $y \geq c$,

(ii) $\mu_H(\pi, p) \neq 1$,

(iii) $p \neq \mathbf{E}\{V|1, \pi, s_1\}$.

If at least one of Conditions (i)-(iii) holds, then there exists a unique $y' \in (c, \overline{v})$ *such that*

$$r_H(\pi, p, \mu_H(\pi, p)) = \int_{y'}^{\overline{v}} (v - c) f_H(v) dv, \tag{2}$$

Furthermore,

$$\int_0^{y'} \sum\nolimits_{s \in B(\mu_H(\pi,p))} \pi(s|v) f_H(v) dv > 0. \tag{3}$$

$$1 - F_H(y') < \int_0^{\overline{v}} \left[\sum_{s \in B(\mu_H(\pi,p))} \pi(s|v) \right] f_H(v) dv \tag{4}$$

$$\frac{1 - F_L(y')}{1 - F_H(y')} < \frac{\sum_{s \in B(\mu_H(\pi,p))} \int_0^{\overline{v}} \pi(s|v) f_L(v) dv}{\sum_{s \in B(\mu_H(\pi,p))} \int_0^{\overline{v}} \pi(s|v) f_H(v) dv} \tag{5}$$

This lemma can be explained in an intuitive way. Consider the following constructed strategy:

$$\pi'(s_1|v) \quad = \quad \begin{cases} 1, & if\ v \geq y' \\ 0, & if\ otherwise \end{cases}, \tag{6}$$

$$p' \quad = \quad \mathbf{E}\{V|1, \pi', s_1\} = \frac{\int_{y'}^{\overline{v}} v f_H(v) dv}{1 - F_H(y')}, \tag{7}$$

Here, the statistical experiment is a monotone binary partition with cutoff y'; the price is set at the expected final value of the product, given that it is higher than y' and the seller is of the high type. Therefore, the right-hand side of (2) is equal to the high-type seller's profit under the constructed strategy if her type is known, i.e., $r_H(\pi', p', 1)$. The left- (right-) hand side of (4) is the trading probability of the high-type seller under the constructed (original) strategy. The left- (right-) hand side of (5) is the ratio of the trading probability between low and high-type seller under the constructed (original) strategy. This lemma shows that for a strategy and associated belief that satisfy certain properties, we can always construct a strategy (π', p') such that the high-type seller obtains the same profit if her type is known. Furthermore, with the constructed strategy, the trading probability of the high-type seller is lower, and the ratio of trading probability between low and high-type seller is also lower.

Let us elaborate on the intuition. Suppose only (iii) holds. This means that the disclosure rule is a monotone binary partition with cutoff, say y, and the belief is being the high type for sure, but the price is lower than the expected value. Then to achieve the same profit for the high-type seller, we can enhance the cutoff to y' and charge a higher price at the same time, i.e., by selling at a higher price but with a lower probability. Clearly, (3) and (4) hold, since $y' > y$. When the disclosure rule is a monotone binary partition, the ratio of trading probability $\frac{1-F_L(y)}{1-F_H(y)}$ is decreasing in y if F_H dominates F_L in hazard ratio as it is harder for the low type seller to generate a higher value. Since likelihood ratio dominance implies hazard ratio dominance, we thus have inequality (5). In the general case, for the same probability of trading, a monotone binary partition maximizes the expected final value of the product, allowing higher price and profit. Therefore, for the same amount of profit, a monotone binary partition requires fewer trading probability, i.e., (4). This reduction of trading probability has a larger impact on the low-type seller, i.e., (5): A higher cutoff and a higher price makes it harder for both types to have a sale, but affects the low type even more due to likelihood ratio dominance.

Define y_H^* as the unique solution of y to the equation:

$$\underline{r}_L = \frac{1-F_L(y)}{1-F_H(y)} \int_y^{\overline{v}} (v-c) f_H(v) dv$$

It will become clear later that the left hand side is the low type seller's equilibrium profit and the right hand side is her revenue if she mimics the high type seller. The following theorem characterizes the unique intuitive equilibrium outcome.

Theorem 1 *The unique equilibrium outcome satisfying the intuitive criterion is separating and can be described as follows.*

1. The low-type seller adopts a monotone binary partition π_L^* with cutoff c, posts a price $p_L^* = \frac{\int_c^{\overline{v}} v f_L(v) dv}{1-F_L(c)}$, and obtains an expected profit $r_L^* = \int_c^{\overline{v}} (v-c) f_L(v) dv$.

2. The high-type seller adopts a monotone binary partition π_H^* with cutoff y_H^*, posts a price $p_H^* = \frac{\int_{y_H^*}^{\overline{v}} v f_H(v) dv}{1-F_H(y_H^*)}$, and obtains an expected profit $r_H^* = \int_{y_H^*}^{\overline{v}} (v-c) f_H(v) dv$.

This equilibrium can be supported by assigning a belief of being the low type for sure upon any off-equilibrium path deviation.

Here is the intuition. Whenever the construction in Lemma 7 is possible, the low-type seller will be strictly worse off regardless of the belief by adopting the constructed strategy. This is because by switching to the constructed strategy, the high-type seller can at most be indifferent, and the low type hurts more than the high-type seller due to likelihood ratio dominance. Given this, upon seeing the constructed strategy, the buyer will think the seller is of the high type for sure by intuitive criterion. As a result, the high-type seller makes the same profit as before. However, since the constructed strategy is a monotone binary partition, we can always lower the cutoff a little so that the high-type seller is strictly better off while keeping the low-type seller strictly worse off.

Thus, none of the three conditions in Lemma 7 should hold in intuitive equilibrium. The opposite of Condition (i) means that the high type has to adopt a monotone binary partition with $y \geq c$. The opposite of Condition (ii) means that the equilibrium has to be separating. The opposite of Condition (iii) means that the price has to be equal to the expected final value of the object, given that it is higher than y' and the seller is of the high type. The reason why the high-type seller has to adopt the cutoff y_H^* is because exclusion is costly and y_H^* is the minimum cost of exclusion to deter the low type's deviation.

While the set of PBE outcomes is quite large, imposing the intuitive criterion eliminates all but one. It is well known in the signaling literature that senders with different types can distinguish themselves via costly signaling, in particular, when the cost of the signal obeys certain single-crossing conditions. In our model, although the seller can choose any statistical experiments without any cost, costly signaling of quality is endogenized via Bayesian persuasion. Furthermore, from this vantage it is clear that the high type will exclude more types of buyers than in the first best, since the signaling mechanism must be (indirectly) costly to be effective.

This equilibrium outcome has many features. First, it is separating, meaning that signaling does arise along the equilibrium path. Second, both types of seller disclose information via monotone binary partition, though with different cutoffs. The advantage of monotone binary partitions is that for the same trading probability, they maximize the buyer's willingness to pay. Third, both types of seller price the product at the expected value, conditional on the value being higher than the cutoff, leaving zero surpluses to the buyer. Fourth, the low-type seller adopts the same strategy as if her type were known to the buyer. The is because the high-type seller has no incentive to mimic the low-type seller. Given that the buyer does not have any information at the beginning, by revealing whether the value is higher or lower than the cost, the low-type seller induces efficient allocation and extracts all of the surplus from trading. Fifth, the high-type seller adopts a monotone binary partition with a cutoff higher than c. Ideally, if the buyer knows the seller's type, the high-type seller also wants to adopt the cutoff c in order to extract all of the surplus from efficient trading. However, because the seller's type is unknown to the buyer, the high-type seller needs to deter the low-type seller from mimicking. The high-type seller's profit has two components: the probability of trading and the selling price. For the high-type seller to distinguish from the low type, she must adopt a higher cutoff. While this lowers the trading probability for both types, it hurts the low-type seller more due to likelihood ratio dominance.

It can be shown that this equilibrium outcome corresponds to Riley's outcome. In the signaling literature, several papers characterize the condition on primitives such that the intuitive equilibrium is guaranteed to be Riley's outcome, examples include Matthews *et al.* (1991), Cho and Sobel (1990) and Eső and Schummer (2009). Instead of solving the intuitive equilibrium outcome by imposing the intuitive criterion over PBE, one may think that an alternative approach is to characterize Riley's outcome directly, and then show that our primitives satisfy those conditions. We do not adopt such an approach for the following reasons. First, our primitives do not satisfy those conditions, and therefore we cannot directly conclude that Riley's outcome is the intuitive equilibrium outcome.[7] Second, solving Riley's outcome by itself is a nontrivial problem in our model, due to the complication of solving the optimal Bayesian persuasion with a continuous buyer's value.

6.1 General signal space and upper censorship

If a general signal space is allowed, we have the following proposition.

PROPOSITION 6 *A PBE in the game with general signal space survives the intuitive criterion if and only if it is outcome equivalent to the one in Theorem* 1.

This means that with general signal space, while there may exist many intuitive equilibria, they all lead to the unique outcome that we have identified in the game with binary signal space. This proposition implies that it is indeed without loss of generality to focus on binary signal space if we are interested in identifying the outcome refined by intuitive criterion.

When a general signal space is allowed, a monotone binary partition with cutoff y can be replaced by another disclosure policy: (1) reveal whether v is greater than y or not, and (2) if v is less than y, then further reveal the true value of v. The reason is that the buyer will not buy the product anyway when v is below y, so the seller does not lose by revealing the true v when it is low. Such a statistical experiment is called upper censorship by Kolotilin *et al.* (2017), or upper-censoring by Alonso and Câmara (2016b). With this interpretation, monotone binary partitions can be ranked in terms of Blackwell informativeness: a higher cutoff is more Blackwell informative. We thus obtain the following result similar to Hedlund (2017).

COROLLARY 1 *The high-type seller implements a Blackwell more informative stochastic experiment than the low-type seller's; The high-type seller implements a Blackwell more informative stochastic experiment than that she would use if her type were public information.*

[7] This may suggest that a more general condition could be found on the equivalency of Riley's outcome and the intuitive equilibrium along this literature.

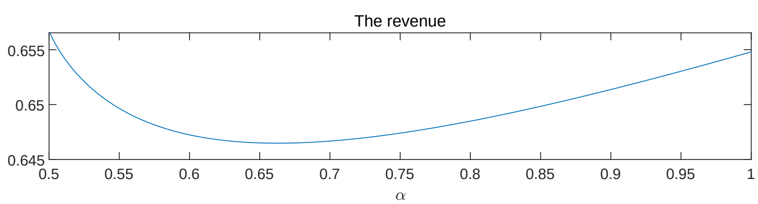
7 Comparative statics

Consider any two distributions $F_1(v)$ and $F_2(v)$ that satisfy the monotone likelihood ratio property: $\frac{f_1(v)}{f_2(v)} > \frac{f_1(v')}{f_2(v')}, \forall v > v'$. Let $F_H(v) = \alpha F_1(v) + (1-\alpha) F_2(v)$, $F_L(v) = \beta F_1(v) + (1-\beta) F_2(v)$, and $F(v) = \mu_H^0 F_1(v) + \mu_L^0 F_2(v)$, for $\alpha \in [\mu_H^0, 1], \beta \in [0, \mu_H^0]$. Then $\frac{f_H(v)}{f_L(v)} > \frac{f_H(v')}{f_L(v')}, \forall v > v'$. By Bayes plausible, $\mu_H^0 \alpha + \left(1-\mu_H^0\right)\beta = \mu_H^0$. Then $\beta = \frac{\mu_H^0(1-\alpha)}{1-\mu_H^0}$. When $\alpha = \mu_H^0$, the seller is unformed; when $\alpha = 1$, the seller is mostly informed. As a result, the parameter α measures how informative the seller's private signal is. We have the following result.[8]

PROPOSITION 7 *In intuitive equilibria, an informed seller's expected profit is lower than that when she is uninformed.*

This result is in line with Alonso and Câmara (2018): private information hurts the sender. One may conjecture that the seller's expected profit monotonically decreases with her private information, i.e., it decreases with $\alpha(> 1/2)$. The following example shows that this is not necessary. The reason is that when α increases, the quality of the high type increases in terms of likelihood ratio, which benefits the seller.

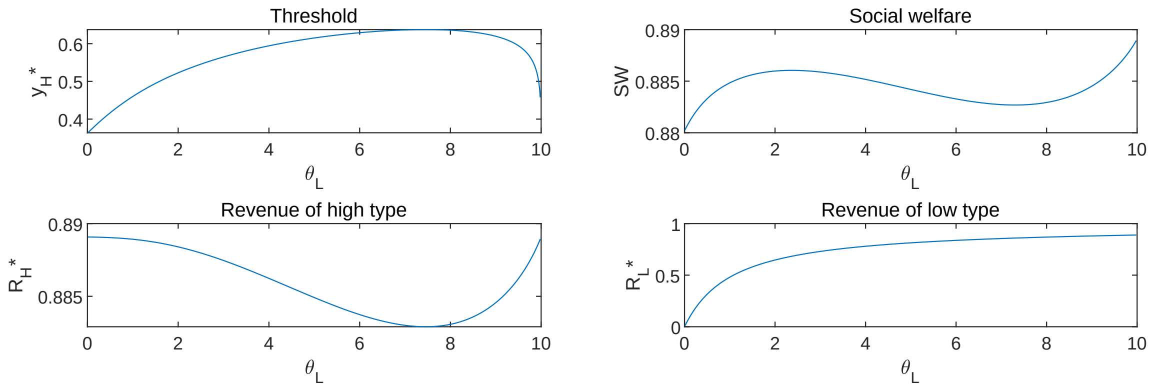
EXAMPLE 1 *Suppose* $F_1(v) = v^5$, $F_2(v) = v$, *on* $[0, 1]$, $c = 0.01$, $\mu_H^0 = 0.5$.



It is also natural to investigate the impact of seller's quality. Here, we investigate how θ_L affects the equilibrium. It turns out that when the low type increases in likelihood ratio, the high-type seller's cutoff and profit, as well as social welfare are not necessary monotone. The following example illustrates that when there is regulation of the minimum quality standard, social surplus could potentially fall.

EXAMPLE 2 *Suppose* $F_\theta(v) = v^\theta, c = 0.02, \theta_H = 10, \mu_H^0 = 0.99,$

[8] We thank one of the referees for pointing out this result.



According to the figure, as θ_L increases, the cutoff of the high-type seller first increases and then decreases dramatically. As a result, the profit of the high type first decreases and then increases. Although the low-type seller's profit always increases with θ_L, the social welfare decreases for $\theta_L \in [2.2\ 7.7]$. If a minimal quality standard is imposed on this range, it could lead to a loss of social welfare.

8 Discussions

8.1 Ex post price

To compare our result with that in Hedlund (2017), we examine the case where the price is automatically adjusted to be equal to the buyer's ex post valuation. We have the following proposition.

PROPOSITION 8 *With ex post price, in any PBE, the high-type seller discloses full information.*

Hedlund (2017) shows that D1 equilibrium is separating or has the high type disclosing full information. Because of the structure of our model, the result is even sharper as full disclosure has to be adopted by the high-type seller in any PBE. While ex post price provides a useful benchmark, it requires the seller to be able to observe the ex post valuation of the buyer. Unfortunately, in reality, the realized valuation is usually observed by the buyer only. For instance, after buyers' trials, sellers can hardly tell whether they like the products or not. In this case, our analysis with ex ante price applies. It turns out that full disclosure does not arise in the equilibrium we characterized in Theorem 1. This demonstrates that pricing strategies play an important role in predicting the outcome.

It is worthy of noting that with ex post price here, we cannot focus on binary signal space, which is consistent with Hedlund (2017). The reason is that even though we can always replace a general signal space with a binary one to obtain the same allocation of the object, the seller's profits and the buyer's payoffs are different due to different ex post prices.

8.2 Perfectly informed seller and ex post verifiable information

One common question is what happens when the seller knows perfectly the buyer's valuation. This means that the only uncertainty for the buyer's valuation is the quality of the product and there is no uncertainty resulting from how well the buyer's idiosyncratic preference matches with the product's characteristics. More specifically, we can assume that, instead of receiving a binary informative signal, the seller knows the buyer's valuation (or equivalently the quality) exactly and privately. Not surprisingly, in this case any PBE leads to the same outcome as if the seller's type were known by the buyer.[9] In other words, when a perfectly informed seller can disclose information through Bayesian persuasion, it turns out that the private information does not result in any distortion and the full information outcome can be achieved. In our main model, the seller is not perfectly informed since the buyers are not homogeneous. With this more general setup, separating can still be achieved but with costs for the high type. To sum up, a take-away message is that, if the seller is perfectly informed, separating arises without cost; if the seller is imperfectly informed, there will be distortion for the high-quality seller.

Another situation to obtain the full information outcome is in Grossman (1981) where the seller can make ex post verifiable disclosures, i.e., the well-known unraveling result. Given the rapid growth of online networks, one prominent source of information the buyer can obtain is from product reviews or word-of-mouth communication. On one hand, if product reviews can fully reveal the quality of a product, the information the seller can reveal becomes ex post verifiable. This applies when there are multiple ex ante identical consumers, and especially if there are many of them.[10] On the other hand, the literature on pricing with social learning and herding demonstrates that such a source of information may not be perfect. First, the seller has incentive to manipulate the buyer's learning process as pointed out by Caminal and Vives (1996, 1999), Bergemann and Välimäki (2000), Vettas (1997), and Bose *et al.* (2008).[11] Second, Caminal and Vives (1999) show that in a duopoly model consumers learn slowly. Third, Bergemann and Välimäki (2000) demonstrates that the learning process may not always be efficient. Finally, Vettas (1997) points out that when consumers' preferences with respect to the product are heterogenous as in our main

[9] A formal proof is available upon request.

[10] We thank the editor and one referee for pointing out this possibility.

[11] In real life example, there are companies who offer to help sellers in Taobao.com, unarguably the most popular online trading platform in China, to manipulate buyers' bad or intermediate reviews at a price of around 28 dollars each. Buyers also report that they are threatened by sellers after leaving a bad review. Furthermore, sellers may also offer rewards for good reviews.

model, learning from others' experiences may not be important.

In our model, we assume away other sources of information. Our result suggests that an alternative for ex post verifiability is to allow perfectly informed sellers to reveal information. In our view, information from the seller and other channels are more like complements rather than substitutes. It would be compelling to have a model to examine the interplays of different sources of information.

9 Conclusion

In this paper, we investigate how the seller can signal her private information through Bayesian persuasion and pricing strategies. We find that signaling cannot arise via one channel alone: different types of sellers adopt different information disclosure strategies and charge different prices in any separating equilibrium. While our model predicts multiple PBE, the intuitive equilibrium outcome is essentially unique. In this equilibrium outcome, the low-type seller's profit is the same as if her type were commonly known; the high type is worse off, since she needs to under serve the buyer to prevent the low type from mimicking. We also demonstrate that a minimum quality standard may not be beneficial for society.

The analysis in our paper can be extended in many different directions. First, we can extend it to allow a continuous seller's type and obtain similar results. We can show that the D1 criterion selects the unique equilibrium outcome. Similarly, this equilibrium outcome is separating. In this equilibrium outcome, all types of sellers adopt a monotone binary partition disclosure rule. The cutoff is strictly increasing in the seller's type, and is determined by a differential equation. All types of sellers charge prices equal to the expected value of the product, conditional on being higher than the cutoff.

Second, we assume that the seller's selling strategy is simple: a take-it-or-leave-it offer after the buyer acquires further information. This resembles many observations in reality: Cosmetic companies usually send out free samples, auto companies usually offer free test drive, and most game or software companies provide a free trial version. However, we can allow the seller to employ a more general pricing strategy: On top of price p, she charges a positive fee w for the buyer to access further information. We can show that, refined by the intuitive criterion, the high-type seller's strategy is unique and remains the same as that in our paper, i.e., $w = 0$. For the low-type seller, his disclosure rule remains the same; however, there is a continuum of combinations of p and w that could arise, which all yield the same expected profit as in our paper.[12]

[12] The proofs for the two extensions are quite lengthy but available upon request from the authors.

Finally, we assume that buyers do not have private information at the beginning. If the seller is not allowed to elicit information from the buyer, similar to Kamenica and Gentzkow (2011), our analysis easily goes through by taking the expectation over the buyer's private information. Challenges arise if the seller is allowed to disclose information conditional on the (reported) type of the buyer, or furthermore, the seller can design a general mechanism to sell the object. This will be left for future investigation.

10 Appendix A: Proofs

Proof of Lemma 3

The proof is by construction. Denote the set of signals that leads to buying and not buying as $S^1 = \{s \in S | \rho(\pi, p, s) = 1\}$ and $S^2 = \{s \in S | \rho(\pi, p, s) = 0\}$, respectively. Let $\pi^D(s_1|v) = \sum_{s \in S^1} \pi(s|v)$, and $\pi^D(s_2|v) = \sum_{s \in S^2} \pi(s|v)$ denote the probabilities of generating signal 's_1' and 's_2'. Then $\pi^D(s_1|v) + \pi^D(s_2|v) = 1$. We maintain the same price. Note that $\rho(\pi, w, p, s) = 1$ if and only if $\int_0^{\bar{v}} v f(v|\mu, \pi, s) dv \geq p$. Therefore, $\rho(\pi^D, p, s_1) = \sum_{s \in S^1} \frac{\int_0^{\bar{v}} \pi(s|v) dv}{\int_0^{\bar{v}} \pi^D(s_1|v) dv} \rho(\pi, p, s) = 1$. Similarly, $\rho(\pi^D, p, s_2) = 0$. Therefore, the purchase decisions are the same. For the buyer with value v, the probability of buying the product given strategy (π, p) is

$$q(v) = \frac{\sum_{s \in S^1} \pi(s|v)}{\sum_{s \in S^1} \pi(s|v) + \sum_{s \in S^2} \pi(s|v)} = \frac{\pi^D(s_1|v)}{\pi^D(s_1|v) + \pi^D(s_2|v)} = q^D(v),$$

Since the price remains the same, the buyer's payoff does not change. The seller's expected profit is

$$r_\theta(\pi, p, \mu_H) = p \left[\sum_{s \in S^1} \int_0^{\bar{v}} \pi(s|v) f_\theta(v) dv \right] = p \int_0^{\bar{v}} \pi^D(s_1|v) f_\theta(v) dv = r_\theta^D(\pi^D, p, \mu_H), \forall \theta.$$

As a result, the seller's expected profit are also the same. **Q.E.D.**

Proof for Lemma 5

For the first statement, we need to show that for π and s leading to non-degenerated belief of v, $\forall$ $\mu_H > \mu_H'$, we have $\frac{\int_0^{\bar{v}} v\pi(s|v) f(v|\mu_H) dv}{\int_0^{\bar{v}} \pi(s|v) f(v|\mu_H) dv} > \frac{\int_0^{\bar{v}} v\pi(s|v) f(v|\mu_H') dv}{\int_0^{\bar{v}} \pi(s|v) f(v|\mu_H') dv}$.

It is easy to show that $\frac{f(v|\mu_H)}{f(v|\mu'_H)}$ is strictly increasing in v. Now,

$$\begin{aligned}
&\frac{\int_0^{\overline{v}} v\pi(s|v)f(v|\mu_H)dv}{\int_0^{\overline{v}} \pi(s|v)f(v|\mu_H)dv} - \frac{\int_0^{\overline{v}} v\pi(s|v)f(v|\mu'_H)dv}{\int_0^{\overline{v}} \pi(s|v)f(v|\mu'_H)dv} \\
= &\frac{1}{\int_0^{\overline{v}} \pi(s|v)f(v|\mu_H)dv} \int_0^{\overline{v}} \left[v - \frac{\int_0^{\overline{v}} v\pi(s|v)f(v|\mu'_H)dv}{\int_0^{\overline{v}} \pi(s|v)f(v|\mu'_H)dv} \right] \pi(s|v) \frac{f(v|\mu_H)}{f(v|\mu'_H)} f(v|\mu'_H)dv \\
> &\frac{1}{\int_0^{\overline{v}} \pi(s|v)f(v|\mu_H)dv} \frac{f(x|\mu_H)}{f(x|\mu'_H)} \underbrace{\int_0^{\overline{v}} (v-x)\pi(s|v)f(v|\mu'_H)dv}_{=0} = 0,
\end{aligned}$$

where $x = \frac{\int_0^{\overline{v}} v\pi(s|v)f(v|\mu'_H)dv}{\int_0^{\overline{v}} \pi(s|v)f(v|\mu'_H)dv}$. The strict inequality holds since $x \in [0, \overline{v}]$ and $\frac{f(v|\mu_H)}{f(v|\mu'_H)}$ is strictly increasing in v. Thus, $E\{V|\mu_H(\pi,p),\pi,s\}$, is strictly increasing in $\mu_H(\pi,p)$. Obviously, when π and s lead to degenerated belief of v, $E\{V|\mu_H(\pi,p),\pi,s\}$ does not change with $\mu_H(\pi,p)$.

For the second statement, we know that $r_\theta(\pi,p,\mu_H(\pi,p))$ is determined by (2). When $\mu_H(\pi,p)$ changes, it only affects the set of signals leading to sale $B(\mu_H(\pi,p))$. Since the expected final value $E\{V|\mu_H(\pi,p),\pi,s\}$ is increasing in $\mu_H(\pi,p)$ according to the first statement, the set $B(\mu_H(\pi,p))$ becomes larger. As a result, $r_\theta(\pi,p,\mu_H(\pi,p))$ is weakly increasing in $\mu_H(\pi,p)$ when $p \geq c$. **Q.E.D.**

Proof for Lemma 6

For the high type,

$$\begin{aligned}
r_H(\pi,p,1) &= \sum\nolimits_{s\in B(1)} \left[(p-c) \int_0^{\overline{v}} \pi(s|v) f_H(v)dv \right] \\
&\leq \sum\nolimits_{s\in B(1)} \left\{ \left[\frac{\int_0^{\overline{v}} v\pi(s|v)f_H(v)dv}{\int_0^{\overline{v}} \pi(s|v)f_H(v)dv} - c \right] \int_0^{\overline{v}} \pi(s|v) f_H(v)dv \right\} \quad (by\ Lemma\ 4) \\
&= \int_0^{\overline{v}} (v-c) \left[\sum\nolimits_{s\in B(1)} \pi(s|v) \right] f_H(v)dv \leq \int_c^{\overline{v}} (v-c) f_H(v)dv,
\end{aligned}$$

Clearly, the described strategy in the lemma achieves the maximum and therefore is optimal.

Now we show that the described strategy in the lemma is the unique maximum. The second inequality above holds if and only if

$$\sum\nolimits_{s\in B(1)} \pi(s|v) = \begin{cases} 1 & if\ v \geq c \\ 0 & if\ otherwise \end{cases}. \tag{8}$$

If $B(1) = \emptyset$, then $\sum_{s \in B(1)} \pi(s|v) = 0$; if $B(1) = \{s_1, s_2\}$, then $\sum_{s \in B(1)} \pi(s|v) = 1$. Both lead to a contradiction to (8). Since we define s_1 (s_2) as the signal leading to a higher (lower) expected value, we must have $B(1) = \{s_1\}$. This has two direct implications. First, (8) becomes $\pi_H(s_1|v)$ described in the lemma. Second, we have $p \leq \frac{\int_c^{\overline{v}} v f_H(v) dv}{1 - F_H(c)}$ according to Lemma 4. Given $\pi_H(s_1|v)$, we have $r_H(\pi, p, 1) = (p - c)[1 - F_H(c)]$ with $p \leq \frac{\int_c^{\overline{v}} v f_H(v) dv}{1 - F_H(c)}$. Therefore, only p_H described in the lemma achieves the maximum. Similar argument follows for the low-type seller. **Q.E.D.**

Proof for Proposition 1

Consider the strategy with $\pi(s_1|v) = \begin{cases} 1, v \geq c, \\ 0, \text{ otherwise,} \end{cases}$ and $p = \frac{\int_c^{\overline{v}} v f_L(v) dv}{\int_c^{\overline{v}} f_L(v) dv}$. Regardless of the belief $\mu_H(\pi, p)$, we have $B(\mu_H(\pi, p)) = \{s_1\}$, and $r_\theta(\pi, p, \mu_H(\pi, p)) = \underline{r}_\theta$. Thus, her profit is independent of the buyer's belief if the seller adopts the above strategy. If the above strategy is on the equilibrium path of type θ, then type θ seller will make a profit equal to $\underline{r}_\theta$. If the above strategy is off the equilibrium path of type θ, the equilibrium condition for PBE implies that type θ seller will make a profit weakly higher than $\underline{r}_\theta$. **Q.E.D.**

Proof for Proposition 3

The proof is by construction. Consider any PBE in the game with general signal space, $(\sigma, \rho, \mu_H(\pi, p))$. We proceed in three steps.

Step 1: By Lemma 3, for any (π, p) with belief $\mu_H(\pi, p)$ in the game with general signal space, there exists an outcome-equivalent strategy in which the statistical experimental has two signals. Pick up anyone, say (π_B, p). Then we can construct a function $\mathcal{K}$ such that $\mathcal{K}(\pi, p) = (\pi_B, p)$. Define the inverse set $A(\pi_B, p) = \{\pi | \mathcal{K}(\pi, p) = (\pi_B, p)\}$. Construct a strategy for the game with binary signal space as follows:

$$\sigma'_\theta(\pi_B, p) = \int_{\pi \in A(\pi_B, p)} \sigma_\theta(\pi, p) \, d\pi,$$

According to Lemma 3, the union of the sets of $A(\pi_B, p)$ cover the whole statistic experimental space with general signal space, thus σ'_θ is a proper strategy.

Step 2: we show that σ' is outcome-equivalent to σ and therefore type-θ seller has no incentive to mimic the other type. Consider any on-equilibrium-path (π_B, p), its belief can be derived from

Bayes' rule:

$$
\begin{aligned}
& \mu_H' \left(\pi_B, p\right) \\
= \; & \frac{\mu_H^0 \sigma_H' \left(\pi_B, p\right)}{\mu_H^0 \sigma_H' \left(\pi_B, p\right) + \left(1-\mu_H^0\right) \sigma_L' \left(\pi_B, p\right)} = \frac{\mu_H^0 \int_{\pi \in A(\pi_B, p)} \sigma_H \left(\pi, p\right) d\pi}{\mu_H^0 \int_{\pi \in A(\pi_B, p)} \sigma_H \left(\pi, p\right) d\pi + \left(1-\mu_H^0\right) \int_{\pi \in A(\pi_B, p)} \sigma_L \left(\pi, p\right) d\pi} \\
= \; & \frac{\int_{\pi \in A(\pi_B, p)} \frac{\mu_H^0 \sigma_H(\pi, p)}{\mu_H^0 \sigma_H(\pi, p) + \left(1-\mu_H^0\right) \sigma_L(\pi, p)} \left[\mu_H^0 \sigma_H \left(\pi, p\right) + \left(1-\mu_H^0\right) \sigma_L \left(\pi, p\right)\right] d\pi}{\mu_H^0 \int_{\pi \in A(\pi_B, p)} \sigma_H \left(\pi, p\right) d\pi + \left(1-\mu_H^0\right) \int_{\pi \in A(\pi_B, p)} \sigma_L \left(\pi, p\right) d\pi} \\
= \; & \frac{\int_{\pi \in A(\pi_B, p)} \mu_H \left(\pi, p\right) \left[\mu_H^0 \sigma_H \left(\pi, p\right) + \left(1-\mu_H^0\right) \sigma_L \left(\pi, p\right)\right] d\pi}{\mu_H^0 \int_{\pi \in A(\pi_B, p)} \sigma_H \left(\pi, p\right) d\pi + \left(1-\mu_H^0\right) \int_{\pi \in A(\pi_B, p)} \sigma_L \left(\pi, p\right) d\pi} = \int_{\pi \in A(\pi_B, p)} \mu_H \left(\pi, p\right) a\left(\pi\right) d\pi,
\end{aligned}
$$

where

$$
a\left(\pi\right) = \frac{\mu_H^0 \sigma_H \left(\pi, p\right) + \left(1-\mu_H^0\right) \sigma_L \left(\pi, p\right)}{\mu_H^0 \int_{\pi \in A(\pi_B, p)} \sigma_H \left(\pi, p\right) d\pi + \left(1-\mu_H^0\right) \int_{\pi \in A(\pi_B, p)} \sigma_L \left(\pi, p\right) d\pi}.
$$

It is easy to show that $\int_{\pi \in A(\pi_B, p)} a\left(\pi\right) d\pi = 1$. Thus, the new belief can be regarded as a linear combination of the original belief in the game with general signal space. Therefore, by utilizing equation (1), the expected value of the object given signal s_1 and the new belief becomes :

$$
\begin{aligned}
& \frac{\int_0^{\bar{v}} v f(v|\mu_H' \left(\pi_B, p\right), \pi_B, s_1) dv}{\int_0^{\bar{v}} f(v|\mu_H' \left(\pi_B, p\right), \pi_B, s_1) dv} = \frac{\int_{\pi \in A(\pi_B, p)} a\left(\pi\right) \int_0^{\bar{v}} v f(v|\mu_H \left(\pi, p\right), \pi, s_1) dv d\pi}{\int_{\pi \in A(\pi_B, p)} a\left(\pi\right) \int_0^{\bar{v}} f(v|\mu_H \left(\pi, p\right), \pi, s_1) dv d\pi} \\
= \; & \frac{\int_{\pi \in A(\pi_B, p)} a\left(\pi\right) \frac{\int_0^{\bar{v}} v f(v|\mu_H(\pi, p), \pi, s_1) dv}{\int_0^{\bar{v}} f(v|\mu_H(\pi, p), \pi, s_1) dv} \int_0^{\bar{v}} f(v|\mu_H \left(\pi, p\right), \pi, s_1) dv d\pi}{\int_{\pi \in A(\pi_B, p)} a\left(\pi\right) \int_0^{\bar{v}} f(v|\mu_H \left(\pi, p\right), \pi, s_1) dv d\pi} \geq p.
\end{aligned}
$$

Therefore, the buyer would buy the object upon seeing signal s_1, i.e., $\rho(\pi_B, p, s_1) = 1$. Similarly, $\rho(\pi_B, p, s_2) = 0$. Thus, the purchase decision is the same. Then with similar arguments as in Lemma 3, the probability of buying the product, the buyer's payoff and the seller's expected profit are all the same.

Step 3: we show that the constructed σ' can be supported as a PBE. Let us construct the off-equilibrium-path belief as follows. For any off-equilibrium-path strategy (π_B, p), let $\mu_H' \left(\pi_B, p\right) = \mu_H \left(\pi, p\right)$ for some $\pi \in A(\pi_B, p)$. Obviously, (π, p) is off-equilibrium-path for the strategy σ. Since σ can be supported as an equilibrium by $\mu_H \left(\pi, p\right)$, $r_\theta \left(\pi_B, p, \mu_H' \left(\pi_B, p\right)\right) = r_\theta(\pi, p, \mu_H(\pi, p)) \leq r_\theta^*$. The claim is verified. The vice versa part is straightforward. **Q.E.D.**

Proof for Proposition 4

In separating equilibria, if the low-type seller adopts the equilibrium strategy, the buyer will infer that the seller is of low type for sure. Given this, an upper bound profit for the low-type seller is the maximum profit she can obtain when her type is observable, i.e., $r_L^* \leq \underline{r}_L$. On the other hand, by Proposition 1, $r_L^* \geq \underline{r}_L$. Therefore, $r_L^* = \underline{r}_L$. By Lemma 6, to achieve $\underline{r}_L$, the low-type seller's strategy is uniquely determined by the prescribed strategy. **Q.E.D.**

Proof for Proposition 5

We first claim that in any PBE, the high-type seller makes a trade with probability strictly higher than zero and strictly lower than one. Let (π_H, p_H) be the high-type seller's equilibrium strategy. If the trading probability is zero, then her profit is zero, which violates Proposition 1. If the trading probability is one, then

$$\begin{aligned} r_H(\pi_H, p_H, 1) &= r_L(\pi_H, p_H, 1) \leq r_L^* \quad (By\ PBE) \\ &= \int_c^{\overline{v}} v f_L(v) dv < \frac{1 - F_H(c)}{1 - F_L(c)} \int_c^{\overline{v}} v f_L(v) dv = r_H(\pi_L^*, p_L^*, 0) \end{aligned}$$

Thus, the high type has an incentive to mimic the low-type seller, a contradiction. As a result, our claim holds, and Part (A) and (B) of the proposition follow directly.

Now we show part (C). In separating equilibria, the low type's strategy is described in Proposition 4. Suppose in separating equilibria, type θ_H adopts strategy (π_H, p_H). Part (A) and (B) imply that the high-type seller makes a trade if and only in the realized signal is s_1, i.e., $B(1) = \{s_1\}$.

First, we show $\pi_H \neq \pi_L^*$ by contradiction. Suppose not and we have $\pi_H = \pi_L^*$. In separating equilibria, $p_H \neq p_L^*$. There are two cases, $p_L^* > p_H$ and $p_L^* < p_H$. In case 1, $p_L^* > p_H$. Then

$$\begin{aligned} r_H(\pi_L^*, p_L^*, 0) &= (p_L^* - c) \int_0^{\overline{v}} \pi_L^*(s_1|v) f_H(v) dv = (p_L^* - c) \int_0^{\overline{v}} \pi_H(s_1|v) f_H(v) dv \\ &> (p_H - c) \int_0^{\overline{v}} \pi_H(s_1|v) f_H(v) dv = r_H(\pi_H, p_H, 1). \end{aligned}$$

Thus, the high type has an incentive to mimic the low type, a contradiction. In case 2, $p_L^* < p_H$. By similar arguments, we can show that the low type has an incentive to mimic the high type, a contradiction.

Second, we show $p_H \neq p_L^*$ by contradiction. Suppose not and we have $p_H = p_L^*$. By the definition of separating equilibrium, the high-type seller has no incentive to mimic the low-type seller:

$$\begin{aligned}
& r_H(\pi_L^*, p_L^*, 0) \le r_H(\pi_H, p_H, 1) \\
\Leftrightarrow \quad & (p_H - c)\int_0^{\overline{v}} \pi_L^*(s_1|v) f_H(v)dv \le (p_H - c)\int_0^{\overline{v}} \pi_H(s_1|v) f_H(v)dv \\
\Leftrightarrow \quad & \int_0^{\overline{v}} \pi_L^*(s_1|v) f_H(v)dv \le \int_0^{\overline{v}} \pi_H(s_1|v) f_H(v)dv \quad (\textit{since } p_H > c \textit{ according to Proposition } 2) \\
\Leftrightarrow \quad & \int_c^{\overline{v}} f_H(v)dv \le \int_0^{\overline{v}} \pi_H(s_1|v) f_H(v)dv. \qquad (9)
\end{aligned}$$

Before proceeding with (9) further, we show $\int_0^c \pi_H(s_1|v) f_H(v)dv > 0$. Suppose not and we have $\int_0^c \pi_H(s_1|v) f_H(v)dv = 0$. Then

$$\int_0^{\overline{v}} \pi_H(s_1|v) f_H(v)dv = \int_c^{\overline{v}} \pi_H(s_1|v) f_H(v)dv \quad < \quad \int_c^{\overline{v}} f_H(v)dv \qquad (10)$$

The strict inequality follows because $\pi_H \neq \pi_L^*$, $\pi_H(s_1|v) = 0, \forall v \in [0, c]$, and π_L^* is a monotone binary partition with cutoff c. However, (10) contradicts with (9) and we must have $\int_0^c \pi_H(s_1|v) f_H(v)dv > 0$.

Now, we can proceed with (9) further:

$$\begin{aligned}
0 \quad & \le \quad \int_0^{\overline{v}} \pi_H(s_1|v) f_H(v)dv - \int_c^{\overline{v}} f_H(v)dv \\
& = \quad \int_0^c \pi_H(s_1|v) \frac{f_H(v)}{f_L(v)} f_L(v)dv - \int_c^{\overline{v}} [1 - \pi_H(s_1|v)] \frac{f_H(v)}{f_L(v)} f_L(v)dv
\end{aligned}$$

$$\begin{aligned}
< \quad & \frac{f_H(c)}{f_L(c)} \left\{ \int_0^c \pi_H(s_1|v) f_L(v)dv - \int_c^{\overline{v}} [1 - \pi_H(s_1|v)] f_L(v)dv \right\} \\
\Leftrightarrow \quad & \int_0^c \pi_H(s_1|v) f_L(v)dv > \int_c^{\overline{v}} [1 - \pi_H(s_1|v)] f_L(v)dv \\
\Leftrightarrow \quad & \int_0^{\overline{v}} \pi_H(s_1|v) f_L(v)dv > \int_c^{\overline{v}} f_L(v)dv \Leftrightarrow \int_0^{\overline{v}} \pi_H(s_1|v) f_L(v)dv > \int_0^{\overline{v}} \pi_L^*(s_1|v) f_L(v)dv. \ (11)
\end{aligned}$$

Given inequality (11), we can show that the low type has an incentive to deviate to the high type's equilibrium strategy (π_H, p_H):

$$\begin{aligned} r_L(\pi_H, p_H, 1) &= (p_H - c)\int_0^{\overline{v}} \pi_H(s_1|v) f_H(v)dv = (p_L^* - c)\int_0^{\overline{v}} \pi_H(s_1|v) f_L(v)dv \\ &> (p_L^* - c)\int_0^{\overline{v}} \pi_L^*(s_1|v) f_L(v)dv = r_L(\pi_L^*, p_L^*, 0). \end{aligned}$$

This completes the proof. **Q.E.D.**

Proof for Lemma 7

By Lemma 6, $r_H(\pi, p, 1) \leq \int_c^{\overline{v}}(v-c)f_H(v)dv$, and the equality holds if and only if $(\pi, p) = (\pi_H, p_H)$ defined in Lemma 6 with $\theta = H$. Given that $(\pi, p) = (\pi_H, p_H)$, $r_H(\pi, p, \mu_H(\pi, p))$ is increasing in $\mu_H(\pi, p)$, and achieves $r_H(\pi, p, 1)$ if and only if $\mu_H(\pi, p) = 1$. To summarize, $r_H(\pi, p, \mu_H(\pi, p)) \leq \int_c^{\overline{v}}(v-c)f_H(v)dv$, and the equality holds if and only if $(\pi, p) = (\pi_H, p_H)$ and $\mu_H(\pi, p) = 1$. Since at least one of conditions (i)-(iii) hold, we have $r_H(\pi, p, \mu_H(\pi, p)) < \int_c^{\overline{v}}(v-c)f_H(v)dv$. Furthermore, it is assumed that $r_H(\pi, p, \mu_H(\pi, p)) > 0$. Since $\int_y^{\overline{v}}(v-c)f_H(v)dv$ is strictly decreasing in $y \geq c$, and is equal to 0 and $\int_c^{\overline{v}}(v-c)f_H(v)dv$ when y is equal to $\overline{v}$ and c, we can always find a unique $y' \in (c, \overline{v})$ such that (2) is satisfied.

Now we show inequalities (3) to (5) hold. For inequality (3), suppose in contrary, we have $\int_0^{y'} \sum_{s\in B(\mu_H(\pi,p))} \pi(s|v) f_H(v)dv = 0$. Then $\sum_{s\in B(\mu_H(\pi,p))} \pi(s|v) = 0, \forall v \in [0, y']$. As a result,

$$\begin{aligned} & r_H(\pi, p, \mu_H(\pi, p)) \\ = \;& (p-c)\int_{y'}^{\overline{v}} \sum\nolimits_{s\in B(\mu_H(\pi,p))} \pi(s|v) f_H(v)dv \\ \leq \;& \sum\nolimits_{s\in B(\mu_H(\pi,p))} \left\{ \int_{y'}^{\overline{v}} \pi(s|v) f_H(v)dv \left[\frac{\int_{y'}^{\overline{v}} v\pi(s|v) f(v|\mu_H(\pi,p))dv}{\int_{y'}^{\overline{v}} \pi(s|v) f(v|\mu_H(\pi,p))dv} - c \right] \right\} \text{ (by Lemma 4)} \\ \leq \;& \sum\nolimits_{s\in B(\mu_H(\pi,p))} \left\{ \int_{y'}^{\overline{v}} \pi(s|v) f_H(v)dv \left[\frac{\int_{y'}^{\overline{v}} v\pi(s|v) f_H(v)dv}{\int_{y'}^{\overline{v}} \pi(s|v) f_H(v)dv} - c \right] \right\} \text{ (by Lemma 5)} \\ = \;& \sum\nolimits_{s\in B(\mu_H(\pi,p))} \int_{y'}^{\overline{v}} (v-c)\pi(s|v) f_H(v)dv \leq \int_{y'}^{\overline{v}} (v-c) f_H(v)dv. \end{aligned}$$

With a similar argument to the proof of Lemma 6, it is easy to show that the equality holds only when π is a monotone binary partition with cutoff y', $\mu_H(\pi, p) = 1$ and $p = \mathbf{E}\{V|1, \pi, s_1\}$. Since at least one of conditions (i), (ii) and (iii) hold, we have $r_H(\pi, p, \mu_H(\pi, p)) < \int_{y'}^{\overline{v}}(v-c)f_H(v)dv$, which contradicts the definition of y'.

For inequality (4), by the definition of y', we have

$$
\begin{aligned}
& \int_{y'}^{\overline{v}} (v-c) f_H(v) dv \\
= \;& \sum\nolimits_{s \in B(\mu_H(\pi,p))} \left[(p-c) \int_0^{\overline{v}} \pi(s|v) f_H(v) dv \right] \\
\leq \;& \sum\nolimits_{s \in B(\mu_H(\pi,p))} \left\{ \left[\frac{\int_0^{\overline{v}} v\pi(s|v) f(v|\mu_H(\pi,p)) dv}{\int_0^{\overline{v}} \pi(s|v) f(v|\mu_H(\pi,p)) dv} - c \right] \int_0^{\overline{v}} \pi(s|v) f_H(v) dv \right\} \quad \text{(by Lemma 4)} \\
\leq \;& \sum\nolimits_{s \in B(\mu_H(\pi,p))} \left\{ \left[\frac{\int_0^{\overline{v}} v\pi(s|v) f_H(v) dv}{\int_0^{\overline{v}} \pi(s|v) f_H(v) dv} - c \right] \int_0^{\overline{v}} \pi(s|v) f_H(v) dv \right\} \quad \text{(by Lemma 5)} \\
= \;& \int_0^{y'} (v-c) \left[\sum\nolimits_{s \in B(\mu_H(\pi,p))} \pi(s|v) \right] f_H(v) dv + \int_{y'}^{\overline{v}} (v-c) \left[\sum\nolimits_{s \in B(\mu_H(\pi,p))} \pi(s|v) \right] f_H(v) dv \\
\Leftrightarrow \;& \int_0^{y'} (v-c) \left[\sum_{s \in B(\mu_H(\pi,p))} \pi(s|v) \right] f_H(v) dv \geq \int_{y'}^{\overline{v}} (v-c) \left[1 - \sum_{s \in B(\mu_H(\pi,p))} \pi(s|v) \right] f_H(v) dv \\
\Rightarrow \;& \int_0^{y'} (y'-c) \left[\sum_{s \in B(\mu_H(\pi,p))} \pi(s|v) \right] f_H(v) dv > \int_{y'}^{\overline{v}} (y'-c) \left[1 - \sum_{s \in B(\mu_H(\pi,p))} \pi(s|v) \right] f_H(v) dv \\
\Leftrightarrow \;& \int_0^{\overline{v}} \left[\sum_{s \in B(\mu_H(\pi,p))} \pi(s|v) \right] f_H(v) dv > 1 - F_H(y')
\end{aligned}
$$

For inequality (5), denote

$$
\begin{aligned}
\int_{y'}^{\overline{v}} \left[\sum_{s \in B(\mu_H(\pi,p))} \pi(s|v) \right] f_H(v) dv &= a, \int_0^{y'} \left[\sum_{s \in B(\mu_H(\pi,p))} \pi(s|v) \right] f_H(v) dv = b, \\
\int_{y'}^{\overline{v}} \left[1 - \sum_{s \in B(\mu_H(\pi,p))} \pi(s|v) \right] f_H(v) dv &= c, \int_{y'}^{\overline{v}} \frac{f_L(v)}{f_H(v)} \left[\sum_{s \in B(\mu_H(\pi,p))} \pi(s|v) \right] f_H(v) dv = ax_1, \\
\int_0^{y'} \frac{f_L(v)}{f_H(v)} \left[\sum_{s \in B(\mu_H(\pi,p))} \pi(s|v) \right] f_H(v) dv &= bx_2, \int_{y'}^{\overline{v}} \frac{f_L(v)}{f_H(v)} \left[1 - \sum_{s \in B(\mu_H(\pi,p))} \pi(s|v) \right] f_H(v) dv = cx_3.
\end{aligned}
$$

Note that by inequality (3) and (4), we have $b > 0$ and $b > c$. Furthermore, Since $a + c = \int_{y'}^{\overline{v}} f_H(v)dv > 0$, we know either $a > 0$ or $c > 0$. Since $\frac{f_L(v)}{f_H(v)}$ is decreasing in v, we have $bx_2 > b\frac{f_L(y')}{f_H(y')}$, and $ax_1 \leq a\frac{f_L(y')}{f_H(y')}$ with equality holds if and only if $a = 0$, and $cx_3 \leq c\frac{f_L(y')}{f_H(y')}$ with equality holds if and only if $c = 0$. Set $x_1 = \frac{f_L(y')}{f_H(y')}$ when $a = 0$, $x_3 = \frac{f_L(y')}{f_H(y')}$ when $c = 0$. Then $x_2 > x_1, x_2 > x_3$. Given those relationships, we have

$$\frac{\int_0^{\overline{v}} \left[\sum_{s\in B(\mu_H(\pi,p))} \pi(s|v)\right] f_L(v)dv}{\int_0^{\overline{v}} \left[\sum_{s\in B(\mu_H(\pi,p))} \pi(s|v)\right] f_H(v)dv} - \frac{\int_{y'}^{\overline{v}} f_L(v)dv}{\int_{y'}^{\overline{v}} f_H(v)dv}$$

$$= \frac{ax_1 + bx_2}{a+b} - \frac{ax_1 + cx_3}{a+c} = \frac{acx_1 + abx_2 + bcx_2 - abx_1 - acx_3 - bcx_3}{(a+b)(a+c)}$$

$$\geq \frac{acx_1 + abx_2 + bcx_2 - abx_1 - acx_2 - bcx_3}{(a+b)(a+c)} \quad (since\ x_2 > x_3)$$

$$= \frac{a(b-c)(x_2 - x_1) + bc(x_2 - x_3)}{(a+b)(a+c)} > 0.$$

The last step holds since $b > c, x_2 > x_1, x_2 > x_3, b > 0,$ *and either* $a > 0$ *or* $c > 0$. **Q.E.D.**

Proof for Theorem 1

We first establish some properties of the intuitive equilibrium in the following lemma.

LEMMA 8 *Any intuitive equilibrium must be separating. Furthermore, the high-type seller sets the information disclosure and price as follows:* $\sigma(\pi, p|\theta_H) > 0$ *only if* $\pi(s_1|v) = \{\begin{smallmatrix}0, & v<y \\ 1, & v\geq y\end{smallmatrix}$ *for some* $y \geq c$, *and* $p = \frac{\int_y^{\overline{v}} vf_H(v)dv}{1-F_H(v)}$.

Proof: Let (π, p) be the high-type seller's equilibrium strategy and $\mu_H(\pi, p)$ be the buyer's belief derived from Bayes' rule in a PBE. Let r_θ be the equilibrium profit, which is strictly positive according to Proposition 1. Suppose $\{(\pi, p), \mu_H(\pi, p)\}$ is not as described in the proposition, then at least one of the conditions (i), (ii) and (iii) in Lemma 7 holds. By Lemma 7, we can construct a strategy according to (6) and (7). Since π' and the price p' can be fully characterized by the cutoff

y', notational wise, we can write the strategy as $(\pi'(y'), p'(y'))$. We have

$$
\begin{aligned}
& r_L(\pi'(y'), p'(y'), 1) \\
= \;& [p'(y') - c] \sum_{s \in B(1)} \int_0^{\overline{v}} \pi'(y') f_L(v) dv \\
= \;& \left[\frac{\int_{y'}^{\overline{v}} v f_H(v) dv}{1 - F_H(y')} - c \right] [1 - F_L(y')] \quad (\textit{only signal } s_1 \textit{ leads to buy}) \\
= \;& \frac{1 - F_L(y')}{1 - F_H(y')} \int_{y'}^{\overline{v}} (v - c) f_H(v) dv \\
= \;& \frac{1 - F_L(y')}{1 - F_H(y')} r_H(\pi, p, \mu_H(\pi, p)) \quad (\textit{according to equality } (2)) \\
< \;& \frac{\sum_{s \in B(\mu_H(\pi,p))} \int_0^{\overline{v}} \pi(s|v) f_L(v) dv}{\sum_{s \in B(\mu_H(\pi,p))} \int_0^{\overline{v}} \pi(s|v) f_H(v) dv} r_H(\pi, p, \mu_H(\pi, p)) \quad (\textit{according to Proposition } 1 \textit{ and Lemma } 7) \\
= \;& \frac{\sum_{s \in B(\mu_H(\pi,p))} \int_0^{\overline{v}} \pi(s|v) f_L(v) dv}{\sum_{s \in B(\mu_H(\pi,p))} \int_0^{\overline{v}} \pi(s|v) f_H(v) dv} (p - c) \sum_{s \in B(\mu_H(\pi,p))} \int_0^{\overline{v}} \pi(s|v) f_H(v) dv \\
= \;& (p - c) \sum_{s \in B(\mu_H(\pi,p))} \int_0^{\overline{v}} \pi(s|v) f_L(v) dv = r_L(\pi, p, \mu_H(\pi, p)) \leq r_L \quad (\textit{By the definition of PBE}).
\end{aligned}
$$

Since $y' > c$ and $r_L(\pi'(y'), p'(y'), 1) = \frac{1-F_L(y')}{1-F_H(y')} \int_{y'}^{\overline{v}} (v - c) f_H(v) dv$ is continuous and decreasing in y', we can find a $\delta > 0$ such that

$$r_L(\pi'(y' - \delta), p'(y' - \delta), 1) < r_L.$$

Therefore, according to Lemma 5, $\forall \mu_H \in [0, 1]$, we have

$$r_L(\pi'(y' - \delta), p'(y' - \delta), \mu_H) \leq r_L(\pi'(y' - \delta), p'(y' - \delta), 1) < r_L,$$

and thus, Condition (a) in Definition 2 is satisfied. Furthermore,

$$
\begin{aligned}
r_H(\pi'(y' - \delta), p'(y' - \delta), 1) &= \int_{y'-\delta}^{\overline{v}} (v - c) f_H(v) dv > \int_{y'}^{\overline{v}} (v - c) f_H(v) dv \\
&= r_H(\pi, p, 1) \geq r_H(\pi, p, \mu_H(\pi, p)) \quad (\textit{according to Lemma } 5)
\end{aligned}
$$

and Condition (b) in Definition 2 is satisfied. Therefore, according to Definition 2, such a PBE is blocked and cannot be an intuitive equilibrium. $\diamondsuit$

This above lemma implies that the intuitive equilibria must be separating and the buyer does not have any uncertainty about the seller's type on the equilibrium path. As a result, the low-type seller's strategy is uniquely determined and described in Proposition 4. The high-type seller's information disclosure must be in the form of monotone binary partition, and the price must be equal to the expected final value of the product, given that the seller is of the high type and the value is higher than the cutoff. Note that the high-type seller could potentially adopt a mixed strategy by randomizing over monotone binary partitions with different cutoffs. One necessary condition for randomization to happen is that different cutoffs yield the same profit. When the cutoff is y, the high-type seller makes a profit $\int_y^{\overline{v}} v f_H(v)dv$. Since $\int_y^{\overline{v}} v f_H(v)dv$ is strictly decreasing in y, it is not optimal for the high-type seller to randomize. Therefore, what is left is to determine the cutoff y.

If the low-type seller deviates to the high-type seller's strategy with cutoff y, her profit is

$$r_L(y) = \frac{1 - F_L(y)}{1 - F_H(y)} \int_y^{\overline{v}} (v - c) f_H(v)dv$$

It is easy to see that $r_L(y)$ is continuous, strictly decreasing on $[c, \overline{v}]$. Furthermore, $r_L(c) = \frac{1-F_L(c)}{1-F_H(c)} \int_c^{\overline{v}}(v - c) f_H(v)dv$ and $r_L(\overline{v}) = 0$. Finally, we have $0 < r_L^* < \frac{1-F_L(c)}{1-F_H(c)} \int_c^{\overline{v}}(v - c) f_H(v)dv$. Therefore, there exists a unique cutoff $y_H^* \in (c, \overline{v})$ such that $r_L^* = r_L(y_H^*)$.

According to Propositions 4 and 8, we only need to show that (I) in intuitive equilibrium, we must have $y = y_H^*$, (II) the proposed strategy can be supported as a separating equilibrium, and (III) the proposed strategy satisfies the intuitive criterion.

(I) The low-type seller's profit if she deviates to the high-type seller's equilibrium strategy is $\frac{1-F_L(y)}{1-F_H(y)} \int_y^{\overline{v}}(v - c) f_H(v)dv$, and her equilibrium path profit is r_L^*. PBE requires that the low type will not want to deviate to the high-type seller's equilibrium strategy, therefore, by the definition of y_H^*, we must have $y \geq y_H^*$.

Now, suppose $y > y_H^*$. Then there exists a positive δ such that $y' = y - \delta > y_H^*$. Define (π', p') such that $\pi'(s_1|v) = \begin{cases} 1, & if\ v \geq y' \\ 0, & if\ otherwise \end{cases}$ and $p' = \frac{\int_{y'}^{\overline{v}} v f_H(v)dv}{1-F_H(y')}$.

Then, $\forall \mu_H \in [0,1]$,

$$\begin{aligned} r_L(\pi', p', \mu_H) &\leq r_L(\pi', p', 1) \quad (\textit{by Lemma } 5) \\ &= \frac{1-F_L(y')}{1-F_H(y')} \int_{y'}^{\overline{v}} (v-c) f_H(v) dv < \frac{1-F_L(y_H^*)}{1-F_H(y_H^*)} \int_{y_H^*}^{\overline{v}} (v-c) f_H(v) dv = r_L^*. \end{aligned}$$

Furthermore, $r_H(\pi', p', 1) = \int_{y'}^{\overline{v}} (v-c) f_H(v) dv > \int_{y}^{\overline{v}} (v-c) f_H(v) dv = r_H(\pi, p, 1)$. Therefore, conditions in Definition 2 are satisfied and the PBE is blocked. This concludes that the only possible PBE that survives the intuitive criterion is as described in the Theorem.

(II) Given the off-equilibrium-path belief being low type for sure, for the proposed strategies to be a separating equilibrium, we need to show that the following conditions hold:

$$r_L^* \geq r_L(\pi_H^*, p_H^*, 1), \tag{12}$$

$$r_L^* \geq r_L(\pi, p, 0), \forall \pi, p \tag{13}$$

$$r_H^* \geq r_H(\pi, p, 0), \forall \pi, p. \tag{14}$$

Condition (12) ensures that the low type will not want to mimic the high type; Condition (13) ensures that the low type will not want deviate to any off-equilibrium-path strategies; Condition (14) ensures that the high type has no incentive to mimic the low type or to deviate to any off-equilibrium-path strategies.

By the definition of (π_H^*, p_H^*), Condition (12) holds. Condition (13) follows Lemma 6. Now we show that Condition (14) holds by contradiction. Suppose in contrary, there exists some (π, p) such that $r_H^* < r_H(\pi, p, 0)$. This implies that $r_H(\pi, p, 0) > 0$ and condition (ii) in Lemma 7 is met. Therefore, we can construct a strategy (π', p') according to (6) and (7) such that $r_H(\pi', p', 1) = r_H(\pi, p, 0)$. As such,

$$r_H^* = \int_{y_H^*}^{\overline{v}} (v-c) f_H(v) dv < r_H(\pi', p', 1) = (p'-c)(1-F_H(y')) = \int_{y'}^{\overline{v}} (v-c) f_H(v) dv,$$

which implies $y' < y_H^*$. Now,

$$\begin{aligned} & r_L(\pi, p, 0) \\ = \ & (p-c) \sum\nolimits_{s \in B(0)} \int_0^{\overline{v}} \pi(s|v) f_L(v) dv \end{aligned}$$

$$
\begin{aligned}
&= \frac{\sum_{s\in B(0)}\int_0^{\overline{v}}\pi(s|v)f_L(v)dv}{\sum_{s\in B(0)}\int_0^{\overline{v}}\pi(s|v)f_H(v)dv}(p-c)\sum_{s\in B(0)}\int_0^{\overline{v}}\pi(s|v)f_H(v)dv \\
&= \frac{\sum_{s\in B(0)}\int_0^{\overline{v}}\pi(s|v)f_L(v)dv}{\sum_{s\in B(0)}\int_0^{\overline{v}}\pi(s|v)f_H(v)dv}r_H(\pi,p,0) = \frac{\sum_{s\in B(0)}\int_0^{\overline{v}}\pi(s|v)f_L(v)dv}{\sum_{s\in B(0)}\int_0^{\overline{v}}\pi(s|v)f_H(v)dv}r_H(\pi',p',1) \\
&> \frac{1-F_L(y')}{1-F_H(y')}r_H(\pi',p',1) \text{ (By Lemma 7)} \\
&> \frac{1-F_L(y')}{1-F_H(y')}\int_{y_H^*}^{\overline{v}}(v-c)f_H(v)dv
\end{aligned}
$$

$$
\begin{aligned}
&> \frac{1-F_L(y_H^*)}{1-F_H(y_H^*)}\int_{y_H^*}^{\overline{v}}(v-c)f_H(v)dv \quad \textit{(by strict monotonicity)} \\
&= r_L^*,
\end{aligned}
$$

which contradiction (13).

(III) To show the proposed PBE satisfies the intuitive criterion, it is sufficient to show that, $\forall(\pi,p)$, if $r_L(\pi,p,1) < r_L^*$, then $r_H(\pi,p,1) < r_H^*$. Suppose in contrary, and there exists some (π,p) such that

$$
r_L(\pi,p,1) = (p-c)\int_0^{\overline{v}}(\sum_{s\in B(1)}\pi(s|v))f_L(v)dv < r_L^*, \tag{15}
$$

$$
r_H(\pi,p,1) = (p-c)\int_0^{\overline{v}}(\sum_{s\in B(1)}\pi(s|v))f_H(v)dv \geq r_H^*. \tag{16}
$$

There are two cases. In case 1, (π,p) takes the following forms:

$$
\pi(s_1|v) = \begin{cases} 1, & if\ v \geq y \\ 0, & if\ otherwise \end{cases},
$$

$$
p = \frac{\int_y^{\overline{v}} vf_H(v)dv}{1-F_H(y)},
$$

with $y \geq c$. Then

$$r_L(\pi, p, 1) = \frac{1 - F_L(y)}{1 - F_H(y)} \int_y^{\overline{v}} (v - c) f_H(v) dv,$$

$$r_H(\pi, p, 1) = \int_y^{\overline{v}} (v - c) f_H(v) dv.$$

On one hand, by (15), and the definition of y_H^*,

$$\frac{1 - F_L(y)}{1 - F_H(y)} \int_y^{\overline{v}} (v - c) f_H(v) dv < \frac{1 - F_L(y_H^*)}{1 - F_H(y_H^*)} \int_{y_H^*}^{\overline{v}} (v - c) f_H(v) dv$$

$$\Rightarrow \quad y > y_H^* \qquad \text{(by strict monotonicity)}$$

On the other hand, by (16),

$$\int_y^{\overline{v}} (v - c) f_H(v) dv \geq \int_{y_H^*}^{\overline{v}} (v - c) f_H(v) dv \Rightarrow y \leq y_H^*,$$

thus, a contradiction.

In case 2, (π, p) is not as described in Case 1, then at least one of condition (i) and (iii) in Lemma 7 holds. Therefore, we can construct a strategy (π', p') according to (6) and (7) with $y' > c$, such that $r_H(\pi, p, 1) = \int_{y'}^{\overline{v}} (v - c) f_H(v) dv$. Now

$$\begin{aligned}
& r_L(\pi', p', 1) \\
= \; & \frac{1 - F_L(y')}{1 - F_H(y')} \int_{y'}^{\overline{v}} (v - c) f_H(v) dv \\
< \; & \frac{\int_0^{\overline{v}} \left[\sum_{s \in B(1)} \pi(s|v)\right] f_L(v) dv}{\int_0^{\overline{v}} \left[\sum_{s \in B(1)} \pi(s|v)\right] f_H(v) dv} \int_{y'}^{\overline{v}} (v - c) f_H(v) dv \quad \textit{(by Lemma 7)} \\
= \; & \frac{\int_0^{\overline{v}} \left[\sum_{s \in B(1)} \pi(s|v)\right] f_L(v) dv}{\int_0^{\overline{v}} \left[\sum_{s \in B(1)} \pi(s|v)\right] f_H(v) dv} \int_0^{\overline{v}} \left[\sum_{s \in B(1)} \pi(s|v)\right] f_H(v) dv (p - c) \\
= \; & (p - c) \int_0^{\overline{v}} (\sum_{s \in B(1)} \pi(s|v)) f_L(v) dv < r_L^*,
\end{aligned}$$

$$r_H(\pi', p', 1) = r_H(\pi, p, 1) \geq r_H^*.$$

Therefore, the constructed (π', p') also satisfies (15)and (16) as in case 1. With the same argument as in case 1, it leads to a contradiction. **Q.E.D.**

Proof for Proposition 6

"⇒": we only need to show that the PBE with general signal space can be supported by assigning a belief of being the low type for sure upon any off-equilibrium path deviation and this belief satisfies the intuitive criterion. This is similar to the proof in Theorem 1, since by Lemma 3 it is sufficient to consider any off-equilibrium-path strategy with binary signal space.

"⇐": suppose not and there exist a PBE with general signal space, denoted as $(\sigma, \rho, \mu_H(\pi, p))$, such that is not outcome equivalent to the one in Theorem 1 and satisfies intuitive criterion. According to Proposition 3, $(\sigma, \rho, \mu_H(\pi, p))$ is outcome equivalent to some other PBE with binary signal space, denoted as $(\sigma', \rho', \mu'_H(\pi, p))$. By Theorem 1, there exists a strategy with binary signal space (π'', p'') that blocks $(\sigma', \rho', \mu'_H(\pi, p))$, but then it also blocks $(\sigma, \rho, \mu_H(\pi, p))$, a contradiction. **Q.E.D.**

Proof for Proposition 7

When the seller is uninformed, it is equivalent to the case of observable type in which the buyer's belief follows $F(v)$. By Lemma 6, the seller's expected profit is $\int_c^{\overline{v}} (v - c) f(v)\, dv$. When the seller is privately informed, the seller's ex ante expected profit is

$$
\begin{aligned}
& \mu_H^0 \int_{y_H^*}^{\overline{v}} (v - c) f(v)\, dv + \left(1 - \mu_H^0\right) \int_c^{\overline{v}} (v - c) f(v)\, dv \\
< \; & \mu_H^0 \int_c^{\overline{v}} (v - c) f(v)\, dv + \left(1 - \mu_H^0\right) \int_c^{\overline{v}} (v - c) f(v)\, dv \qquad \text{(since } y_H^* > c) \\
= \; & \int_c^{\overline{v}} (v - c) f(v)\, dv.
\end{aligned}
$$

Q.E.D.

Proof for Proposition 8

First, for high-type seller,

$$
\begin{aligned}
r_H^* &= \int_{s\in S}\int_0^{\overline{v}} \pi\left(s|v\right) f_H\left(v\right) dv \left(E\{V|\mu_H\left(\pi\right), \pi, s\} - c\right) ds \\
&\leq \int_{s\in S}\int_0^{\overline{v}} \pi\left(s|v\right) f_H\left(v\right) dv \left(E\{V|1, \pi, s\} - c\right) ds \text{ (by Lemma 5)} \\
&= \int_{s\in S}\int_0^{\overline{v}} \left(v-c\right) \pi\left(s|v\right) f_H\left(v\right) dv ds \\
&\leq \int_c^{\overline{v}} \left(v-c\right) f_H\left(v\right) dv, \qquad (17)
\end{aligned}
$$

with equality holds only if $E\{V|\mu_H\left(\pi\right), \pi, s\} = E\{V|1, \pi, s\}$. Consider the strategy

$$
\pi_F\left(s_i|v\right) = \begin{cases} 1 & if\ v = i \\ 0 & if\ v \neq i \end{cases}
$$

Then for any belief u,

$$
r_H\left(\pi_F, \mu_H\right) = \int_c^{\overline{v}} \left(v-c\right) f_H\left(v\right) dv.
$$

Therefore, $r_H^* = \int_c^{\overline{v}} \left(v-c\right) f_H\left(v\right) dv$. Second, for the low-type seller,

$$
\begin{aligned}
& \mu_H^0 r_H^* + \left(1-\mu_H^0\right) r_L^* \leq \mu_H^0 \int_c^{\overline{v}} \left(v-c\right) f_H\left(v\right) dv + \left(1-\mu_H^0\right) \int_c^{\overline{v}} \left(v-c\right) f_L\left(v\right) dv \\
\Leftrightarrow\ & r_L^* \leq \int_c^{\overline{v}} \left(v-c\right) f_L\left(v\right) dv,
\end{aligned}
$$

with equality holds only if the seller sells with certainty if and only if $v \geq c$. The first inequality follows from that the seller's maximal expected profit cannot exceed maximal social welfare. Since for any belief μ,

$$
r_L\left(\pi_F, \mu_H\right) = \int_c^{\overline{v}} \left(v-c\right) f_L\left(v\right) dv,
$$

$r_L^* = \int_c^{\overline{v}} \left(v-c\right) f_L\left(v\right) dv$. Third, suppose the high-type seller adopts π in which there exists a signal s with positive measure and value $v_1 > v_2$ such that $\pi\left(s|v_1\right) \pi\left(s|v_2\right) > 0$. Since equality of (17) holds only if $E\{V|\mu_H(\pi),\pi,s\} = E\{V|1,\pi,s\}$, $\mu_H(\pi) = 1$. Therefore

$$
\begin{aligned}
r_L(\pi,\mu_H(\pi,p)) &= \int_{s\in S}\int_0^{\overline{v}} \pi(s|v) f_L(v)\,dv\,(E\{V|1,\pi,s\}-c)\,ds \\
&> \int_{s\in S}\int_0^{\overline{v}} \pi(s|v) f_L(v)\,dv\,(E\{V|0,\pi,s\}-c)\,ds \\
&= \int_c^{\overline{v}} (v-c) f_L(v)\,dv.
\end{aligned}
$$

Contradiction. The claim is verified. **Q.E.D.**

Nanjing Audit University
University of Technology Sydney

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