How Does Automated Market Design Affect the Outcomes and Behavior of Liquidity Providers?

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Abstract

Automated Market Makers (AMMs) are an emerging market design, unique to Decentralized Finance (DeFi), that automate the market making process using smart contracts. Despite managing over USD $50 billion in trades monthly, AMMs struggle to effectively manage adverse selection cost (ASC), otherwise known as impermanent loss (IL). This thesis investigates three different AMM designs and how they impact ASC and the behavior of liquidity providers (LPs). Using empirical data from 6 asset pairs over a sample period of 12-18 months, I find that the level of ASC these markets experience is indeed driven by market design, however different designs handle assets of different properties differently. However, I find that the level of ASC bears little effect on the behavior of LPs and market design has an inconsistent effect on this relationship. My findings imply a key result that AMM design is not a "one-size-fits-all" approach in terms of managing ASC.

Certificate of Original Authorship

I certify that the work in this thesis has not previously been submitted for a degree nor has it been submitted as part of requirements for a degree except as fully acknowledged within the text. I also certify that the thesis has been written by me. Any help that I have received in my research work and the preparation of the thesis itself has been acknowledged. In addition, I certify that all information sources and literature used are indicated in the thesis.

Signature of Student: Oliver Cao[1]

Date: 23/11/2023

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1 Introduction

Decentralized Finance (DeFi) is built on the notion of creating a completely ”trustless” financial system. Consequently, the development of markets that allow the trade of assets without the need of a trusted third party was a natural progression in the space. Decentralized Exchanges (DEXs) have been developed to achieve this goal, the most unique being Automated Market Makers (AMMs). AMMs are a distinctive market type within DeFi that prices assets using algorithms, based on the availability of supply, rather than bid and ask quotes set by market makers as seen in traditional limit-order book (LOB) markets (Wang, 2020). AMMs use smart contracts to enable individuals to become liquidity providers (LPs) by depositing their assets into "liquidity pools". Traders can then buy or sell the assets in these pools, paying fees on each trade which are distributed to LPs as passive income. In allowing any individual to become a market maker, AMMs form part of the "democratization of finance" of which DeFi's purpose is built upon.

Impermanent Loss (IL) is one of the primary challenges faced by LPs in the DeFi ecosystem. At its core, IL represents a potential reduction in the value of assets provided by LPs to AMMs compared to simply holding those assets outside the AMM. This loss arises when there is a price discrepancy between the price of an asset within the AMM and its market value elsewhere. As LPs cannot set the price at which the AMM sells their assets, this creates a situation akin to a market where sellers unknowingly price their assets below market value. When arbitrageurs spot these price discrepancies, they buy the undervalued asset from the AMM, leaving LPs with a greater proportion of the overvalued asset. Over time, this can erode the overall value of LPs' holdings. This phenomenon is reminiscent of the adverse selection cost (ASC) in traditional finance, where those with better information exploit market inefficiencies to the detriment of less-informed participants. In the context of AMMs, IL can be viewed as a form of ASC, where LPs bear the cost of these market inefficiencies.

Market designers are tasked with managing ASC by either minimizing IL or compensating their LPs with sufficient fee revenue to provide them with attractive returns. Both approaches come with costs and benefits, and market designers face a fundamental trade-off between the complexity and expressiveness[2] of their exchanges (Milonis, Moallemi, & Roughgarden, 2023). While the simplest forms of AMMs sacrifice expressiveness for simplicity, LOBs represent the other side of the coin, using complex mechanisms to optimize the expressiveness of their demand curves. Since their introduction, there have been many innovations to AMM design that aim to find a balance between complexity and expressiveness and deliver better outcomes for their LPs.

[2]"Expressiveness", in the context of exchange mechanisms, refers to the ability of a system to accurately represent a wide range of demand curves, reflecting the diverse preferences and demands of market participants.

This study examines three AMM designs to determine which design best protects its LPs against IL and how this influences LP behavior. The objective of this study is to shed light on the mechanics behind AMM and its influence on the behavior and outcomes of LPs, by answering the following research questions:

1. How does AMM design affect IL?
2. Which AMM design has the most ”active” LPs?
3. How does AMM design affect LPs’ response to volatility in centralized markets?

The three AMM designs that I compare in this study are: i) Constant Product Market Makers (CPMM) via Uniswap, ii) Constant Mean Market Makers (CMMM) via Balancer, and iii) Proactive Market Makers (PMM) via DODO.

The CPMM model was the pioneering approach to the AMM. Characterized by its simple formula $xy = k$, where x and y represent the quantities of the two assets in a liquidity pool and k is a constant, CPMMs operate by maintaining the value of the two assets within a pool in an equal ratio. Popularized by platforms such as Uniswap, CPMMs offer a decentralized and permissionless mechanism for traders and LPs. CPMMs have remained as the most popular AMM design with Uniswap, the largest provider, facilitating over USD$39b in monthly trading volume as of October 2023 (The Block, 2023).

CMMM pools represent an evolution of the foundational CPMM model. While CPMMs rely on a simple product formula, CMMMs introduce variable weights to assets in a liquidity pool. This weighted approach allows for greater flexibility in asset distribution by allowing one asset in the pool to maintain a higher fixed weight. This change in pool structure changes the dynamics of IL across differing price changes, with the key theoretical benefit being that a weighted pool experiences lower IL for large, positive price changes in the asset with the higher weight. Balancer is the most popular application of the CMMM model, maintaining USD$9.17b in monthly trading volume as of October 2023 (Dune Analytics, 2023).

DODO’s PMM is an innovative twist on traditional AMM designs. Unlike conventional AMMs which rely solely on mathematical formulas to determine asset prices, PMMs proactively adjust market prices using external price oracles[3]. This integration of real-time price feeds allows PMMs to reduce the risk of IL, bridging the gap between decentralized and traditional market-making mechanisms. Despite the proposed protection to IL, PMMs have not maintained the same level

[3] A price oracle is a mechanism that draws accurate and up-to-date asset price information from external sources, outside the blockchain, to provide timely and reliable data for uses within the blockchain.

of popularity as protocols such as the CPMM or the CMMM, facilitating USD$2.97b in monthly trading volume as of October 2023 (Dune Analytics, 2023).

I compare how AMM design affects IL and LP behavior across three asset pair categories: i) Stable-Stable pairs: pairs that trade exclusively ”stable-coins[4]”, ii) Stable-Risky pairs: pairs that trade some non-pegged crypto-asset for a stable-coin, and iii) Risky-Risky pairs: pairs that trade one non-pegged crypto-asset for another.

I answer the three research questions first by calculating IL and LP activeness from empirical data, employing statistical tests to determine if there is a statistically significant difference in these measures across AMM design. I then employ a pooled Ordinary Least Squares (OLS) regression to estimate the relationship between centralized market volatility and AMM liquidity for each asset pair within the sample, comparing this relationship across AMM designs.

I find that the difference in IL across CPMM and PMM protocols is inconsistent across asset pair types, with the PMM protocols experiencing higher IL in pairs which trade exclusively stable-coins. The opposite result is observed in pools that trade risky assets, in which I find that the PMM pools experience significantly lower IL. This is the expected result, as the PMM pools leave less room for arbitrage opportunities due to the AMM price deviating less from the market price. However, the result for the stable-stable pools implies some inconsistencies in how the PMM mechanism handles low volatility assets, highlighting the benefits of the simpler AMM designs in handling this asset type. Furthermore, I find that the CMMM pools see little statistically significant difference in terms of IL when compared to the CPMM pools. This highlights that the theoretical benefits of using fixed weights are not seen in the empirical data.

In terms of LP activeness, I find no significant link between the level of IL within a pool and the activeness of its LPs. I find that the pools with the highest IL often experience the least activeness within each asset pair, subverting the expectation that LPs would manage their positions more closely in the face of ASC. Moreover, I find that the CMMM pool has a very stable liquidity base when compared to CPMM and PMM pools. These results imply that LPs in these markets are likely to expect some level of IL when investing in these pools, which highlights the risk profile of LPs in these markets and implies a level of risk-aversion that is much lower than that seen in traditional markets.

The final phase of this study finds little statistically or economically significant effect of volatility in centralized markets on the liquidity of AMMs. This reinforces previously established evidence that AMMs achieve higher levels of liquidity provision during periods of high market volatil-

[4]“Stable-coins” are cryptocurrencies which have their value pegged to the US Dollar and as such, are extremely low volatility assets.

ity (O'Neill et al., 2023). Furthermore, I find little difference in the effect across AMM designs, implying that LPs in different AMMs act similarly when it comes to asset price volatility.

The results of this study have implications for future research and provide useful insights for practitioners in the space. Academically, the findings of this study advance our understanding of the relation between AMM design, IL and LP behavior, specifically shedding light on the key differences across asset pair types. This will allow for future research into these differences as the DeFi space continues to grow and new AMM designs are established. For practitioners, the outcomes of the IL study show that different AMM designs act differently when handling different asset types. These findings will allow for the development of more robust AMM designs which can cater to specific asset types and reduce the ASC of their LPs. Finally, for investors, the findings demonstrate which AMMs perform better across asset types – providing a framework for the AMMs in which to invest different assets.

My research builds on a subset of research into IL in AMMs that largely focuses on CPMMs (Aigner & Dhaliwal, 2021; Loesch et al., 2021; Angeris et al., 2019) by providing insight into the differences between market type on identical asset pairs and sample periods. Furthermore, this paper provides evidence on the dynamics of IL in PMM pools, enhancing our understanding of how this novel and unexplored design operates in practice.

This paper also extends upon research into the characteristics of LPs in AMMs (e.g., Aigner & Dhaliwal, 2021). I reinforce the notion that, fundamentally, LPs in AMMs are much less risk-averse than those in traditional markets, and that this notion holds across AMM design. Finally, this research relates to ongoing research into liquidity provision during market volatility in AMMs (e.g., O'Neill et al., 2023), providing further evidence that LPs in AMMs are resilient to volatility shocks. This relationship does not differ significantly across AMM design, demonstrating that this relationship is likely due to the frictions faced by LPs in rebalancing their position – rather than market design.

2 Literature Review

With AMMs managing over $50 billion USD in trade volume every month, they are a rapidly growing market type in the DeFi space. Ongoing research has found that AMMs can significantly reduce the trading costs incurred by investors in markets with high turnover and low volatility (e.g., O'Neill et al., 2023), illustrating their potential as the future of market design. Particularly, these findings demonstrate how the unique nature of stable-coins make them the perfect assets for the simple design of many popular AMMs.

Despite their rapid growth, the sustainability of AMMs is hindered by the issue of IL. IL is a prominent issue for LPs, acting as the major deterrent to providing liquidity to the market. IL occurs "due to a change in the reserve ratio of the pool resulting from a divergence in the unit price of the pool tokens" (Dos Santos, et al., 2022). As the price of an asset within an AMM diverges from the fair market price, arbitrageurs purchase the undervalued assets from the pool, altering its ratio of tokens. LPs liquidity also changes in ratio, leading to them holding a larger portion of the undervalued asset(s) within the pool. Loesch et al. (2021) examine IL within Uniswap v3 and find IL of 23.37% across 17 pools studied over a 6-month sample period, concluding that "overall and for almost all pools, both the minimal and actual IL surpass the fees earned by LPs during the period". Similarly, Boueri (2022) studies the returns of various Uniswap pools over an 8-month sample period and finds that the median pool experiences a low to zero net ROI over the sample period. This research demonstrates the diminutive nature of IL and its effect on LP returns, highlighting it as the primary issue facing LPs. It should be noted that IL is "impermanent" in nature, in that it only eventuates if an LP withdraws their liquidity at a price different from the price at which they initially deposited their liquidity. If the price returns to its initial state, the IL is negated.

While research has demonstrated IL can prove costly for LPs, it is an essential part of AMM market design as it facilitates efficient price discovery. As IL occurs when the exchange rate of the two assets within a pool changes from its initial state when liquidity was deposited, price discovery inherently causes IL for LPs in these markets. Hansson (2023) finds that, like traditional markets, price discovery in AMMs is driven largely by a small group of sophisticated traders. Furthermore, it finds that most of the price discovery is driven by large trades, for which their price impacts are more persistent when performed by informed traders. Due to the public nature of trading on DEXes, where all information on transactions is accessible publicly, this creates a unique market where informed traders are forced to display their intentions to the market. Capponi et al. (2023) studies this unique situation and finds that informed traders employ a "jump-bidding" strategy wherein they place a high initial fee for their trade to ward off potential competitors. This research demonstrates the fundamental necessity of IL in AMMs. Thus, finding the optimal market design that minimizes IL while still facilitating efficient price discovery is crucial to the sustainability of AMMs as a competitor to LOBs.

Another important dimension of the IL dilemma is fees. A high fee will reduce the impact of IL on LP returns, as it provides a passive income to cover any potential IL. However, studies have demonstrated how higher fees will lead to larger price divergence from an efficient, reference market (Angeris et. al, 2019). This demonstrates that the issue of IL cannot be solved by increasing the fee level of a market, as it disincentivizes arbitrageurs, hindering price discovery. Accordingly, market design is the key factor in striking the optimal balance between IL and efficient price discovery.

CMMMs offer a novel solution to this trade-off between IL and price discovery by allowing for a pricing mechanism which replicates the CPMM mechanism in terms of price discovery but allows for differing dynamics of IL for its LPs. Tiruviluamala et al. (2022) creates a mathematical framework for IL in Geometric Mean Market Makers (G3Ms), such as CPMMs and CMMMs, and demonstrates the difference in IL dynamics across pools with differing weights. CMMMs that hold one asset in a higher weight will experience lower IL for positive price changes in the higher-weighted assets, and higher IL for negative price changes. This relationship is magnified by the level of weighting within the pool. This creates an alternate option for LPs, where they can benefit from lower IL if the price of their holdings increases.

PMMs offer an alternate solution, essentially outsourcing price discovery to a centralized market. This allows for an AMM where price discovery within the AMM is less important and allows for lower levels of IL as a result. To further this benefit, the DODO PMM exchange incentivizes arbitrage trade only to the level that LPs' liquidity remains in the same ratio as it was deposited. Consequently, the DODO PMM exchange offers an alternative option to the traditional AMM that benefits from decreased IL (DODO, 2020).

Given the above theoretical benefits of each exchange, I hypothesize that the PMM pool will experience the lowest IL of the three studied markets, due to the decreased opportunity for arbitrage trade to alter the internal AMM price. Furthermore, I hypothesize that the CMMM pools will have the second lowest IL due to the reduced IL for positive price changes. Finally, I hypothesize that the CPMM pools will have the highest IL, given their simple design and the necessity for IL to achieve efficient price discovery.

In addition to the trade-off between IL and price discovery, market designers face a trade-off between the complexity of their market design and the expressiveness of demand curves (Milonis et al., 2023). This trade-off is particularly important for DEXes, where the complexity of market design increases the computational power needed to maintain the exchange. CPMMs represent one end of this spectrum, with a design that sacrifices the expressiveness of demand curves for computational efficiency. LOB markets are the opposite, with complex mechanisms for liquidity provision but high expressiveness of demand curves. However, LOBs have been proven to be an inadequate solution in a decentralized context, as the cost of market making becomes prohibitively expensive due to the ”gas” fees charged for each action[5]. As such, the optimal AMM design must strike the optimal balance in terms of computational efficiency and the expressiveness of LP demand.

Traditional market microstructure literature suggests that LPs would increase their bid-ask spread in the face of new information or changing market conditions (Madhavan, 2000). However, for LPs in AMMs, this is not an option. As such, it would be expected that LPs would respond to these conditions by changing the level of liquidity they provide to the market. As such, I hypothesize that markets with higher IL will have a higher “activeness” in their LPs, as they re-balance their portfolios by transferring their holdings to pools with less volatile assets. However, there are several key differences between LPs in traditional markets and AMMs that should be considered. Heimbach et al. (2021) analyze the behavior of LPs across the Uniswap ecosystem. They find that the majority of LPs contribute their liquidity to only one pool, with only a few LPs contributing to more than 10 pools. Additionally, they find that movement between pools is rare, unless driven by external motivations such as mining benefits provided by an alternate pool. As such, I hypothesize that while IL should increase the “activeness” of LPs, the impact should not be as exacerbated as one would expect in a traditional market.

Despite the demonstrated rigidness of liquidity in AMMs, I hypothesize that increases in volatility in the pools’ assets will likely lead to a decrease in liquidity. Volatility creates a risk for LPs on two fronts. First, volatility leads to increased opportunity for arbitrage trade within the AMM, leading to increased risk of IL. Additionally, heightened volatility amplifies price uncertainty for LPs’ holdings, incentivizing them to withdraw their liquidity to mitigate potential losses. This hypothesis aligns with the evidence from traditional markets, where it is seen that “market makers simultaneously remove their liquidity during unfavorable market conditions” (Anand & Venkataram, 2016). While this is likely driven by price uncertainty, like AMMs, it is also driven by demand determinants such as mutual fund outflows and investor sentiment. As a result, commonality in liquidity increases substantially during periods of high market volatility (Karoyli et

[5]Gas fees are the costs incurred to execute an action on the blockchain. In the context of LOBs, gas is charged each time a user creates, amends or removes an order from the book.

al., 2012). This commonality of liquidity is the key difference I expect to see between the studied AMMs and traditional markets. I hypothesize that those pools which handle IL better, and in turn have less active LPs, will have less response to volatility as the underlying risks of price uncertainty are lesser for these pools.

However, much like the “activeness” of LPs, I anticipate the effect of volatility on liquidity to be weak across the markets, as evidence has shown that AMMs achieve higher levels of liquidity provision during periods of extreme volatility than traditional markets (O’Neill, et al., 2023). Furthermore, Heimbach et al. (2021) find that LPs exhibit little reaction in their liquidity to price changes, indicating that LPs are either insensitive to IL or are hoping for the price to revert to the initial state, negating any losses.

This weak relationship is likely caused by the frictions LPs face in adding/removing their liquidity, as well as the risk profile of LPs in general. These frictions include gas fees and slippage. For LPs, gas represents the cost to add or withdraw liquidity from the market. Gas fees increase with the complexity of the action being executed. As such, gas does not scale with position size, charging the same fee irrespective of the size of the position being added or withdrawn. This makes it more expensive to re-balance smaller positions. Similarly, slippage increases with the size of the position, causing friction in rebalancing larger positions. This is particularly true when trying to rebalance via the AMM itself (Loesch et al., 2021). Lower gas fees prove beneficial for LPs, not only in reduced costs but in increased fee revenue. With lower gas fees, LPs can re-balance more frequently, allowing them to concentrate their liquidity around the market price, maximizing the rewards they receive from traders (Caparros et. al, 2023). This is a major benefit of new blockchains such as Polygon, which sacrifice security for computational efficiency.

The risk profile of LPs also likely plays a significant role in the resilience of AMM liquidity. Research has shown that cryptocurrency investors are driven largely by social influence and risk-seeking behavior (Almeida & Gonçalves, 2023). In contrast with traditional markets, one of the key properties of an LP is that they are “willing to own all of either one of the assets at either their lower price limit or zero” (Aigner & Dhaliwal, 2021). Other research about investor behavior in cryptocurrency markets has shown that crypto-investors “evaluate volatility risk fundamentally different than in traditional financial markets” and “price volatility positively, seeing it as an opportunity for higher returns” (Nadler & Guo, 2020). This further validates the fundamental differences between AMMs and traditional markets, as market makers in traditional markets are generally highly informed, rational entities. I aim to build upon this research and determine if the risk profile of LPs differs across AMM designs by examining how LPs across the three studied AMMs respond to volatility.

3 Data and Measures

I collect data for the three specified AMMs and a centralized market, across six asset pairs. From the AMMs, I collect prices, total value locked (TVL)[6], token balances, and fee revenue at a daily frequency over the sample period of 1 January 2022 to 14 July 2023, where possible. Additionally, I collect TVL data on an hourly level for the same period. For the LOB markets, I gather prices at a 5-minute interval to construct a volatility measure and at daily frequency to construct a daily return and volatility measure, over the same sample period.

3.1 Pairs and Comparisons

I conduct my analysis across three pair types; stable-stable pools which allow trading between stable-coins, stable-risky pools which allow trading between non-pegged crypto-assets and stable-coins, and stable-risky pools which allow trading between two non-pegged crypto-assets.

To ensure a rigorous analysis, the assets within the pools need to be comparable. Table 1 outlines the pairs I examine across the different AMMs. Pairs are selected on the basis that they appear in two or more of the AMMs under analysis.

Table 1: Pairs and Comparisons

This table shows the asset pairs that are examined within each of the AMMs and Binance, the centralized market. Pairs that are being analyzed in the respective markets are represented by a tick mark. Pairs are categorized into three categories: Stable-Stable, Stable-Risky, and Risky-Risky. The chosen sample period is 1 January 2022 to 14 July 2023.

Pair Type	Pair	Uniswap	DODO	Balancer	Binance
Stable-Stable	USDC/USDT	✓	✓		✓
	DAI/USDT	✓	✓		✓
Stable-Risky	BTC/USD	✓	✓		✓
	ETH/USD	✓	✓	✓	✓
Risky-Risky	BAL/ETH	✓		✓	✓
	GNO/ETH	✓		✓	✓

[6]Total Value Locked (TVL) refers to the total value of all assets within a pool, in USD terms. It is used as a measure of the liquidity of a pool for the purpose of this study.

To identify which AMMs trade the different pair types most frequently, I collect data on the volume, in USD, traded across pair types within the top 20 pairs on each exchange. Table 2 demonstrates that most of DODO's trading volume comes from stable-stable pools, with 92.9% of its trading volume throughout the sample period occurring on this pair type. The remainder occurs on Stable-Risky pairs, and it experiences no trade in its top 20 pairs occurring on risky-risky pairs. This deviates from my initial expectation that DODO's oracle pricing mechanism would encourage trading of more volatile assets due to the benefits of using an oracle to minimise IL. Furthermore, I find that Uniswap experiences the most stable-risky trading volume of the three, with 76.5% of volume within its top 20 pairs coming from this pair type, with stable-stable and risky-risky making up merely 23.5% of trading volume over the sample period. Like DODO, most of Balancer's trading volume comes from stable-stable pairs. With the other pair types making up 25.3% of volume across the top 20 pairs. This is consistent with my preliminary assumption that these more "simple" pool mechanisms would operate more efficiently with stable-coin trading.

Recent research into the stable-coin volume on the DODO exchange may explain the surprisingly high amount of volume in this asset class. Eigenphi (2023) finds that sandwich attacks[7] make up over 60% of trading volume in the DAI/USDT and USDC/USDT trading pairs. This likely explains the unusually high amount of stable-coin trade, as well as the outliers seen in the stable-coin data, as will be discussed in Section 3.5.

Table 2: Pair Type by Volume

This table shows the volume traded within the top 20 pairs on each AMM within the sample period 1 January 2022 – 14 July 2023. Pairs are split into three categories: Stable-Stable, Stable-Risky, and Risky-Risky pools. Exchanges are listed as "Name (AMM Design)". Volume is denoted in billions of US dollars (*Volume*) and percentage of volume of the top 20 pairs (%). The most traded pair type on each exchange is emboldened.

	Uniswap (CPMM)		DODO (PMM)		Balancer (CMMM)	
Pair Type	Volume	%	Volume	%	Volume	%
Stable-Stable	84.00	12.94	**74.22**	**92.92**	**25.70**	**74.64**
Stable-Risky	**496.69**	**76.49**	5.65	7.08	3.24	9.41
Risky-Risky	68.66	10.57	0.00	0.00	5.49	15.94

[7] A sandwich attack is a type of front-running attack that occurs when a trader places orders before and after a victim's trade to manipulate the asset's price, usually to the victim's disadvantage.

3.2 Sample Period

I use a sample period that spans from January 1st 2022, to 14th July 2023. I choose this sample period for several key reasons. Firstly, 2022 witnessed heightened activity in the cryptocurrency and DeFi markets, making it a particularly relevant and eventful year to examine. Moreover, Figure 1 illustrates how this sample period encompasses phases of relative market stability, as well as instances of extreme price volatility driven by market events such as the Terraluna (LUNA) crash and the FTX controversies[8]. Consequently, this sample period provides an opportune environment for analyzing LPs' behavior across different market conditions, covering both stability and volatility. I extend the sample period past the end of 2022 to accommodate the DAI/USDT pair, wherein data availability begins on 14 July 2022. Hence, I use a 12-month sample period of 14 July 2022 to 14 July 2023 for this pool. However, I contain the sample period to 2022 for the stable-risky pools, due to issues with data past this period on the DODO exchange.

3.3 Measures

To measure the activeness of LPs, I use the logarithm of the change in TVL[9]. I then create a measure of "activeness" by calculating the standard deviation of the hourly difference in the logarithm of TVL within each day.

Similarly, in my analysis of the effect of volatility on liquidity, I calculate the daily difference in the logarithm of TVL to create a scale-neutral metric for the daily change in liquidity across pools. For the volatility measure, I use the daily historical volatility, calculated as the standard deviation of 5-minute logarithmic returns within each day. This creates a daily measure which represents the volatility of the asset price within each calendar day. I then take the 3-day rolling sum of the daily difference in this volatility measure to provide a measure which captures the multi-day trend of volatility in the markets under consideration.

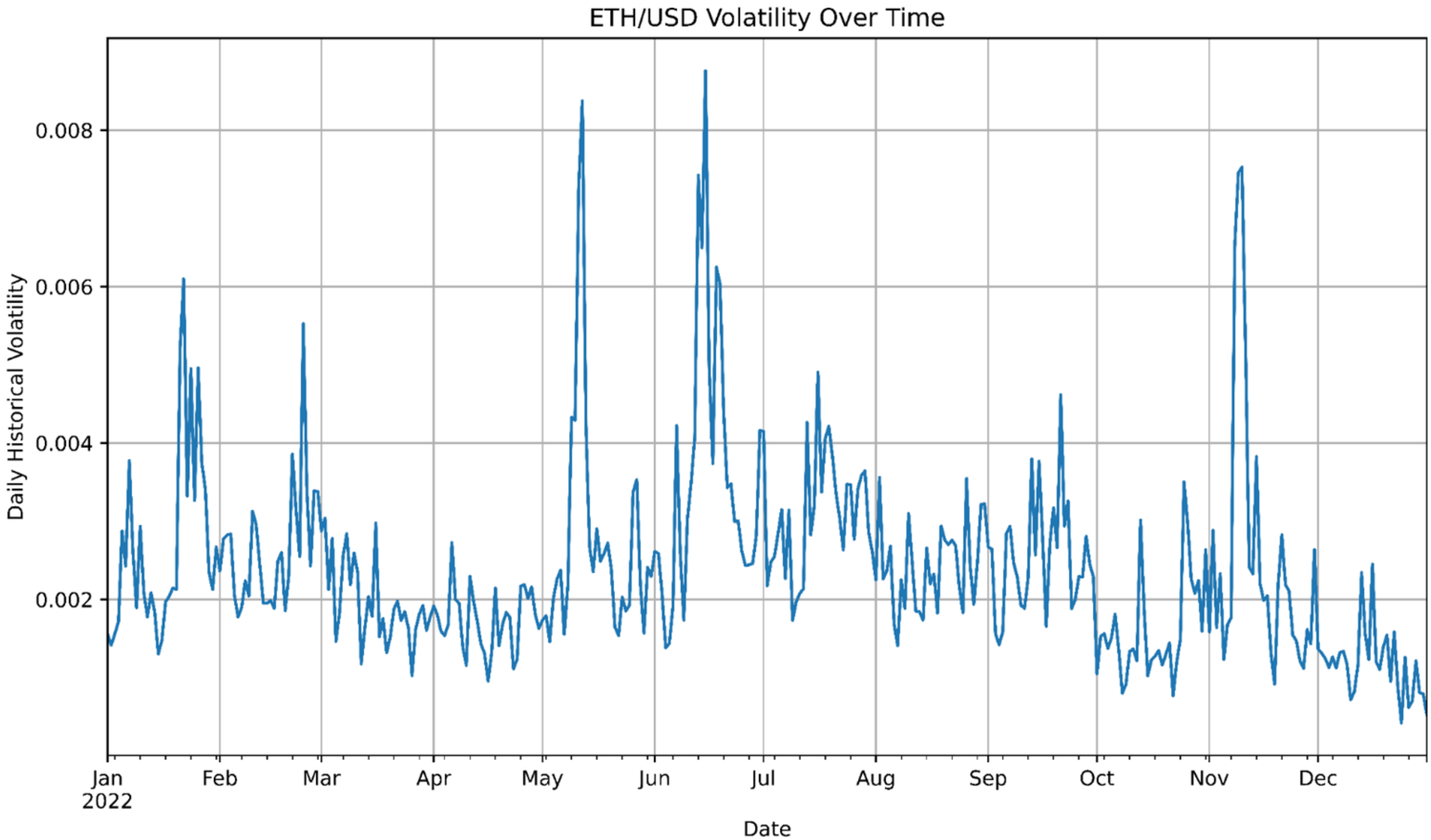
For the controls variables, I use gas prices, fee revenue and daily returns. I take the gas price in "gwei", a unit that represents one billionth of an Ether. For fee revenue, I take the logarithm of fee revenue in USD terms, to neutralize the scale across pools. Similarly, I take the logarithm of daily returns to ensure a normalized distribution of returns and stabilize my regression. For

[8] The LUNA crash refers to the de-pegging of the "Terraluna" stable-coin. The FTX controversy refers to the liquidation of FTX, the second largest cryptocurrency exchange at the time. Both of these events caused significant volatility in cryptocurrency markets.

[9] Using the logarithm of the change in TVL helps stabilize variance in data, especially when comparing pools of different sizes. The logarithmic transformation compresses the scale, reducing disparities in raw TVL values.

Figure 1: Ethereum Volatility During 2022

This graph shows the daily historical volatility of the Ethereum/USDT market on Binance over the sample period of 2022. USDT refers to the TetherUSD stable-coin. Volatility is calculated as the standard deviation of 5-minute logarithmic returns within each calendar day. Volatility is calculated on the price of Ethereum in USDT.



the risky-risky pools, wherein the value of the pool is largely determined by both tokens within the pool, I use the average of the logarithmic returns of the two tokens in the pool as the control variable. This step is not necessary for the stable-stable or stable-risky pools, as the daily returns of the stable-coin component of these pools is not economically significant.

3.4 Data Sources

To calculate IL in the CPMM and CMMM pools, I require token prices at a daily frequency. I source these data from The Graph[10] . I use the Uniswap v3 subgraph[11] to collect token prices at each daily close within the sample period.

Similarly, I use the Messari[12] subgraph for Balancer to collect token balance data at daily close. From these token balances, I infer the pool price for each asset using the pricing formula as seen in the Balancer whitepaper (Martinelli & Mushegian, 2019). For the PMM pools on DODO, I use the DODO v2 subgraph to collect pool prices and token balances in order to calculate IL. To construct the "activeness" measure, I collect hourly TVL data from the same three subgraphs used for the daily data. However, due to issues with the Uniswap subgraph, I am unable to collect hourly data for the stable-risky Uniswap pools.

The analysis of liquidity's sensitivity to volatility requires daily TVL and fee revenue data from the AMMs, daily volatility and returns from the centralized market, and gas data from the Ethereum network. I use the subgraphs to obtain the AMM-level data. For the centralized exchange data, I use Binance's Application Programming Interface (API), which allows querying of price data for all Binance markets. I collect price data at daily and 5-minute closes. Finally, I use Dune Analytics[13] to obtain a daily average gas price. To achieve this, I use a DuneSQL[14] query that takes the average gas price across all transactions conducted on the Ethereum network in "gwei", within each day[15].

[10]The Graph is an indexing and query protocol for blockchain data that enables developers to access and retrieve data from various decentralized networks.

[11]A subgraph refers to a specific application on The Graph designed to index and query data from a particular blockchain or exchange.

[12]Messari refers to a crypto analytics and market intelligence platform.

[13]Dune is a data analytics platform tailored for blockchain data.

[14]DuneSQL is a platform that allows users to query Ethereum blockchain data using SQL syntax.

[15]Additional details on the specific queries used for data collection can be found in Appendix A.

3.5 Data Analysis

Figure 2 shows the daily AMM prices over the sample period for each market within each pair before any data manipulation. Upon visual inspection, it becomes clear that the DODO pools contain some outliers, particularly in the stable-coin pools. These outliers can be due to several factors, including data collection issues, erroneous data, or issues with the pool itself. While these outliers could be due to issues with the design of the DODO pools such as oracle manipulation, slippage, or pool attacks, their severity necessitates some treatment of the data to reduce their impact evident on the analysis. Furthermore, it is evident in the ETH/USD pool that the ETH price is near identical in the CPMM and CMMM pools, which highlights the mathematical similarities in their pricing functions.

3.6 Data Treatment and Manipulation

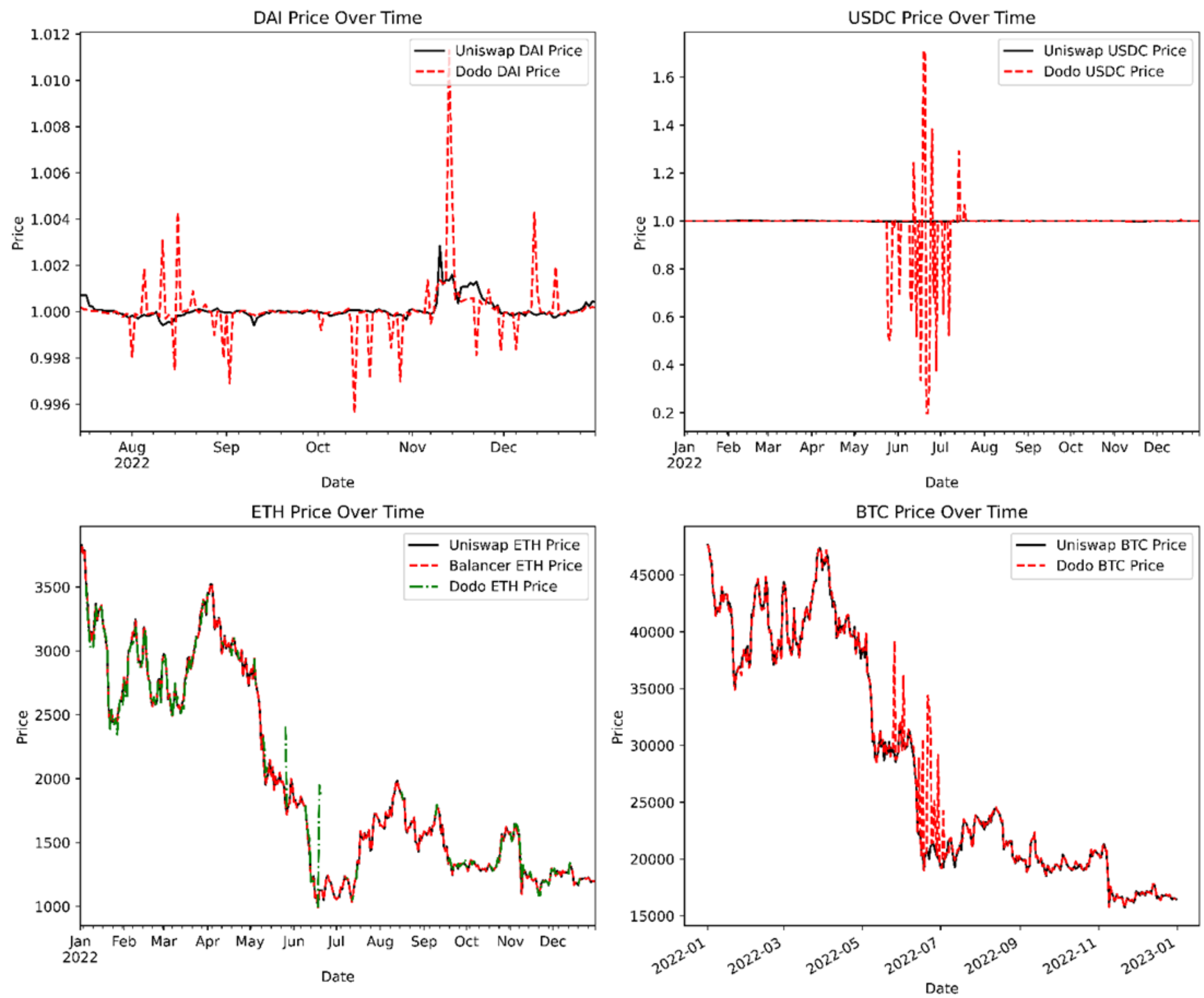
To mitigate the effect of the outliers in the DODO price data, as seen in Figure 2, I winsorize the price data at the 5% and 95% level to reduce their significance, while still maintaining some of their explanatory power. I do this for all phases of analysis, including calculating pool values for the IL analysis, as well as calculating TVL for the activeness and regression analysis. These outliers are not observed in the Uniswap or Balancer pools, likely as they are much more liquid pools and do not experience the magnitude of sandwich attack issues as seen in the DODO pools (Eigenphi, 2023).

The token balances collected for the DODO pools encompass both the total reserves of the pool as well as the 'LP' tokens. LP tokens refer to tokens minted that represent an LP's liquidity in the pool. In the DODO pools, LP tokens are split between the two assets. As such, one LP token represents one token deposited into the pool for that asset. To formulate a token balance which represents the number of tokens in the pool, ignoring the addition/removal of liquidity, I take the token balance at $t = 0$, and sum the change in token balances caused by trade onto the token balances. This provides an adjusted token balance for each day which reflects the change in token balance caused purely by trade and not LPs adding/removing tokens. This step is essential to the IL analysis, as liquidity events such as addition/removal have a material effect on the value of the pool which would alter the IL calculations[16].

[16]Further explanation for this step can be found in Appendix B.

Figure 2: Asset Prices within AMMs Over Time

This graph shows the daily close price of each asset, in USD, within each AMM in the stable-stable and stable-risky pairs. Sample period is 1 January 2022 to 14 July 2023 for the USDC prices, 14 July 2022 to 14 July 2023 for DAI prices, and 2022 for the ETH and BTC prices. DAI and USDC prices are represented in USDT terms, while ETH and BTC prices are represented in USDC terms. USDC and USDT are stable-coins, i.e., tokens pegged to the US Dollar.



4 Research Design and Findings

4.1 Impermanent Loss

The first stage of this study is an empirical evaluation of how each AMM design influences IL. I evaluate this by calculating both daily and cumulative IL within each pool and comparing these figures for each pair across pools, as listed in Table 1.

To calculate IL, I first categorize the two tokens within each pool into “base” and “quote” tokens. The “quote” token refers to the token in which we measure IL. For example, the quote token in the ETH/USD pool is USD, while in the BAL/ETH pool it is ETH. For the stable-stable pools, I use USDT as the quote currency. To measure day-to-day IL within the CPMM pools, I use the formula commonly seen in the literature (e.g., Aigner & Dhaliwal, 2021; O’Neill, et al., 2023):

$$IL = \sqrt{R} - \frac{1}{2}(R+1) \tag{1}$$

Where: $R = \frac{P_t}{P_{t-1}}$

P_t = The price of the base token at time t.

P_{t-1} = The price of the base token at time $t-1$.

Equation 1 does not work with the weighted nature of Balancer’s CMMM pools. As such, a formula which incorporates the fixed weights of the tokens is required. Balancer provides a formula to measure IL, which calculates IL as the ratio of the value of the assets after the price change, assuming no change in the weights ($Hold\ Ratio$) and the value of the pool after price change and arbitrage ($Invariant\ Ratio$). This formula is expressed as follows:

$$Hold\ Ratio = w_A \times R_A + w_B \times R_B \tag{2}$$

$$Invariant\ Ratio = \frac{(B_A \times R_A)^{w_A} \times (B_B \times R_B)^{w_B}}{B_A^{w_A} \times B_B^{w_B}} \tag{3}$$

$$IL = \frac{Invariant\ Ratio}{Hold\ Ratio} - 1 \tag{4}$$

Where: w_A is the weight of token A, w_B is the weight of token B, B_A is the balance of token A, B_B is the balance of token B, R_A is the relative price change of token A, R_B is the relative price change of token B.

Assuming all inputs (R, w, B) are positive and measuring IL using token B as the quote currency, I simplify this formula to the following:

$$IL = \frac{R_A^{w_A}}{w_A \times R_A + w_B} - 1 \tag{5}$$

I use Equation 5 to calculate IL for each of the weighted pools within this study. Further information on this formula can be found in Appendix C.

As both formulas use the relative price change of the assets to measure IL, we can pre-determine how the IL curve changes over this measure, similar to what is seen in Tiruviluamala et al. (2022). Figure 3 demonstrates that IL in the weighted pools increases at a slower rate for positive price changes. However, it also shows that the difference in IL between the CPMM/CMMM is minimal for relative price changes between 0 and 2. As such, I aim to determine if the theoretical benefits of the weighted pools, as seen in Figure 3, are observed in the empirical data or if the price changes do not vary to a point where the IL is significantly different.

Finally, for the PMM pools, calculating IL using relative price changes will not sufficiently capture the nature of the pricing mechanism of the DODO protocol. As such, I calculate IL using the value of the assets within the pool. IL is measured as follows:

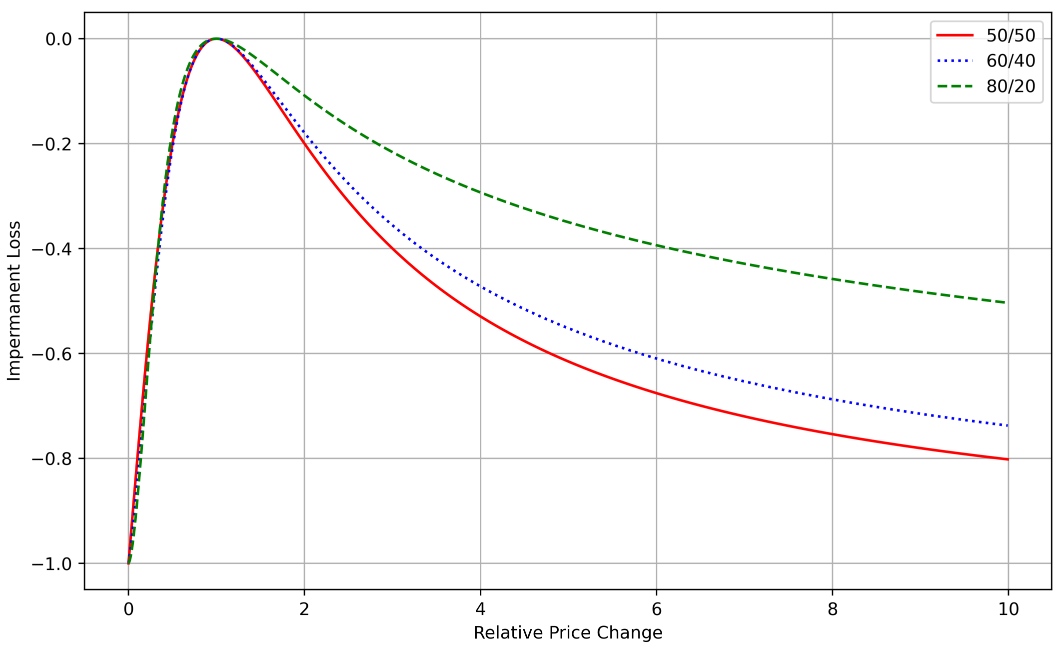
$$IL = Pool\,Value - Hold\,Value \tag{6}$$

Where: $Pool\,Value$ is the value of the assets within the pool at time t, and $Hold\,Value$ is the value of the assets if held outside the pool at time t.

The above formula allows me to compare the value of the assets within the pool, and if held outside the pool, before and after arbitrage.

Figure 3: IL for Different Pool Compositions

This graph shows the IL curves over relative price changes from 0 to 10 for pools of different weights, specifically pools that hold the two assets in a 50-50 ratio, denoted by the solid red line; 60-40 ratio, denoted by the blue dotted line; and an 80-20 ratio, denoted by the green dashed line. IL is denoted in % terms, with the negative sign denoting a loss. Relative price change is measured as the exchange rate of the two tokens in a pool at time t, divided by the exchange rate at time $t-1$.



For all markets, I calculate both day-to-day IL and cumulative IL over the sample period. Day-to-day IL is calculated as the IL from one day to the next, using R as defined in Equation 1. Cumulative IL is the level of IL experienced by the pool from the beginning of the sample period, where R is defined as follows:

$$R = \frac{P_t}{P_{t=0}} \tag{7}$$

Where: P_t is the price of the base token at time t, and $P_{t=0}$ is the price of the base token at the beginning of the sample period.

To observe the statistical significance of the IL results, I use a Mann-Whitney U test (Mann & Whitney, 1947). This is a non-parametric test which tests the null hypothesis that for two randomly selected observations from two datasets, X and Y, the probability of X being greater than Y equals the probability of Y being greater than X. I run this test for each pairwise set of markets within each asset pair to determine if the difference in IL is statistically significant.

4.1.1 Findings

Within the stable-stable pools, I find that the PMM pools experience higher levels of IL across the sample periods. For the USDC/USDT pool, I find that the PMM pool experiences 0.03 basis points of cumulative IL over the sample period of 2022, with the CPMM pool having 0.0002 basis points of IL. Similarly, I find that the PMM's median daily IL is significantly higher than the CPMM's at 0.10 basis points and 0.00005 basis points, respectively. This result is consistent when comparing PMM and CPMM for the DAI/USDT pool, in which the PMM has a higher cumulative IL at 0.11 basis points compared to the CPMM's 0.01 basis points, with the medians reflecting the same result, as seen in Table 3. The results for both pools are statistically significant at the 99.9% confidence level. These results are contrary to the assumption that oracle pricing reduces IL and indicate that the PMM mechanism worsens LP returns in these pools, potentially uncovering an issue with the mechanism in handling assets with very minor price movements.

For the stable-risky pools, I find the opposite, and expected, result that the PMM pool experiences lower IL. In the ETH/USD pool, the PMM pool experiences no cumulative IL over the sample period, compared to the CPMM's 3.66% IL. The CMMM pool, which contains 60% ETH, experiences higher cumulative IL than the CPMM equivalent with 5.18% cumulative IL. On the day-to-day level, DODO experiences much lower daily median IL at 0 basis points compared to the CPMM's 0.48 and CMMM's 0.49. This result is consistent in the other stable-risky pool, where

I find that the PMM also experiences cumulative IL of 0 basis points while the CPMM experiences cumulative IL of 1.76%. Table 4 shows that the difference in daily IL between the PMM and the CPMM/CMMM is statistically significant at the 99.9% level, however the difference between the CPMM and CMMM is not statistically significant. The results from the stable-risky pools demonstrate the strength of the PMM protocol in handling volatile assets, maintaining a much lower IL than its CPMM/CMMM counterparts. The stark difference in IL between the PMM and other protocols across these two pair types implies that the properties of the assets within a liquidity pool play a significant role in how market design affects IL, with pools that trade less volatile assets having superior performance on the CPMM protocol. As such, I find that simplicity in pool design is key for stable-stable pools, whereas the use of a price oracle can significantly reduce IL for more volatile assets, where the magnitude of a price discrepancy is going to be much higher.

Finally, for the risky-risky pools, I measure IL using ETH as the quote currency. The results demonstrate an unanticipated result where the CMMM pool with 80% BAL experiences lower IL than the 50–50 CPMM pool. I find that the CMMM pool suffers cumulative IL of 3.61% over the sample period, while the CPMM pool suffers 4.89%. The medians reflect a similar result, with Uniswap experiencing 0.52 basis points of daily median IL, and Balancer experiencing 0.26 basis points. These results are statistically significant at the 99.9% confidence level. The GNO/ETH pool, where I compare a CPMM pool with a CMMM pool with 80% GNO and 20% ETH, experiences the same result. I find the CPMM pool experiences higher cumulative IL over the sample period of 5.87% compared to 4.27% in the CMMM weighted pool. In terms of median IL, the difference is marginal at 0.001 basis points. The results of these risky-risky pools demonstrate an interesting result that a pool that holds the quote currency in a lower weighting can reduce IL, even in volatile periods. However, I find that overall, the IL in only one of the three studied CMMM pools is statistically significant from that of its CPMM counterpart, indicating that on a day-to-day level the pools likely perform quite similarly.

Figure 4 visualizes the cumulative IL over the sample period for the studied pools, demonstrating an interesting result in that the PMM pools (DODO) experience both impermanent loss and gain. As discussed by Labadie (2022), this is likely due to slippage, which is captured by the LPs as impermanent gain. This result highlights that the PMM mechanism is maintaining the IL of its pools close to, if not, zero, however the magnitude of the intermittent losses/gains is amplified in the stable-stable pools.

Table 3: IL Results

This table shows the daily IL calculations for each pair within each AMM studied. The sample period is 1st January 2022 to 14 July 2023 for the Stable-Stable and Risky-Risky pools, and 2022 for the Stable-Risky pools, with the exception of the DAI/USDT pool which uses a sample period of 14 July 2022 to 14 July 2023. *Exchange* refers to the protocol on which the pool exists, with *Type* referring to the AMM design of this exchange. *Mean* and *Median* daily IL figures are shown, as well as *Cumulative* over the sample period. IL is expressed in basis points, with the negative sign denoting a loss and the positive sign a gain. The lowest values within each asset pair are emboldened.

Pair Category	Pair	Exchange	Type	Mean (bps)	Median (bps)	Cumulative (bps)
Stable-Stable	USDC/USDT	Uniswap	CPMM	**-0.000018**	**-0.000006**	**-0.000204**
		DODO	PMM	-0.28	-0.10	-0.03
	DAI/USDT	Uniswap	CPMM	**-0.000031**	**-0.000008**	**-0.001246**
		DODO	PMM	-0.027	-0.0068	-0.11
Stable-Risky	ETH/USD	Uniswap	CPMM	-1.31	-0.48	-365.76
		Balancer	CMMM	-1.24	-0.49	-518.50
		DODO	PMM	**1.14**	**0.00**	**0.00**
	BTC/USD	Uniswap	CPMM	-0.72	-0.22	-177.54
		DODO	PMM	**1.19**	**0.00**	**0.00**
Risky-Risky	BAL/ETH	Uniswap	CPMM	-0.52	-0.16	-488.85
		Balancer	CMMM	**-0.26**	**-0.07**	**-360.56**
	GNO/ETH	Uniswap	CPMM	-0.064	-0.011	-587.30
		Balancer	CMMM	**-0.044**	**-0.010**	**-427.3**

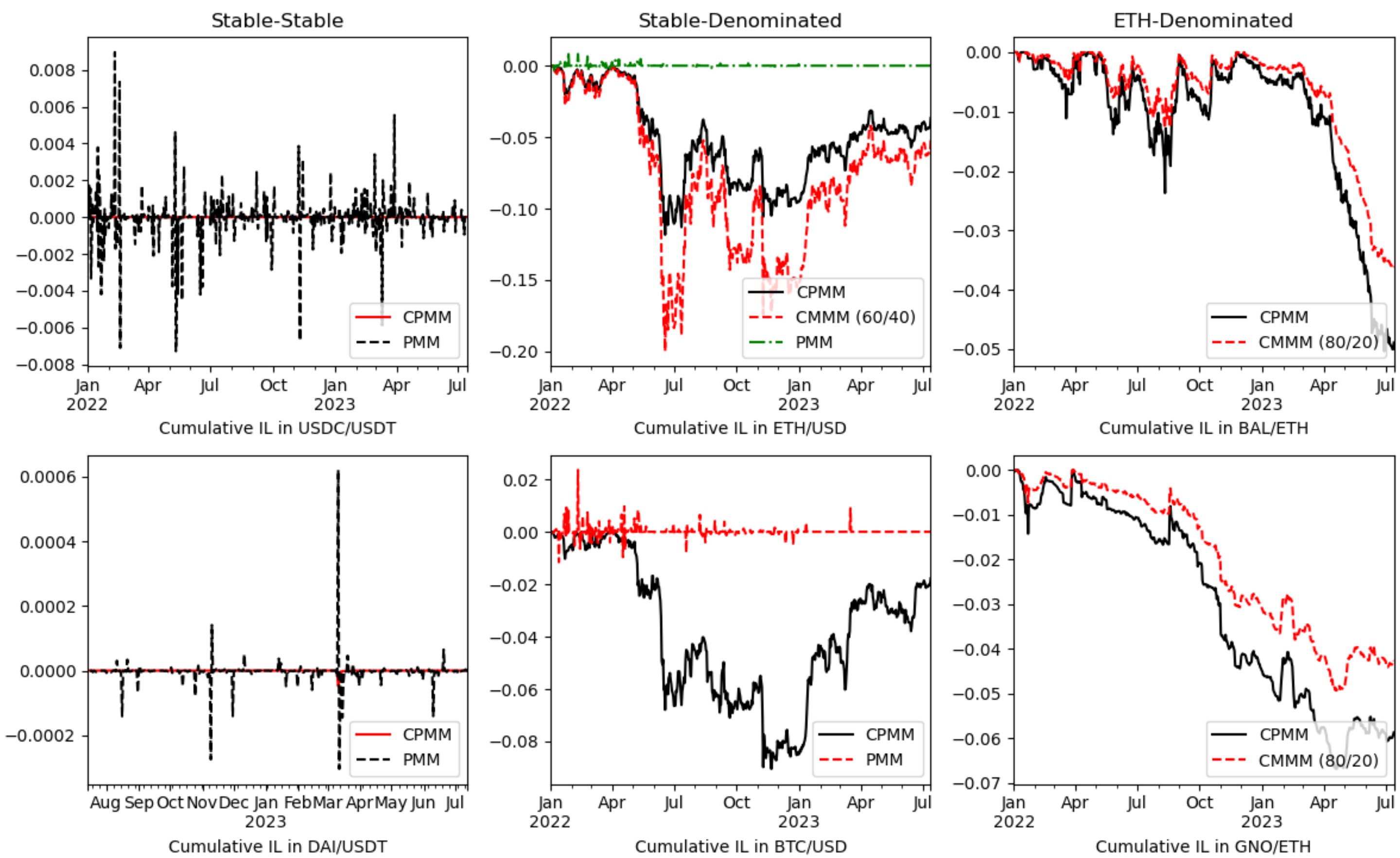
Table 4: Mann-Whitney U Test for IL Results

This table shows the results of the Mann-Whitney U test for the IL results for each pairwise set of AMMs within each asset pair. The *P-Value* represents the probability of rejecting the null hypothesis, being that the difference in the daily IL values in *Exchange 1* and *Exchange 2* is not statistically significant. $p<0.05$*, $p<0.01$**, $p<0.001$***. Statistically significant values are emboldened.

Pair Category	Pair	Exchange 1	Exchange 2	P-Value
Stable-Stable	USDC/USDT	Uniswap	DODO	**0.0000****
	DAI/USDT	Uniswap	DODO	**0.0000****
Stable-Risky	ETH/USD	Uniswap	Balancer	0.6720
		Uniswap	DODO	**0.0000****
		Balancer	DODO	**0.0000****
	BTC/USD	Uniswap	DODO	**0.0000****
Risky-Risky	BAL/ETH	Uniswap	Balancer	**0.0001****
	GNO/ETH	Uniswap	Balancer	0.1916

Figure 4: Cumulative IL Over Time

This graph shows the cumulative IL of each liquidity pool within each asset pair, for each AMM design, over time. Each line within each graph represents the different AMM designs. The sample period is 1 January 2022 to 14 July 2023, except for the DAI/USDT pair, which is 14 July 2022 to 14 July 2023, and the ETH/USD and BTC/USD pools, which is the year of 2022. IL is calculated in percentage terms, with the negative sign denoting a loss and the positive sign a gain. Plots are labelled by pair and pair categories.



4.2 LP Activeness

From the results of the first phase, I hypothesize that LPs that experience higher IL are more likely to re-balance their positions (i.e., higher activeness, to minimize the losses.). As I do not have access to granular data on liquidity events within each liquidity pool, I measure LP activeness as the "daily volatility" in the TVL of each pool. I calculate this as the standard deviation of the hourly change in the logarithm of TVL within each day, creating a metric that reflects the magnitude at which LPs within each market are re-balancing their position on an hour-to-hour basis. To adjust for any differences in asset price volatility, which would affect the TVL, I compare this metric across the pools/pairs as outlined in Table 1. To test for statistical significance, I run the Mann-Whitney U test on the LP activeness results for each pair of markets within each asset pair.

4.2.1 Findings

Table 5 outlines the mean and median levels of activeness within each pool. I find for the stable-stable pools that the level of activeness is not consistent with the level of IL within these pools, in that the PMM pools have lower activeness for both pools than their CPMM counterpart. Similarly, I find the difference in activeness between the CPMM and PMM pools is the opposite for the stable-risky pools, where the PMM pools experience higher activeness, on average. Finally, I find that the CMMM pool in the ETH/USD pair is the lowest of all the pools in this pair. Table 6 shows that the difference between pools is statistically significant for all pools/pairs, with the exception of the BTC/USD pool, where I find no statistically significant difference between the CPMM/PMM pool.

The opposite relationship between the IL of the pools and their activeness suggests that there is no intrinsic link between the ASC that an LP faces and the rate at which they re-balance their portfolio. This speaks to the established notion that the risk profile of LPs in these markets is fundamentally different than those in traditional markets, and implies that LPs likely expect some level of IL when investing in these pools. Furthermore, I find that the CMMM pool has a very stable liquidity base, implying that the risk profile of the LPs in this market differs from those in the other pools. This finding could be a symptom of the dynamics of IL in these pools, where large positive price changes incur less IL than CPMM pools. Consequently, it is possible that LPs within these pools have some positive expectations about the future price of the assets within the pool and do not plan on re-balancing their position much, if at all.[17]

[17] Due to data constraints, the activeness study does not cover all of the studied pairs. Hence, the findings are not as robust as those in the other phases of the study. However, the key result remains robust across the 4 studied pools.

Table 5: LP Activeness Results

This table shows the results of the "activeness" measurements for each AMM within each asset pair. Pairs are categorised by pair "type". *Exchange* refers to the protocol on which the pool exists, with *Type* referring to the AMM design of this exchange. Activeness is measured as the standard deviation of the difference in the natural logarithm of hourly Total Value Locked (TVL), within each calendar day. *Mean* and *Median* daily activeness figures for each pool are reported. The lowest figures within each pair are emboldened. Figures are reported × 10^3.

Pair Category	Pair	Exchange	Type	Mean	Median
Stable-Stable	USDC/USDT	Uniswap	CPMM	12.57	2.53
		DODO	PMM	**5.78**	**0.87**
	DAI/USDT	Uniswap	CPMM	12.27	7.84
		DODO	PMM	**7.78**	**1.17**
Stable-Risky	ETH/USD	Uniswap	CPMM	8.92	5.34
		Balancer	CMMM	**3.70**	**0.22**
		DODO	PMM	33.37	9.00
	BTC/USD	Uniswap	CPMM	**16.82**	**6.29**
		DODO	PMM	18.39	6.32

Table 6: Mann-Whitney U Test for LP Activeness Results

This table shows the results of the Mann-Whitney U test for the LP activeness results. The *P Value* represents the probability of rejecting the null hypothesis, being that the difference in the activeness values in *Exchange 1* and *Exchange 2* is not statistically significant. $p<0.05$*, $p<0.01$**, $p<0.001$***. Statistically significant values are emboldened.

Pair Category	Pair	Exchange 1	Exchange 2	P Value
Stable-Stable	USDC/USDT	Uniswap	DODO	**0.0000*****
	DAI/USDT	Uniswap	DODO	**0.0000*****
Stable-Risky	ETH/USD	Uniswap	Balancer	**0.0000*****
		Uniswap	DODO	**0.0000*****
		Balancer	DODO	**0.0000*****
	BTC/USD	Uniswap	DODO	0.6014

4.3 Liquidity's Sensitivity to Volatility

I analyze how AMM design affects liquidity's sensitivity to volatility in each of the studied markets using an OLS regression of the daily change in liquidity on the three-day rolling sum of the daily change in volatility of the base asset, in terms of the quote asset, in a centralized market. Specifically, I aim to determine if changes in the volatility of a pool's assets in an external reference market affects the liquidity decisions of its LPs. Additionally, I aim to determine if market design plays a role in this effect. The hypothesis for this phase is that those pools with the most active LPs will be more responsive to volatility in the underlying asset prices, as these LPs are already managing their positions more frequently and will likely be quicker to respond to changes in market conditions, as well as respond by a larger margin.

The measure of liquidity I use for the dependent variable in this regression is the TVL. I measure the daily change in liquidity as the daily difference in the natural logarithm of the TVL within each pool. The key independent variable in this regression is the 3-day rolling sum of the daily change in the volatility of the base asset on Binance, a centralized LOB market. Using the three-day rolling sum of daily volatility change allows for an analysis of how the three-day trend of volatility affects the level of liquidity within each market.

I also include several controls that could also affect liquidity. I control for the logarithm of fee revenue earned by the pool, as the level of fees will likely influence the level of liquidity within a pool as it provides some financial incentive for LPs. As informed traders only act on inefficiencies if their profits outweigh the cost of transacting, fees can act as a proxy for the level of uninformed trades within the market, which can balance the negative effect that informed traders have on LP returns.

Moreover, I control for the price of executing a transaction, otherwise known as "gas", on the Ethereum network, through the daily average price of gas in "gwei". "Gwei" is the unit in which gas is priced and represents one billionth of an Ethereum token. The gas price refers to the number of "gwei" that one unit of gas costs. As mentioned in Section 2, gas prices create frictions in an LPs ability to feasibly add or withdraw liquidity and, as such, is a relevant variable in this analysis. I include the interaction of gas and the volatility measure to sufficiently capture the potential relationship between market volatility and the influence of gas prices on LP's liquidity decisions. I expect this relationship to be opposite to that of volatility and liquidity, implying that the cost of adding/withdrawing liquidity reduces the magnitude by which LPs respond to volatility.

Finally, I include dummy variables for the CMMM and PMM markets, as well as their interactions with the volatility measure to measure the difference in liquidity's sensitivity to volatility by AMM design. Uniswap is the reference market in this analysis, and its relationship between volatility and liquidity is found in the "volatility" coefficient.

This regression is expressed as follows:

$$\Delta \ln TVL_{i,t} = \alpha + \beta_1 \Delta\, VOLA_{i,t} + \beta_2 D_{i,t} + \beta_3 D_{i,t} \times \Delta \ln VOLA_{i,t} + \beta_4 + \ln FEES_{i,t}$$
$$+ \beta_5 GAS_t + \beta_6 GAS_t \times \Delta \ln VOLA_{i,t} + \beta_7 RETURN_{i,t} + \varepsilon_t$$

Where: $\Delta \ln TVL_{i,t}$ is the daily change in the logarithm of TVL in market i at time t, $\Delta VOLA_{i,t}$ is the 3 day rolling sum of the daily change in volatility in market i at time t, $D_{i,t}$ is the dummy variable for market i at time t, $\ln FEES_{i,t}$ is the logarithm of fee revenue, in USD terms, in market i at time t, GAS_t is the gas price at time t, and $RETURN_t$ is the logarithmic return of the base asset in the centralized market at time t.

I run this regression as a pooled regression for each asset pair, using data from each AMM design. The key result comes from the differences in liquidity's sensitivity to volatility across AMMs, which can be observed by comparing the β_1 and β_3 coefficients.

I use heteroskedasticity and autocorrelation-consistent (HAC) standard errors to correct for possible heteroskedasticity and autocorrelation within the dataset. I also select a lag of 5 to account for any weekly effects that appear in the volatility or return measures of the studied markets.

4.3.1 Findings

Table 7 outlines the estimates of the regression. By observing the volatility coefficients, I find that there is a statistically significant effect of volatility on liquidity in only three of the six studied markets. Furthermore, I find that the effect is positive for both stable-coin markets. This is a surprising result, implying that when the volatility of the stable-coin increases, LPs increase their liquidity in these pools. However, this result is weakly significant for both pools. By observing the interaction of the PMM dummy and volatility, I find that the difference in this relationship between the CPMM and PMM markets is not consistent across the stable-coin pairs. While the PMM $\times$ Volatility coefficient in the DAI/USDT market suggests that the positive effect of volatility on liquidity is almost entirely negated in the PMM market, there is no statistically significant difference between the two markets in the USDC/USDT pair. Furthermore, the economic significance of these coefficients is low. For example, if the volatility measure in the DAI/USDT pair were to double from its mean, this would lead to an approximate increase in the TVL of 1.34%. Similarly, if the volatility measure were to double from its mean in the USDC/USDT pool, this would lead to an increase in TVL of approximately 0.5%. Overall, the findings of the stable-stable pool are surprising and subvert the expectations set out in the hypothesis. The positive effect of volatility in liquidity implies that LPs may actually increase their liquidity in stable-stable pools during periods of high volatility, potentially to earn increased fee revenue caused by informed trade. While informed traders will be causing IL, the IL in these pools is extremely minimal and LPs may not see this as an issue when choosing to increase their liquidity in these markets.

In the stable-risky markets, I find that only the ETH/USD pair has a statistically significant effect of volatility on liquidity. The coefficient of the volatility variable in this market demonstrates the expected result that an increase in volatility will lead to a decrease in liquidity. However, the interaction between the CMMM dummy and volatility implies that this effect is positive for the CMMM pool by a similar margin to the CPMM pool. The economic significance of this coefficient is also minimal, suggesting a decrease in the TVL of 1.4% if the volatility were to double from its mean. The coefficient of the CMMM $\times$ volatility variable reinforces the risk profile that was observed in the activeness study, indicating that LPs in this market are not affected by the risk of ASC.

For the remaining three markets, I find no statistically significant effect of volatility on liquidity. The weak statistical significance across the studied pools suggests that there is little effect of volatility on the liquidity in these markets. This implies that LPs in these markets are much more resilient to volatility shocks than those in traditional markets, reinforcing previous research on the risk profile of LPs, such as Aigner & Dhaliwal (2021), which suggests that the risk profile of LPs in AMMs is vastly different to that of market makers in traditional markets. Furthermore, I find that the difference in this relationship is not significant across the different AMM designs, suggesting that market design has little effect on how LPs perceive the risk of volatility.

Table 7: Regression Results

This table reports the estimates from the pooled OLS regression. The dependent variable is the daily change in the logarithm of the total value locked (TVL) within each pool. The independent variable, *Volatility*, is the 3-day sum of the daily difference in the daily historical volatility in the centralized market. *Return* is the daily logarithmic return of the assets within the pool. *Fees* is the logarithm of the fee revenue earned by the pool, in USD terms. *Gas* is the cost of executing an action on the blockchain. *CMMM* and *PMM* are dummy variables representing those AMM designs. $p < 0.05$*, $p < 0.01$**, $p < 0.001$***.

Dep. Variable			Change in Log TVL			
	USDC/USDT	DAI/USDT	ETH/USD	BTC/USD	BAL/ETH	GNO/ETH
Volatility	55.800	**121.460***	**-5.836****	0.078	-1.924	3.441
	(30.436)	**(57.274)**	**(2.332)**	(3.212)	(7.227)	(3.270)
Return	-3.544	-3.517	**0.443*****	**0.642*****	**1.258*****	**0.449****
	(5.429)	(7.226)	**(0.076)**	**(0.065)**	**(0.197)**	**(0.156)**
Fees	**0.022****	-0.005	-0.006	-0.002	**0.017***	-0.0005
	(0.010)	(0.009)	(0.004)	(0.001)	**(0.008)**	(0.001)
Gas	-0.0001	-0.0003	0.00006	-0.00004	-0.00005	0.0001
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
Gas*Volatility	**-1.323***	**-1.964***	-0.002	-0.046	-0.035	0.018
	(0.637)	**(0.984)**	(0.035)	(0.040)	(0.167)	(0.069)
PMM	**0.189***	-0.042	-0.054	-0.010		
	(0.088)	(0.047)	(0.033)	(0.008)		
PMM*Volatility	7.455	**-111.475****	-5.247	-5.778		
	(23.973)	**(42.571)**	(4.763)	(4.796)		
CMMM			-0.042		**-0.062***	-0.006
			(0.024)		**(0.031)**	(0.005)
CMMM*Volatility			**10.201*****		-2.744	-3.310
			(2.911)		(5.500)	(3.728)
Obs.	784	727	928	708	1074	1100
Cov. Estimator	HAC	HAC	HAC	HAC	HAC	HAC
R-Squared	0.139	0.043	0.061	0.122	0.063	0.067

4.3.2 Alternative Regression Model

To adjust for potential discrepancies in how LPs perceive market volatility, I run the same regression using the daily range volatility as an alternative volatility measure. This measure captures the range of daily prices, which may more directly influence liquidity decisions in AMMs.

I calculate daily range volatility as:

$$\text{Range Volatility} = \left| \frac{\text{Close} - \text{Open}}{\left(\frac{\text{Close}+\text{Open}}{2}\right)} \right| \tag{8}$$

This regression is expressed as follows:

$$\begin{aligned}\Delta \ln TVL_{i,t} = \alpha + \beta_1 \Delta\, VOLA_{i,t} + \beta_2 D_{i,t} + \beta_3 D_{i,t} \times \Delta \ln VOLA_{i,t} + \beta_4 + \ln FEES_{i,t} \\ + \beta_5 GAS_t + \beta_6 GAS_t \times \Delta \ln VOLA_{i,t} + \beta_7 RETURN_{i,t} + \varepsilon_t\end{aligned}$$

Where: $\Delta \ln TVL_{i,t}$ is the daily change in the logarithm of TVL in market i at time t, $\Delta VOLA_{i,t}$ is the daily change in range volatility in market i at time t, $D_{i,t}$ is the dummy variable for market i at time t, $\ln FEES_{i,t}$ is the logarithm of fee revenue, in USD terms, in market i at time t, GAS_t is the gas price at time t, and $RETURN_t$ is the logarithmic return of the base asset in the centralized market at time t

4.3.3 Alternative Regression Findings

Table 8 outlines the estimates of the alternative regression. The results are quite similar to those found in the first regression, with a few key differences. Firstly, the results of the stable-stable pools show stronger statistical significance and demonstrate a negative effect of volatility on liquidity for both PMM pools, as observed in the coefficient of the interaction of the PMM dummy and volatility. This suggests the range volatility measure is a more practical volatility measure for the stable-coin pools, which is unsurprising given the small price changes in these assets, which likely would not be captured by 5-minute returns as used for the first volatility measure.

Furthermore, the results of the stable-risky and risky-risky pools hold, in that there is little statistically significant effect of volatility on liquidity, except for the ETH/USD pool where I find again that there is a negative relationship for the CPMM/PMM pool and positive relationship for the CMMM pool.

The findings of this regression reinforce those of the first regression, providing some stronger statistical significance for some coefficients, thus proving the robustness of the first regression model. Furthermore, they demonstrate that analysing volatility in stable-coin markets at a daily frequency is likely more explanatory than shorter time intervals.

Table 8: Alternative Regression Results

This table reports the estimates from the alternative pooled OLS regression. The dependent variable is the daily change in the logarithm of the total value locked (TVL) within each pool. The independent variable, *Volatility*, is the daily difference in the range volatility in the centralized market. *Return* is the daily logarithmic return of the assets within the pool. *Fees* is the logarithm of the fee revenue earned by the pool. *Gas* is the cost of executing an action on the blockchain. *CMMM* and *PMM* are dummy variables representing those AMM designs. $p < 0.05$*, $p < 0.01$**, $p < 0.001$***.

Dep. Variable	Change in Log TVL					
	USDC/USDT	DAI/USDT	ETH/USD	BTC/USD	BAL/ETH	GNO/ETH
Volatility	-8.645	**42.420****	**-0.205***	-0.038	0.211	-0.270
	(9.267)	**(15.406)**	**(0.091)**	0.083	(0.455)	(0.247)
Return	1.394	-22.807	**0.447*****	**0.646*****	**1.255*****	**0.281****
	(4.735)	(18.367)	**(0.066)**	**(0.063)**	**(0.208)**	**(0.104)**
Fees	0.023	-0.002	-0.006	-0.003	**0.016***	-0.0002
	(0.015)	(0.010)	(0.004)	(0.002)	**(0.007)**	(0.001)
Gas	-0.0003	-0.0003	0.00005	-0.00004	-0.000002	0.0001
	(0.000)	(0.000)	(0.000)	(0.0004)	(0.000)	(0.000)
Gas*Volatility	0.034	**-0.270***	-0.001	-0.0003	0.005	-0.0004
	(0.107)	**(0.132)**	(0.002)	(0.001)	(0.006)	(0.003)
PMM	0.199	-0.024	-0.059*	-0.016		
	(0.135)	(0.055)	(0.033)	(0.008)		
PMM*Volatility	**-5.364*****	**-13.554*****	0.270	-0.255		
	(1.679)	**(4.070)**	(0.171)	(0.145)		
CMMM			-0.043*		-0.058	-0.005
			(0.024)		(0.031)	(0.005)
CMMM*Volatility			**0.384***		-0.551	-0.453
			(0.180)		(0.424)	(0.263)
Obs.	788	728	932	720	1077	1098
Cov. Estimator	HAC	HAC	HAC	HAC	HAC	HAC
R-Squared	0.043	0.045	0.058	0.121	0.075	0.069

4.4 Limitations

Due to data accessibility, I was unable to include fee revenue in the IL analysis to formulate an LP return metric, as this would require granular analysis of the fees earned by individual LPs which is not feasible for the scope of this study. Using the fee revenue data for the entire pool would not suffice, as this would lead to unreliable results due to the fees including those earned by LPs that added their liquidity during the sample period. Thus, the IL analysis does not fully capture how the profits of LPs evolve over the sample periods.

The robustness of the second and third phase of this study is limited by the lack of data availability. The initial challenge of collecting data for this study is pair selection, as there are not many pairs which trade frequently on all three exchanges. As such, this study is limited to only examining 6 asset pairs. Additionally, due to the varying ages of the liquidity pools, data is not available for all pools on the same sample period. This weakens the comparison across pairs due to a lack of consistency in market conditions observed over their respective sample periods. Finally, both the Binance API and AMM subgraphs provide slightly incomplete datasets for some pools. While the incompleteness of the datasets across the study does reduce both the breadth of the study and the reliability of the results, there is still sufficient data to draw some preliminary conclusions about the differences in AMM designs across the 6 studied pairs.

4.5 Implications for Future Research

This paper provides an introductory evaluation of the fundamental differences between AMM designs in terms of IL and LP behavior. However, as the popularity of DEXs continues to grow and data availability improves, a more comprehensive analysis of these differences could be explored to further solidify the variations in pool types, specifically when it comes to differing asset classes. The inclusion of new AMM designs as they are launched would likely also prove interesting, specifically research into the risk profile of LPs in brand-new exchanges. Additionally, including fee revenue in the study of IL could prove useful to further understand which AMM designs are more profitable for LPs. Finally, due to the constraints of this study, pools with more than two assets were excluded. As such, research into these pool types and how they compare to the AMMs studied in this paper would also prove extremely interesting, particularly how the inclusion of more than two assets impacts IL.

5 Conclusion

While AMMs are a unique and rapidly growing market type in the DeFi ecosystem, the optimal design that balances ASC, price discovery and computational efficiency has yet to be uncovered. I contribute to existing literature on AMMs and AMM design by analysing three of the most popular designs on their handling of ASC and its impact on the behaviors of LPs within these markets.

Using empirical data, I find that different AMM designs handle assets of different properties differently when it comes to ASC. Particularly, I find that AMMs that employ a more simplistic approach to pricing their assets experience lower IL for assets with a lower volatility, while those that expend more computational power on minimising ASC are able to do so for highly volatile assets. Furthermore, I find that AMM design plays little role in LP behavior, both in terms of how LPs manage their positions and how they respond to volatility in the price of their holdings.

These findings have several key implications for academics, practitioners and investors alike. The difference in ASC across market design and asset pairs demonstrates that practitioners should not adopt a uniform approach to automated market making. As such, the design of markets that cater to certain asset classes may prove beneficial to the growth and sustainability of AMMs. The absence of a measurable impact of ASC on LP behavior across the markets reinforces previous findings on LP behavior and suggest a rigidity in liquidity in these markets that is not seen elsewhere, providing a basis for further research into the effect of volatility in these markets. Finally, this study provides investors with valuable insight into the performance of different asset classes across the three AMM designs, creating a rudimentary framework of where to invest certain assets in these markets.

Appendix A: Additional Details on Data Collection

To collect daily AMM data, I use the following queries on each of the respective AMM's subgraphs:

- Uniswap: "poolDayDatas"
- Balancer: ”liquidityPoolDailySnapshots”
- DODO: ”pairDayDatas

These queries take a snapshot of the liquidity pools at the timestamp of the last trade made within each calendar day.

Similarly, I use the following queries for the hourly AMM data:

- Uniswap: ”poolHourDatas”
- Balancer: ”liquidityPoolHourlySnapshots”
- DODO: ”pairHourDatas

These queries act the same as the daily queries, at the hourly level. Specific details on the smart contract addresses of each pool can be found in Table 1A.

To collect the gas data from Dune Analytics, I use a query that accesses the ”ethereum.transactions” database and averages the amount of gas paid in gwei per transaction, for each calendar day.

Table 1A: Smart Contract Addresses for Liquidity Pools

This table shows the smart contract addresses of the pools in this study. These addresses are used to identify the pools within the blockchain using the specified subgraph queries.

Exchange	Pair	Address
Uniswap	USDC/USDT	0x3416cf6c708da44db2624d63ea0aaef7113527c6
	DAI/USDT	0x48da0965ab2d2cbf1c17c09cfb5cbe67ad5b1406
	BTC/USD	0x99ac8ca7087fa4a2a1fb6357269965a2014abc35
	ETH/USD	0x8ad599c3a0ff1de082011efddc58f1908eb6e6d8
	BAL/ETH	0xdc2c21f1b54ddaf39e944689a8f90cb844135cc9
	GNO/ETH	0xf56d08221b5942c428acc5de8f78489a97fc5599
DODO	USDC/USDT	0xc9f93163c99695c6526b799ebca2207fdf7d61ad
	DAI/USDT	0x3058ef90929cb8180174d74c507176cca6835d73
	BTC/USD	0x2109f78b46a789125598f5ad2b7f243751c2934d
	ETH/USD	0x75c23271661d9d143dcb617222bc4bec783eff34
Balancer	ETH/USD	0x0b09dea16768f0799065c475be02919503cb2a35
	BAL/ETH	0x5c6ee304399dbdb9c8ef030ab642b10820db8f56
	GNO/ETH	0xf4c0dd9b82da36c07605df83c8a416f11724d88b

Appendix B: IL Calculations for DODO Pools

To calculate IL in the DODO pools, I first create a figure for the token balance at each day that is adjusted for any liquidity events (additions/withdrawals) throughout each day.

The DODO data provides data on the total reserves of the pool for each token (”baseTokenReserve” & ”quoteTokenReserve”), and the amount of ”LP” tokens in the pool (”baseLpTokenTotalSupply” & ”quoteLpTokenTotalSupply”). One LP token represents one token deposited into the pool by an LP.

To create a metric for token balances that adjusts for LP additions/withdrawls, I first create a metric that measures the daily change in the token balances caused only by trade. I calculate this as follows:

$$Adjusted\,Change\;in\;Balance = \Delta Token\;Reserves - \Delta LP\;Token\;Supply$$

I sum these values on to the token balances, creating a figure that represents the token balances at each day that excludes changes caused by LP additions/withdrawals, and only includes changes caused by trade.

To calculate IL, I calculate the ”Pool Value” and ”Hold Value” as follows:

$$Pool\,Value = Token\;Balance_{A_t} \times P_{A,t} + Token\;Balance_{B,t} \times P_{B,t}$$

$$Hold\,Value = Token\;Balance_{A_t-1} \times P_{A,t} + Token\;Balance_{B,t-1} \times P_{B,t}$$

$$IL = \frac{Pool\,Value_t - Hold\,Value_t}{Pool\,Value_{t-1}}$$

Appendix C: IL Formula for Balancer Pools

I manipulate the Balancer IL formula (Balancer, 2023) to work under the assumption that IL is being calculated in terms of token B within a pool, and simplify this formula to take R_A and the weights of the tokens as the inputs.

I achieve this as follows:

$$Hold\ Ratio = w_A \times R_A + w_B \times R_B$$
$$Invariant\ Ratio = \frac{(B_A \times R_A)^{w_A} \times (B_B \times R_B)^{w_B}}{B_A^{w_A} \times B_B^{w_B}}$$
$$IL = \frac{Invariant\ Ratio}{Hold\ Ratio} - 1$$

Since I am measuring IL in terms of token B, assume R_B = 1.

$$Hold\ Ratio = w_A \times R_A + w_B$$
$$Invariant\ Ratio = \frac{(B_A \times R_A)^{w_A} \times B_B^{w_B}}{B_A^{w_A} \times B_B^{w_B}}$$
$$= R_A^{w_A}$$
$$IL = \frac{R_A^{w_A}}{w_A \times R_A + w_B} - 1$$

Glossary

Adverse Selection Cost (ASC) Costs arising from a situation where there is assymetric information between two parties, where one party has an advantage over the other.

Automated Market Maker (AMM) A type of decentralised exchange that prices assets using algorithms.

Blockchain A decentralized digital ledger where transactions are recorded, verified, and maintained across a network of computers.

Cryptocurrency A digital or virtual form of currency that uses cryptography for security.

Decentralized Exchange (DEX) An exchange that allows the trade of cryptocurrencies without the need for an intermediary, using smart contracts.

Decentralized Finance (DeFi) An emergent financial system that uses secure distributed ledgers, eliminating intermediaries and enabling faster, direct peer-to-peer exchanges, lending, and borrowing.

Gas Fees The cost of validating a transaction on the Ethereum blockchain.

Impermanent Loss (IL) The risk for liquidity providers of seeing the value of their reserved tokens decrease in comparison to holding the assets (Heimbach et al., 2021).

Liquidity Pool A reserve of cryptocurrencies held by a smart contract that uses the assets to facilitate trade via an Automated Market Maker.

Liquidity Provider (LP) A user that provides their cryptocurrencies to a liquidity pool, earning fees from trade volume.

Oracle In the context of DeFi, an oracle is a mechanism that draws information from external sources, outside the blockchain, for uses within the blockchain..

Smart Contract A digital protocol that automatically executes an event or transaction according to the terms of a contract or an agreement.

Stable-coin A cryptocurrency which has its value pegged to another asset, usually a fiat currency such as the US Dollar.

Total Value Locked (TVL) The total value of all assets ”staked” within a DeFi protocol e.g., a liquidity pool.

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