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***'Optimal Reserve Prices in  
Auctions with Expectations-Based  
Loss-Averse Bidders'***

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# Optimal Reserve Prices in Auctions with Expectations-Based Loss-Averse Bidders\*

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## Abstract

We characterize the optimal reserve price in first-price and second-price auctions with independent private values when bidders are expectations-based loss averse. Under “unacclimating personal equilibrium” (UPE), whereby bidders keep their expectations fixed when choosing their bids, the optimal public reserve price can be lower than under either risk neutrality or risk aversion. Moreover, secret and random reserve prices raise more revenue than public ones since, by giving every bidder a small chance to win the auction, the seller exposes all bidders to the “attachment effect”. In contrast, under “choice-acclimating personal equilibrium” (CPE), whereby a bid determines both the reference lottery and the outcome lottery, the optimal reserve price is public and it differs across the two auction formats. Furthermore, the seller excludes more types compared to the risk-neutral and risk-averse benchmarks.

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**Keywords:** Reference-Dependent Preferences; Loss Aversion; Reserve Price; First-Price Auction; Second-Price Auction; Personal Equilibrium.

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# 1 Introduction

Reserve prices are a prevalent tool auctioneers use to raise their expected revenue. A reserve price acts as an additional bid placed by the auctioneer since, in order to win, buyers must also outbid the reserve. Thus, a reserve price increases the competitiveness of an auction, which might lead buyers to bid more aggressively. Yet, this comes at a cost for the auctioneer because no trade happens if no buyer bids at least the reserve price. Indeed, a reserve price excludes buyers with relatively low valuations from the auction and reduces the overall probability of trade. Seminal theoretical contributions by Myerson (1981) and Riley and Samuelson (1981) have characterized the revenue-maximizing reserve price as the solution to this trade-off between decreasing the probability of trade and amplifying competitive pressure. In particular, they show that with risk-neutral bidders having independent private values, the optimal reserve price is (i) deterministic and public, (ii) the same in all standard auction formats, and (iii) under mild conditions on the distribution of bidders' values, independent of the number of bidders. However, not all of these features are empirically observed. For instance, in real-world auctions sellers often use shill bidding and secret reserve prices.<sup>1</sup> Moreover, several studies show that reserve prices are often significantly lower than what these classical models predict; see Paarsch (1997), Haile and Tamer (2003), Davis et al. (2011) and Gonçalves (2013). This evidence suggests that sellers may face additional trade-offs not captured by the classical risk-neutral and/or risk-averse model.<sup>2</sup>

In this paper, we analyze reserve prices in first-price and second-price auctions where symmetric bidders have independent private values (IPV) and are expectations-based loss averse á la Kőszegi and Rabin (2006, 2007, 2009). We derive the revenue-maximizing reserve price for each format and highlight how loss aversion modifies the seller's trade-off between increasing competitive pressure and reducing the probability of trade. In particular, we show that loss aversion can rationalize reserve prices that (i) are secret, (ii) vary with the number of bidders, (iii) differ between auction formats, and (iv) are lower than what the theory predicts for risk-neutral and risk-averse bidders.

Section 2 introduces the auction environment and bidders' preferences, and describes the solution concepts. We consider a standard symmetric environment with independent private values and analyze two canonical sealed-bid auction formats: the first-price auction (FPA) and the second-price auction (SPA). Following Kőszegi and Rabin (2006), we posit that, in addition to classical material utility, a bidder also experiences "gain-loss utility" when comparing her material outcomes to a reference point equal to her expectations regarding those same outcomes, with losses being more painful than equal-size gains are pleasant.

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<sup>1</sup>While Ashenfelter (1989) provides some early examples of shill bidding in art auctions, this phenomenon appears to occur at a substantially higher rate in online auctions, where sellers can use multiple accounts to bid on their own items; see Grether et al. (2015). Secret reserve prices are documented by Elyakime et al. (1994) and Li and Perrigne (2003) in timber auctions, and by Bajari and Hortaçsu (2004) in internet auctions.

<sup>2</sup>With private values, Hu et al. (2010) show that risk aversion can explain low reserve prices in the FPA but not in the SPA; if in addition bidders have interdependent values, Hu et al. (2019) show that risk aversion can explain low reserve prices also in the SPA.

The way that reference dependence affects bidding incentives depends on the extent to which loss-averse bidders incorporate their bid into their reference point. We consider two alternative formulations introduced by Kőszegi and Rabin (2007). In a “choice-acclimating personal equilibrium” (CPE), bidders choose the strategy that maximizes their payoff given that the strategy determines both the distribution of the reference point and the distribution of outcomes; hence, when contemplating whether to deviate from her equilibrium bid, a bidder adjusts her reference point accordingly. In an “unacclimating personal equilibrium” (UPE), instead, bidders choose the strategy that maximizes their payoff keeping expectations fixed, and the distribution of outcomes so generated must coincide with the expectations; hence, when deviating from her equilibrium bid, a bidder holds her reference point fixed. Both specifications are sensible from a theoretical perspective and have been applied in various economic settings.<sup>3</sup> We refrain from taking a stand on which specification is more appropriate, but point out that the two yield different predictions for bidder behavior.

We begin our analysis in Section 3 by deriving the public revenue-maximizing reserve price in the FPA under UPE.<sup>4</sup> For this solution concept, the “attachment effect” (Kőszegi and Rabin, 2006) is the main force driving the bidding behavior of loss-averse bidders. In particular, the higher the probability with which a bidder, in equilibrium, expects to win the auction, the bigger the loss she endures if she ends up losing it. Hence, the bidder has an incentive to increase her bid, so as to win more often and avoid experiencing the loss. Thus, the attachment effect induces an upward pressure on the equilibrium bidding strategy, ensuring that a bidder’s cost from an upward deviation is higher than the described benefits. Importantly, the attachment effect only affects the behavior of a bidder who expects to win the auction with strictly positive probability — however small — and is thus exposed to potential losses in equilibrium; yet, it does not affect those bidders who abstain from the auction since they do not incur a loss when not winning it.

The fact that bidders who do not expect to win are not subject to the attachment effect has several implications for the characterization of the revenue-maximizing reserve price. First, it puts downward pressure on the optimal reserve price. Indeed, by increasing the reserve price, the seller excludes a larger set of bidder types from the auction. As a bidder’s attachment increases in her type, the higher the marginally excluded type, the larger the attachment effect that the seller forgoes. Thus, with expectations-based loss aversion, increasing the reserve price is more costly for the seller compared to a situation where the attachment effect is not present; e.g., with risk-neutral bidders. This finding is especially relevant for those empirical studies that first estimate the distribution of bidders’ values and then use the theoretical insights of Myerson (1981) and

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<sup>3</sup>In particular, UPE has been extensively used to study firms’ pricing and advertising decisions; see, for instance, Heidhues and Kőszegi (2008, 2014) Karle and Peitz (2014, 2017), Karle and Möller (2020), Karle and Schumacher (2017) and Rosato (2016). On the other hand, CPE has been mainly applied to analyze questions in contract theory and mechanism design; see Herweg et al. (2010), Lange and Ratan (2010), Eisenhuth (2019), Meisner and von Wangenheim (2021) and Benkert (2022).

<sup>4</sup>As shown by Balzer and Rosato (2021), under UPE the FPA and SPA are revenue equivalent; therefore, our results on the reserve price for the FPA carry over to the SPA.



Riley and Samuelson (1981) to estimate the revenue-maximizing reserve price as the minimum bid that excludes all bidders with a “virtual value” lower than the seller’s value; see Paarsch (1997), and Haile and Tamer (2003). Yet, as our first result implies, because of the attachment effect, it may be optimal to include in the auction also bidders with virtual values lower than the seller’s own value. While risk aversion can rationalize such a low reserve price in the FPA (see Hu et al., 2010), it cannot explain it for the SPA.<sup>5</sup>

Second, the optimal reserve price varies with the number of bidders in the auction. Indeed, the more bidders are present, the less optimistic each one of them is about her chances of winning, which in turn decreases their attachment. Therefore, raising the reserve price becomes less costly when the number of bidders increases.<sup>6</sup> With risk aversion, instead, the optimal reserve price (in the FPA) is decreasing in the number of bidders; see Vasserman and Watt (2021).

Third, the fact that the attachment effect does not operate on excluded bidders, who rationally expect to lose the auction with certainty, makes secret and random reserve prices revenue superior to public and deterministic ones. To see why, notice that with a secret reserve price each bidder type expects to win the auction with strictly positive — albeit potentially arbitrarily small — probability. In such an auction, therefore, every bidder is exposed to potential losses and thus has an incentive to bid more aggressively in order to avoid them. Hence, by transforming the public reserve price into a secret one, the seller can ensure that every bidder experiences the attachment effect, which enhances revenue. On the other hand, by doing so, the seller reduces the competitive pressure on the buyers’ side, which could potentially harm revenue since those low-type bidders excluded under a public reserve would be participating now. However, the seller can choose a distribution for the (secret) reserve price that puts large probability mass on relatively high prices and arbitrarily small mass on low ones. Such a distribution ensures that, while the seller exposes every bidder to the attachment effect, the competitive pressure is almost the same as under a public reserve price.

As argued by Bajari and Hortaçsu (2004), secret reserve prices are common in internet auctions.<sup>7</sup> Moreover, an alternative way of implementing a secret and random reserve price is via “shill bidding”, a prominent phenomenon in real-world auctions whereby a dummy buyer submits pre-specified bids on behalf of the seller; see Ashenfelter (1989). Thus, the attachment effect provides a

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<sup>5</sup>However, beyond risk aversion, several, different explanations for low reserve prices in both the SPA and FPA have been proposed. These include correlated types (Levin and Smith, 1996), interdependent values (Quint, 2017; Hu et al., 2019), endogenous entry (McAfee, 1993; Levin and Smith, 1994; Peters and Severinov, 1997), bidders’ selection neglect when sellers are privately informed about the quality of the objects they sell (Jehiel and Lamy, 2015), level-k bidders (Crawford et al., 2009) and taste projection (Gagnon-Bartsch et al., 2021).

<sup>6</sup>Menicucci (2021) obtains a similar result in the classical IPV risk-neutral model when the bidders’ virtual values are not monotone; in contrast, our result holds also for the regular case of increasing virtual values.

<sup>7</sup>Secret reserve prices on Ebay are deterministic; yet, from the bidders’ perspective, the secret reserve price might appear random if they do not observe how precisely the seller chooses it. Indeed, the distribution that we characterize for the optimal secret reserve price puts a large mass on the upper bound of the support and arbitrarily little mass everywhere else, consistent with the notion of bidders expecting the seller to make small “mistakes”. In timber auctions, secret reserve prices are “truly” random, as argued by Li and Perrigne (2003).

novel rationale for secret *and* random reserve prices. Rosenkranz and Schmitz (2007) provided an earlier rationale for secret (but deterministic) reserve prices. In their model, publicly announcing the reserve price plants a reference point in the bidders’ minds, making it less attractive to win at a price higher than the reserve. Secret reserve prices can also be rationalized under risk aversion and in common-value auctions *if* the seller’s value is privately known.<sup>8</sup> Furthermore, secret reserve prices can emerge with uninformed bidders who learn their value as the auction unfolds (Hossain, 2008) or with competing sellers if not all buyers hold correct expectations about the distribution of the reserve price across sellers (Jehiel and Lamy, 2015).

In Section 4, we derive the revenue-maximizing reserve price under CPE where, differently from UPE, bidders’ reference points immediately adjust to the bid they submit. Thus, when raising her bid, a bidder simultaneously changes the lottery over material outcomes — the probability of winning and the payment conditional on winning — as well as her reference lottery; this, in turn, modifies the seller’s incentives for the choice of the optimal reserve price. First, a public reserve price now outperforms a secret one. The reason is that under CPE even a bidder who, in equilibrium, does not participate in the auction would be exposed to losses if she were to deviate and bid (at least) the reserve price; hence, a secret reserve price is not needed in order to expose all bidders to potential losses.

Moreover, differently from UPE, the optimal reserve price under CPE always excludes more types compared to the risk-neutral benchmark. This is because the marginal bidder — the bidder who in equilibrium submits a bid exactly equal to the reserve price — is exposed to potential losses (in the good’s dimension) when participating in the auction, but avoids them when abstaining. Hence, such bidder is willing to participate only if the reserve price is strictly below her value. Therefore, the revenue that the seller extracts from the marginal bidder is lower than in the risk-neutral benchmark; this, in turn, leads the seller to select a marginal bidder with a higher type, and thus to exclude more types compared to the risk-neutral case.

Finally, under CPE the optimal reserve price differs between the FPA and the SPA. In particular, for low levels of loss aversion over money, the reserve price in the SPA is larger than that in the FPA, while the opposite holds for high levels of loss aversion over money. The reason is that the SPA exposes bidders to additional payment risk compared to the FPA, and this risk varies with the choice of the reserve price. Indeed, a higher reserve price increases the expected payment of participating bidders in both the FPA and SPA; yet, in the SPA there is a partially mitigating effect since the reserve price “replaces” uncertain bids, thereby reducing bidders’ payment risk conditional on winning. This reduction in payment risk, which is not present in the FPA, increases bids and generates an upward pressure on the revenue-maximizing reserve price in the SPA. How-

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<sup>8</sup>Li and Tan (2017) show that a seller with a privately known value may prefer a secret reserve price to a public one when facing risk-averse buyers. The reason is that, as the optimal reserve price depends on the seller’s value, the fact that the seller is privately informed makes the reserve price random from the buyers’ perspective. However, if the seller’s value was commonly known, as in our model, a public reserve price would then be optimal. A similar argument holds in common-value auctions with risk-neutral bidders; see Vincent (1995).

ever, unlike the SPA, the FPA exposes high-type bidders to less risk than low-type bidders. Indeed, the bidder with the highest type rationally expects to win the auction with certainty and therefore does not experience any dis-utility from loss aversion over money. Thus, increasing the reserve price in the FPA increases the expected payments of rather high-type bidders by more than that of low types. This effect, which is not present in the SPA, also creates an upward pressure on the revenue-maximizing reserve price. For moderate levels of loss aversion over money, the first effect dominates the second one and the optimal reserve price in the SPA is larger than that in the FPA; however, the second effect prevails for high levels of loss aversion over money so that the FPA’s optimal reserve price exceeds the SPA’s one.

Section 5 concludes the paper by summarizing the results of our model and discussing some further implications. Proofs are relegated to Appendix A and Supplementary Appendix B.

## 2 The Model

In this section, we introduce the auction environment and the bidders’ preferences, and provide formal definitions of the solution concepts — CPE and UPE — in the context of sealed-bid auctions.

### 2.1 Environment

A seller auctions off an item to  $N \geq 2$  bidders via a sealed-bid auction. Each bidder  $i \in \{1, 2, \dots, N\}$  has a private value  $t_i$  independently drawn from the support  $[\underline{t}, \bar{t}]$ , with  $\bar{t} > \underline{t} = 0$ , according to the same cumulative distribution function  $F$ .<sup>9</sup> We assume that  $F$  is continuously differentiable, with strictly positive density  $f$  on its support. Further, we impose the standard assumption that  $F$  has a monotone hazard rate; i.e.,  $\frac{f(x)}{1-F(x)}$  is increasing for all  $x \in [\underline{t}, \bar{t}]$ . This, in turn, implies that bidders’ “virtual values” are increasing; i.e.,  $V(t_i) \equiv t_i - \frac{1-F(t_i)}{f(t_i)}$  is increasing in  $t_i$ . The seller has a commonly-known value  $t^S \in [0, \bar{t}]$ .

We consider two canonical selling mechanisms: the first-price sealed-bid auction (FPA) and the second-price sealed-bid auction (SPA). We restrict attention to symmetric equilibria in increasing strategies.<sup>10</sup> In such equilibria, the bidder with the highest type wins the auction, conditional on placing a bid above the reserve price. We denote by  $F_1$  the cumulative distribution function of the highest order statistic among  $N - 1$  draws, and by  $f_1$  its corresponding density. Furthermore, let  $q(t_i)$  be the probability with which a type- $t_i$  bidder receives the good *in equilibrium*. Finally, let  $r_{RN}$  denote the revenue-maximizing reserve price with risk-neutral bidders and notice that  $r_{RN} > 0$  since  $\underline{t} = 0$ .

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<sup>9</sup>We assume that  $\underline{t} = 0$  to economize the presentation. Under this assumption, a seller facing risk-neutral bidders would always choose a non-trivial reserve price; i.e., there are no corner solutions.

<sup>10</sup>Throughout the paper, we restrict attention to symmetric (i.e., non discriminatory) auction mechanisms; for a recent analysis of asymmetric auctions with expectations-based loss-averse bidders, see Muramoto and Sogo (2022).

## 2.2 Bidders' Preferences

Bidders have expectations-based reference-dependent preferences as formulated by Kőszegi and Rabin (2006). Bidder  $i$ 's utility function has two components. First, for a given auction price  $p$ , her *consumption utility* is given by  $q(t_i - p)$ , where  $q \in \{0, 1\}$  and  $q = 1$  if she wins the auction. Second, the bidder also derives utility from comparing her actual consumption to a reference consumption outcome given by her recent expectations (probabilistic beliefs).<sup>11</sup> Hence, given a consumption outcome  $(q, p)$  and a deterministic reference point  $(r^g, r^m) \in \{0, 1\} \times \mathbb{R}$ , a bidder's total utility is

$$U[(q, p) | (r^g, r^m), t_i] = q(t_i - p) + \mu^g(qt_i - r^g t_i) + \mu^m(r^m - p) \quad (1)$$

where

$$\mu^k(x) = \begin{cases} \eta^k x & \text{if } x \geq 0 \\ \eta^k \lambda^k x & \text{if } x < 0 \end{cases}$$

is *gain-loss utility*, with  $\eta^k \geq 0$  and  $\lambda^k > 1$ , and  $k \in \{g, m\}$ . The parameter  $\eta^k$  captures the weight a bidder attaches to gain-loss utility while  $\lambda^k$  is the coefficient of loss aversion. Hence, losses receive a larger weight than equal-size gains. Moreover, according to (1), a bidder assesses gains and losses separately over each dimension of consumption utility, with different gain-loss parameters.<sup>12</sup>

Because in many situations expectations are stochastic, Kőszegi and Rabin (2006) allow for the reference point to be a pair of probability distributions  $H^g$  and  $H^m$  over the two dimensions of consumption utility; then, a bidder's total utility from the outcome  $(q, p)$  can be written as

$$U[(q, p) | (H^g, H^m), t_i] = q(t_i - p) + \int_{r^g} \mu^g(qt_i - r^g t_i) dH^g(r^g) + \int_{r^m} \mu^m(r^m - p) dH^m(r^m).$$

In words, for each utility dimension a bidder compares the realized outcome to all possible outcomes in the reference lottery, each one weighted by its probability.

## 2.3 Solution Concepts

A bidder learns her type before submitting a bid and, hence, maximizes her interim expected utility. If the distribution of the reference points is  $H = (H^g, H^m)$  and the distribution of consumption outcomes is  $G = (G^g, G^m)$ , the interim expected utility of a bidder with type  $t_i$  is

$$EU[G|H, t_i] = \int_{\{q,p\}} \int_{\{r^g, r^m\}} U[(q, p) | (r^g, r^m), t_i] dH(r^g, r^m) dG(q, p).$$

<sup>11</sup>Banerji and Gupta (2014), Rosato and Tymula (2019) and Eisenhuth and Ewers (2020) provide experimental support for the Kőszegi and Rabin (2006)'s model in the context of sealed-bid auctions.

<sup>12</sup>We allow for different parameters of gain-loss utility on the good and money dimensions because the two have different implications for bidding in auctions. In particular, our formulation is rich enough to capture situations where bidders are loss averse only regarding the consumption dimension. Such case applies if bidders' income is subject to large background risk, as argued by Kőszegi and Rabin (2009); in a similar vein, Novemsky and Kahneman (2005) argue that money given up in purchases may not be subject to loss aversion.

A strategy for bidder  $i$  is a function  $\beta_i : [\underline{t}, \bar{t}] \rightarrow \mathbb{R}_+$ . Fixing all other bidders' strategies,  $\beta_{-i}$ , the bid of bidder  $i$  with type  $t_i$ ,  $\beta_i(t_i)$ , induces a distribution over the set of final consumption outcomes. Let  $\Gamma(\beta_i(t_i), \beta_{-i})$  denote this distribution. In a sealed-bid auction, uncertainty is resolved after all bids are submitted. Thus, holding her opponents' strategies fixed, a bidder's strategy affects the distribution over consumption outcomes. As pointed out by Kőszegi and Rabin (2007), when a person makes a committed decision long before outcomes occur, she affects the reference point with her choice so that  $G \equiv H$ . This is what Kőszegi and Rabin (2007) call choice-acclimating personal equilibrium (CPE):

**Definition 1.** *A strategy profile  $\beta^*$  constitutes a Choice-Acclimating Personal Equilibrium (CPE) if for all  $i$  and for all  $t_i$ :*

$$EU [\Gamma(\beta_i^*(t_i), \beta_{-i}^*) | \Gamma(\beta_i^*(t_i), \beta_{-i}^*), t_i] \geq EU [\Gamma(b, \beta_{-i}^*) | \Gamma(b, \beta_{-i}^*), t_i]$$

for any  $b \in \mathbb{R}_+$ .<sup>13</sup>

In a CPE, if a bidder deviates to a different strategy, her reference point (i.e., her probabilistic beliefs about consumption outcomes) changes accordingly. Yet, Kőszegi and Rabin (2007) also notice that when a decision is made shortly before outcomes realize, the reference point is fixed by past expectations; then, when the decision maker chooses the strategy that maximizes her expected utility, she takes the reference point as given. Being fully rational, therefore, she can plan to submit a bid only if she is willing to follow it through, given the reference point determined by the expectation to do so. This is what Kőszegi and Rabin (2007) call unacclimating personal equilibrium (UPE):

**Definition 2.** *A strategy profile  $\beta^*$  constitutes an Unacclimating Personal Equilibrium (UPE) if for all  $i$  and for all  $t_i$ :*

$$EU [\Gamma(\beta_i^*(t_i), \beta_{-i}^*) | \Gamma(\beta_i^*(t_i), \beta_{-i}^*), t_i] \geq EU [\Gamma(b, \beta_{-i}^*) | \Gamma(\beta_i^*(t_i), \beta_{-i}^*), t_i]$$

for any  $b \in \mathbb{R}_+$ .

Therefore, differently from CPE, under UPE if a bidder deviates to a different strategy, her reference point does not change. Notice that there might exist multiple UPEs; that is, multiple bids that the bidder is willing to follow through. In this case, following Kőszegi and Rabin (2006, 2007), we assume that the bidders select the UPE that provides them with the highest expected utility among all symmetric UPEs; that is, bidders select their (symmetric) Preferred Personal Equilibrium (PPE).<sup>14</sup>

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<sup>13</sup>As shown by Dato et al. (2017), focusing on pure-strategy equilibria is without loss of generality under CPE.

<sup>14</sup>See also Heidhues and Kőszegi (2014), Rosato (2016), Freeman (2019), and Balzer and Rosato (2021).

Let  $\Lambda^k \equiv \eta^k (\lambda^k - 1)$ , for  $k \in \{g, m\}$ . The following assumption, maintained for the remainder of the paper, guarantees the existence of a symmetric equilibrium in increasing strategies:

**Assumption 1.** (*No dominance of gain-loss utility in the good dimension*)  $\Lambda^g \equiv \eta^g (\lambda^g - 1) \leq 1$ .

For given  $\lambda^g$ , Assumption 1 places an upper bound on  $\eta^g$  (and vice versa).<sup>15</sup> This bound ensures that a bidder's expected utility is increasing in her type by imposing that the weight on gain-loss utility does not (strictly) exceed the weight on consumption utility.<sup>16</sup> Finally, notice that risk neutrality is embedded in the model as a special case (for either  $\eta^g = \eta^m = 0$  or  $\lambda^g = \lambda^m = 1$ ).

### 3 Unacclimating Personal Equilibrium

This section characterizes the optimal reserve price under unacclimating personal equilibrium (UPE). First, we focus on the optimal deterministic reserve price; then, we show that the seller can achieve an even higher revenue by employing a secret and random reserve price.

#### 3.1 Deterministic Reserve Prices

In the FPA, without a reserve price, the highest bidder wins the good and pays her bid. Hence, in a symmetric equilibrium, it holds that  $q(t) = F_1(t)$  for all  $t \in [t, \bar{t}]$ . However, with a (binding) reserve price  $r$ , in a symmetric equilibrium there is a threshold type  $t_r$  such that  $q(t) = 0$  for all  $t \in [t, t_r)$  and  $q(t) = F_1(t)$  for  $t \in [t_r, \bar{t}]$ ; that is, all bidders with types below  $t_r$  prefer not to participate in the auction whereas the opposite holds for all bidders with larger types.

Fix a symmetric and increasing bidding strategy,  $\beta_I : [t, \bar{t}] \mapsto \mathbb{R}_+$ . Moreover, fix  $r$  and the implied  $t_r$ , to be determined shortly, and consider a type- $t$  bidder who mimics a *larger* type  $\tilde{t} > t$ .<sup>17</sup> With a slight abuse of notation, denote her expected payoff by  $EU(\tilde{t}, t)$ ; this is given by

$$\begin{aligned} EU(\tilde{t}, t) &= q(\tilde{t})(t - \beta_I(\tilde{t})) - \eta^g \lambda^g (1 - q(\tilde{t}))q(t)t + \eta^g (1 - q(t))q(\tilde{t})t \\ &\quad - \eta^m \lambda^m (1 - q(t))q(\tilde{t})\beta_I(\tilde{t}) + \eta^m (1 - q(\tilde{t}))q(t)\beta_I(t) - \eta^m \lambda^m q(\tilde{t})q(t)(\beta_I(\tilde{t}) - \beta(t)) \end{aligned} \quad (2)$$

where  $q(t) = F_1(t)$  if  $t \geq t_r$  and 0 otherwise.

<sup>15</sup>Assumption 1 is relevant for the derivation of the equilibrium bids under CPE, but it is not needed under UPE; nonetheless, we maintain it throughout the paper as this makes it easier to compare equilibrium bids across the two specifications.

<sup>16</sup>Herweg et al. (2010) first introduced Assumption 1 and referred to it as “no dominance of gain-loss utility”. This assumption, which has been used also by Lange and Ratan (2010), Eisenhuth (2019) and Benkert (2022), ensures that a loss-averse agent does not select first-order stochastically-dominated options; see also Masatlioglu and Raymond (2016). Using data from first-price and all-pay auctions, Eisenhuth and Ewers (2020) obtain an estimate for  $\Lambda^g$  of 0.42 (with a standard error of 0.16); similarly, using data from a BDM-like auction, Banerji and Gupta (2014) obtain an estimate for  $\Lambda^g$  of 0.283 (with a standard error of 0.08).

<sup>17</sup>Recall that we are interested in characterizing the Preferred Personal Equilibrium (PPE). As shown by Balzer and Rosato (2021), in such equilibrium the bidders' upward incentive constraints are the binding ones.

The first term in the first line of equation (2) is a bidder's expected material payoff,  $q(\tilde{t})(t - \beta_I(\tilde{t}))$ . The second and third terms represent the (expected) gains and losses in the item dimension for a bidder who planned to bid  $\beta_I(t)$  and hence expected to win with probability  $q(t)$ , but then deviates and bids to  $\beta_I(\tilde{t})$  and hence wins with probability  $q(\tilde{t})$ . Whenever she loses, the bidder experiences a loss of  $\eta^g \lambda^g q(t)t$  weighted by the probability of losing the auction,  $1 - q(\tilde{t})$ . Similarly, if the bidder expected to lose the auction with probability  $1 - q(t)$  but ends up winning it, she experiences a gain of  $\eta^g(1 - q(t))t$  weighted by the probability with which that gain occurs,  $q(\tilde{t})$ . The terms in the last line of equation (2) capture the respective (expected) gains and losses in the money dimension. The first term represents the psychological loss of a bidder who expected to lose the auction and pay nothing with probability  $1 - q(t)$  but ends up winning it and paying her bid,  $\beta_I(\tilde{t})$ , which occurs with probability  $q(\tilde{t})$ ; thus, the bidder's expected loss is  $\eta^m \lambda^m (1 - q(t))q(\tilde{t})\beta_I(\tilde{t})$ . The second term represents the psychological gain for a bidder who expected to win the auction with probability  $q(t)$  and pay her bid, but because of the deviation she ends up losing the auction, which occurs with probability  $1 - q(\tilde{t})$ ; thus, the bidder's expected gain is  $\eta^m(1 - q(\tilde{t}))q(t)\beta_I(t)$ . Finally, the third term stems from the losses a bidder incurs, when winning the auction, by comparing her actual winning bid with the expected one.

Now consider a bidder with type  $t \in [\underline{t}, t_r)$ . In equilibrium, such a bidder does not want to mimic the threshold type  $t_r$  implying that

$$\begin{aligned} EU(t_r, t) &\leq EU(t, t) \\ \Leftrightarrow F_1(t_r)(1 + \eta^g)t &\leq F_1(t_r)(1 + \eta^m \lambda^m)r. \end{aligned} \quad (3)$$

To understand condition (3) note that, in equilibrium, bidders whose types are in  $[\underline{t}, t_r)$  do not participate in the auction and thus expect to win the good with zero probability. If one of these bidders deviates and mimics type  $t_r$ , she then wins the auction with probability  $F_1(t_r)$ . Thus, her expected gains from deviating entail a material gain of  $F_1(t_r)t$  and a psychological gain of  $F_1(t_r)\eta^g t$ , since she expected to lose the auction for sure; hence, the terms on the left-hand side of (3) represent the benefits from deviating and submitting a bid equal to the reserve price. Similarly, the terms on the right-hand side of (3) capture the expected costs from such a deviation. Indeed, the expected costs of deviating and placing a bid equal to  $r$  entail a material cost of  $F_1(t_r)r$  and a psychological loss of  $\eta^m \lambda^m F_1(t_r)r$ . Letting  $t \rightarrow t_r$  from below, and making (3) hold with equality, we obtain the following relationship between the reserve price,  $r$ , and the type of the marginal bidder:

$$r = \left( \frac{1 + \eta^g}{1 + \eta^m \lambda^m} \right) t_r. \quad (4)$$

For given  $t'$ , in the following we denote the solution to (4) by  $r(t')$  (i.e., the reserve price assuring that  $t_r = t'$ ). Using the relationship between the threshold type  $t_r$  and the reserve price, we then apply the standard logic from auction theory: equilibrium behavior shapes the bidding

function up to a constant, which is pinned down by type  $t_r$ 's expected payment,  $F_1(t_r)r$ . The next lemma formally states a bidder's expected payment in the PPE for a given  $t_r$ .

**Lemma 1.** *Consider the PPE of a first-price auction with deterministic reserve in strictly increasing and symmetric strategies. Let  $t_r$  be the lowest type that receives the good with strictly positive probability. The expected payment from a bidder with type  $t \geq t_r$  is:*

$$F_1(t)\beta_I^*(t) = \int_{t_r}^t \frac{[1 + \eta^g \lambda^g F_1(x) + \eta^g(1 - F_1(x))]e^{\frac{\Delta^m[F_1(t) - F_1(x)]}{1 + \eta^m \lambda^m}}}{1 + \eta^m \lambda^m} f_1(x) dx + \frac{(1 + \eta^g)F_1(t_r)t_r}{1 + \eta^m \lambda^m} e^{\frac{\Delta^m[F_1(t) - F_1(t_r)]}{1 + \eta^m \lambda^m}} \quad (5)$$

and 0 for any  $t < t_r$ .

Notice that expressions (4) and (5), and hence  $\beta_I^*(t)$ , reduce to their well-known, risk-neutral analogues if either  $\eta^g = \eta^m = 0$  or  $\eta^g = \eta^m > 0$  but  $\lambda^g = \lambda^m = 1$ .

The next result compares the effect of the optimal reserve price under loss aversion,  $r^*$ , on the probability of a trade with that under risk neutrality for the case when bidders are loss averse only in the good's dimension.

**Proposition 1.** *Fix  $\eta^g > 0$  and  $\eta^m = 0$ . Under PPE, the optimal threshold type in the FPA and SPA with deterministic reserve price,  $t_r^*$ , is smaller than that under risk neutrality,  $t^{RN}$ . Moreover, there exists a  $\bar{\lambda}^g$  such that the optimal reserve price is larger than in the risk-neutral benchmark if and only if  $\lambda^g \leq \bar{\lambda}^g$ . In particular, for  $t^S > 0$ , and sufficiently large  $\lambda^g$ , the optimal reserve price is strictly smaller than  $t^S$ .*

According to Proposition 1, a seller facing loss-averse bidders (in the good's dimension only) excludes fewer types than one facing risk-neutral bidders. To gain intuition, note that Lemma 1 implies that for  $\eta^m = 0$  types with  $t \geq t_r$  bid according to

$$\beta_I^*(t) = \left( \int_{t_r}^t \{1 + \eta^g \lambda^g F_1(x) + \eta^g(1 - F_1(x))\} x f_1(x) dx + F_1(t_r)t_r(1 + \eta^g) \right) / F_1(t).$$

This bidding function highlights that the attachment effect only affects types above  $t_r$ . For these types, next to the material benefits from winning marginally more often,  $f_1(t)t$ , there is an additional incentive to bid aggressively in order to reduce potential losses; i.e., the term  $f_1(t)\eta^g \lambda^g F_1(t)$ . Indeed, a bidder who expected to win with probability  $F_1(t)$  experiences a loss equal to  $\eta^g \lambda^g F_1(t)t$  if she loses. If such bidder raises her bid marginally, she reduces the probability of incurring the loss. Similarly, the term  $\eta^g(1 - F_1(t))$  stems from the bidder's incentives to raise her bid in order to increase the probability of experiencing a gain. Losses, however, are more painful than equal-size gains are pleasant (i.e.,  $\lambda^g > 1$ ) and, therefore, the incentive to avoid losses dominates that to



increase gains. Importantly, only a bidder who expects to win the auction with strictly positive probability has an incentive to reduce her losses. That is, the term  $\eta^g \lambda^g F_1(t)t$  does not amplify the marginal benefit from winning for a bidder whose type is below  $t_r$ . Indeed,  $\beta_I^*(t_r) = t_r(1 + \eta^g) = r$ , highlighting that the minimal bid is not affected by the attachment effect. Due to this wedge between the minimum bid and larger bids, raising the reserve price is more costly for a seller under loss aversion than under risk neutrality.

Moreover, according to Proposition 1, the optimal reserve price with loss-averse preferences can be either larger or smaller than the one under risk neutrality. Indeed, as equation (4) reveals, for the same threshold type, the reserve price under loss aversion is larger than that under risk neutrality. Thus, whether the risk-neutral reserve price is below the loss-averse one depends on how much lower  $t_r^*$  is compared to  $t_{RN}$ ; this, in turn, depends on  $\lambda^g$ . In particular, for sufficiently large  $\lambda^g$ , the seller sets a reserve price lower than his valuation; indeed, the seller might even opt for an “absolute auction” with  $r^* = 0$ . Recall from Myerson (1981) that with risk-neutral bidders, the optimal reserve price is such that those bidders with “virtual values” below the seller’s own value are excluded. Hence, the attachment effect implies that it can be optimal for a seller to include also bidders with a virtual value below  $t^S$ . Notice that while risk aversion can explain such low reserve prices in the FPA, it cannot in the SPA (see Hu et al., 2010). The next result describes how the reserve price and the probability of no trade vary with the number of bidders.

**Proposition 2.** *Suppose that  $\eta^m = 0$ . The probability of no trade and the reserve price depend on the number of bidders. If  $t^S = 0$  (resp.  $t^S > 0$ ), as  $N \rightarrow \infty$ , the no-trade probability converges to (resp. is strictly lower than) its risk-neutral counterpart. Moreover, for any  $t^S \geq 0$ , if  $N \rightarrow \infty$  the optimal reserve price is larger than under risk neutrality.*

To see the intuition for this result, suppose first that  $t^S = 0$ . As  $N \rightarrow \infty$ , every type below  $\bar{t}$  expects to win the auction with (almost) zero probability. Hence, the seller does not affect any such type’s expectations of winning when raising the reserve price. Moreover, the limit probability of no trade under loss aversion is lower than its risk-neutral counterpart if  $t^S > 0$ . The reason is that loss-averse buyers bid more aggressively than risk-neutral ones; i.e., the seller raises more revenue from a loss-averse buyer than from a risk-neutral buyer with the same type. Hence, the seller has a weaker incentive to exclude them. Nevertheless, the lower probability of no trade under loss aversion does not imply that the reserve price is lower compared to risk neutrality. Finally, note that also under risk aversion the optimal reserve price in the FPA depends on the number of bidders, if the virtual values are increasing. Yet, under risk aversion, the reserve price in the SPA is constant in  $N$ ; see Hu et al. (2010).

The next result concerns the comparative statics of loss aversion on the probability of no trade and the reserve price when  $\eta^m > 0$ .

**Proposition 3.** *The following comparative statics hold for the optimal public reserve price in the PPE.*

1. For any  $\eta^m$  and  $t^S$ , the probability of no trade decreases in  $\eta^g$ .
2. For any given  $\eta^g \geq 0$  there exist  $\bar{N}$ ,  $\underline{N}$ ,  $\underline{t}^S$  and  $\bar{\eta}^m$  such that
  - (a) if  $N > \bar{N}$ , the probability of no trade increases in  $\eta^m$ ;
  - (b) if  $N < \underline{N}$  and  $t^S < \underline{t}^S$ , the probability of no trade decreases in  $\eta^m$  if  $\eta^m < \bar{\eta}^m$ .
3. As  $N \rightarrow \infty$ ,
  - (a) the probability of no trade converges to its risk-neutral counterpart if  $t^S = 0$ ;
  - (b) the limit probability of no trade is strictly larger than its risk-neutral counterpart if and only if  $\eta^m \lambda^m > \eta^g$  and  $t^S > 0$ ;
  - (c) the optimal reserve price,  $r^*$ , is strictly larger than its risk-neutral counterpart if and only if  $\eta^m \lambda^m < \eta^g$ .

Proposition 3 shows that the effects of  $\eta^g$  and  $t^S$  continue to hold when buyers are also loss averse over money, i.e.,  $\eta^m > 0$ . Moreover, the probability of no trade increase in  $\eta^m$  if  $N$  is sufficiently large. However, for small  $N$  and  $t^S$  the opposite may hold. To gain intuition, suppose that  $\eta^m > 0$  but  $\eta^g = 0$ . In this case, equation (5) becomes:

$$F_1(t)\beta_I^*(t) = \int_{t_r}^t \frac{e^{\frac{\Lambda^m[F_1(t)-F_1(x)]}{1+\eta^m\lambda^m}}}{1+\eta^m\lambda^m} f_1(x)xdx + \frac{F_1(t_r)t_r}{1+\eta^m\lambda^m} e^{\frac{\Lambda^m[F_1(t)-F_1(t_r)]}{1+\eta^m\lambda^m}}. \quad (6)$$

Suppose the seller raises the reserve price, thereby increasing the competitive pressure on the bidders' side; this has two opposing effects on the bidding incentives. First, there is a direct “competitive effect” which implies that bidders behave more aggressively because of the increased competition; in turn, the expected payment of all types above  $t_r$  increases.<sup>18</sup> If  $\eta^m > 0$ , however, there is also an indirect psychological effect at play since the increase in the winning bids (due to the competitive effect) exposes bidders to additional psychological dis-utility when winning; indeed, the larger the winning bid, the larger the loss a bidder experiences when (unexpectedly winning and) having to pay that bid. This “dis-utility effect” mitigates the competitive effect to some degree. Importantly, the dis-utility effect affects low types more than high types, as low types expect to win less often and thus experience higher (monetary) losses when winning. As a result, low-type bidders are less responsive to an increase in the reserve price compared to high-type bidders.

There are two ways in which the seller can react to the dis-utility effect. First, she might reduce the no-trade probability compared to the risk-neutral case. Indeed, the dis-utility effect

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<sup>18</sup>This competitive effect is also present under risk neutrality. Thus, if  $\eta^m = 0$  it is the sole effect.

makes increasing the threshold type — and hence the reserve price — less attractive than under risk neutrality. Alternatively, she can increase the threshold type compared to the risk-neutral benchmark. The rationale for the second option is that the opportunity cost of the no-trade event is lower under loss aversion than under risk neutrality since low types' bids increase by less than those of high types. Proposition 3 shows that if  $N$  is rather small, implying that it is relatively likely that the winning bidder has a low type, and if the opportunity cost of the no-trade event is rather large (i.e., small  $\eta^m$  and  $t^S$ ), the seller prefers the first option; by contrast, the second option is preferred for  $N$  and  $t^S$  large enough.

Yet, Proposition 3 also states that the dis-utility effect affects all bidder types to the same degree as  $N$  goes to infinity. This is intuitive since, as  $N$  grows large, almost all types expect to receive the good with the same probability; i.e., zero. Thus, when  $N$  is very large, the increase in the payments of high-types bidders when raising the reserve price is proportional to that of low-type ones. In this situation, the opportunity cost of the no-trade event for a risk-neutral seller facing loss-averse bidders is larger than with risk-neutral bidders if and only if  $t^S > 0$  and the effect of loss aversion over money outweighs the attachment effect.

Finally, note that all the results of this section hold for the SPA. Indeed, Balzer and Rosato (2021) show that both auction formats yield the same expected revenue to the seller. Moreover, since the threshold type  $t_r$  does not face any risk in her payment conditional on winning, the relationship between  $r$  and  $t_r$  satisfies (4) also in the SPA. Hence, the optimal reserve price and the optimal threshold type in the SPA coincide with those in the FPA.

### 3.2 Secret Reserve Prices and Shill Bidding

We now show that the seller can achieve an even higher revenue using a random and secret reserve price. Recall that the attachment increases the seller's opportunity cost of raising the reserve price. Because secret and random reserve prices mitigate this cost, they allow the seller to raise more revenue than under a deterministic reserve price, as the following proposition states.

**Proposition 4.** *Consider either a FPA or SPA. Under PPE, for any  $\eta^k, \lambda^k$  with  $k \in \{g, m\}$ , and any public reserve price, there exists a distribution of random and secret reserve prices that yields a higher revenue than the public one.*

To gain some intuition, suppose that  $\eta^m = 0$  and consider a FPA with a secret reserve price drawn from a commonly known distribution over some interval  $[\underline{r}, \bar{r}]$ ; then, in a symmetric equilibrium in increasing strategies, for each possible realization of the reserve price,  $r \in [\underline{r}, \bar{r}]$ , there exists a corresponding threshold type,  $\tilde{t}_r$ , whose bid coincides with  $r$ . Hence, a bidder with a type below the highest such threshold type,  $\tilde{t}_{\bar{r}}$ , wins the good if both (i) her type is larger than that of all other bidders, and (ii) she bids higher than the secretly drawn reserve price. In what follows, it will prove convenient to directly work with the implied distribution of threshold types. That is,

suppose that the seller first draws  $\tilde{t}_r \in [\tilde{t}_r, \tilde{t}_r]$  according to some distribution  $F_0$ . Then, the seller computes the reserve price  $r(\tilde{t}_r)$  that matches the equilibrium bid of a bidder with type  $\tilde{t}_r$ , who expects to win the good with probability  $q(t) = F_1(\tilde{t}_r)F_0(\tilde{t}_r)$ .<sup>19</sup>

Note that the above framework nests a public reserve price as a special case that occurs if all probability mass is on one threshold type, say,  $\tilde{t}_r^p \in [\tilde{t}_r, \tilde{t}_r]$ ; that is,  $F_0(\tilde{t}_r) = 0$  if  $\tilde{t}_r < \tilde{t}_r^p$  and  $F_0(\tilde{t}_r) = 1$  otherwise. However, as argued in Section 3.1, under such an extremely discontinuous threshold-type distribution, the seller forgoes the attachment effect of the marginal bidder with type  $\tilde{t}_r^p$ . Indeed, the implied winning probability,  $q$ , jumps from 0 to  $F_1(\tilde{t}_r^p)$  at the threshold type and, in a PPE, that type has the same attachment level as a type that loses for sure. More generally, consider an arbitrary threshold-type distribution that might have discontinuous jump points. At each such jump point, the corresponding threshold type has the attachment level of the type immediately below her. As the seller’s revenue increases in the bidders’ attachment level, the seller prefers continuous distributions that smooth out jump points (see the left panel of Figure 1).

In fact, we show in the proof of Proposition 4 that the seller can achieve a revenue arbitrarily close to a strict upper bound when using a continuous distribution of threshold types. This distribution is such that every bidder type expects to win the auction with strictly positive probability — as tiny as that might be. That is,  $\tilde{t}_r = t$ . More precisely, the distribution is such that bidders with types strictly below  $\tilde{t}_r$  expect to win the auction with a small probability, and this probability steeply increases in a bidder’s type for types in a neighborhood below  $\tilde{t}_r$ ; see Figure 1 for a concrete example. Moreover, the seller chooses  $\tilde{t}_r$  such that the virtual value of the largest threshold type is equal the seller’s value; i.e.,  $\tilde{t}_r = (1 - F(\tilde{t}_r))/f(\tilde{t}_r) + t^S$ . One way for the seller to implement such a distribution of threshold types is to use the CDF  $F_0(\tilde{t}_r) = (\frac{\tilde{t}_r}{t_r})^K$ , with  $K \in \mathbb{R}_+$  and “large”. The right panel of Figure 1 depicts  $F_0$  when  $\tilde{t}_r = 0.5$  and  $K = 30$ .

Therefore, under a secret and random reserve price, all bidders with type below the largest threshold type  $\tilde{t}_r$  expect to win with strictly positive probability and are exposed to (potential) losses. In particular, the steep increase of  $q(t)$  from (almost) zero to (almost)  $F_1(\tilde{t}_r)$  in the neighborhood below  $\tilde{t}_r$  ensures that types slightly below  $\tilde{t}_r$  have an incentive to bid aggressively in order to reduce their potential losses.<sup>20</sup>

A second rationale behind the steep increase in the reserve price follows from the usual rationing effect that is also present under risk neutrality: by imposing a larger minimal bid (the reserve price), the seller increases competitive pressure on the bidders’ side at the cost of decreasing the probability of trade. When ignoring the role of the attachment effect, described in Section 3.1, the optimal resolution of this trade-off entails excluding all types with virtual values below the seller’s own value. With the above-described secret and random reserve price, the attachment effect does not affect the seller’s trade-off as all bidder types are exposed to the attachment effect. Moreover,

<sup>19</sup>The equilibrium bid depends on the distribution of the reserve prices itself. Equation (7) below provides a closed-form solution for the implied reserve price.

<sup>20</sup>The intuition for this effect is reminiscent of the one behind the random-sales results of Heidhues and Kőszegi (2014) and Rosato (2016).

whether the probability of receiving the good jumps from 0 to  $F_1(\tilde{t}_r)$  at the threshold type (as with a public reserve price) or it increases more smoothly (as under the distribution of the secret reserve price described above) is of second-order importance for the goal of intensifying the competitive pressure on the bidders' side.

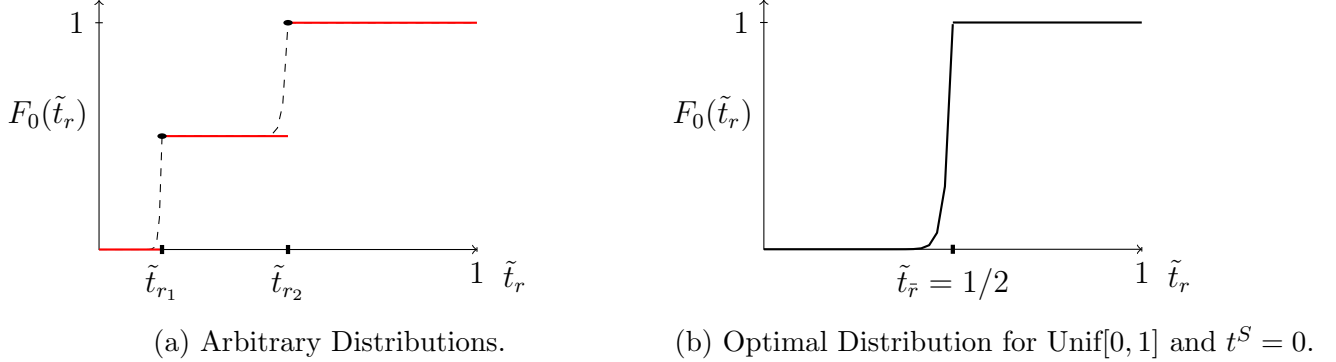


Figure 1: The solid red lines in the left panel depict the distribution of threshold types under an (arbitrary) secret reserve price scheme with two reserve prices,  $r_1$  and  $r_2$ . The dashed curve is a continuous approximation that leaves the seller with strictly larger revenue and uses infinitely many reserve prices. The right panel depicts the optimal CDF of the threshold types when bidders' types are distributed according to a Unif[0, 1]. The induced distribution of secret reserve prices,  $\beta_I^*(\tilde{t}_r)$ , can be obtained from (7).

In order to obtain the distribution of the secret reserve price, start by fixing  $F_0$ , the distribution of the threshold types  $\tilde{t}_r$ . Then, substitute the drawn  $\tilde{t}_r$  into the equilibrium bidding function that applies without a reserve price, but where a type- $t$  bidder wins the auction with probability  $q(t) = F_0(t)F_1(t)$ ; that is, for  $\tilde{t}_r \in [t, \tilde{t}_r]$ , we have

$$\beta_I^*(\tilde{t}_r) = \frac{\int_t^{\tilde{t}_r} \frac{[1+\eta^g \lambda^g q(x) + \eta^g (1-q(x))] e^{\frac{\Lambda^m [q(\tilde{t}_r) - q(x)]}{1+\eta^m \lambda^m}} q'(x) x dx}{q(\tilde{t}_r)}}{q(\tilde{t}_r)}. \quad (7)$$

In equilibrium, bidders correctly anticipate that the seller implements reserve price  $r = \beta_I^*(\tilde{t}_r)$  when drawing  $\tilde{t}_r$  according to CDF  $F_0(\tilde{t}_r)$ , and their optimal response is given by the bidding function in (7), replacing  $\tilde{t}_r$  with a buyer's type  $t$ .

Secret reserve prices are common in internet auctions; see Bajari and Hortag̈su (2004). However, different from our result, these secret reserve prices are usually deterministic. Yet, from the bidders' perspective, the secret reserve price might appear as random if they do not observe how precisely the seller chooses the reserve price. Indeed, as shown in Figure 1, the distribution of the secret reserve price has most of the mass on the upper bound of its support, and arbitrarily little mass everywhere else. Thus, our characterization applies if bidders place a small probability on the event that the seller makes a “mistake” when choosing the secret reserve price. Moreover, our secret-reserve-price scheme could also be implemented via “shill bidding”, a prominent phenomenon in real-world auctions, whereby a dummy bidder submits pre-specified bids on behalf of the seller; see Ashenfelter (1989).

Our final result in this section concerns the comparative statics with respect to the intensity of loss aversion in the money dimension.

**Proposition 5.** *Under the optimal secret reserve price, we have the following comparative statics.*

1. *If  $t^S = 0$ , the probability of no trade*
  - (a) *is arbitrarily close to that under risk neutrality if  $\eta^m = 0$  and strictly larger if  $\eta^m > 0$ ;*
  - (b) *converges to that under risk neutrality as  $N \rightarrow \infty$ .*
2. *If  $t^S > 0$  and  $\eta^m = 0$ , the probability of no trade*
  - (a) *is strictly smaller than that under risk neutrality;*
  - (b) *converges to a limit value smaller than that under risk neutrality as  $N \rightarrow \infty$ .*

The results of Proposition 5 and the intuition behind them are similar to those in Proposition 3 under a deterministic reserve price. Indeed, because with secret reserve prices the attachment effect does not affect the optimal no-trade probability anymore, differences with the risk-neutral benchmark are driven solely by loss aversion over money,  $\eta^m$ . Finally, we re-emphasize that the analysis and the results in this section carry over to the SPA.

## 4 Choice-Acclimating Personal Equilibrium

This section analyzes the optimal reserve price for both the FPA and SPA under CPE. Our first result is that, differently from under UPE, public reserve prices are optimal.

**Proposition 6.** *In both the FPA and SPA, under CPE, a public reserve price outperforms secret (and random) ones.*

Under CPE, a bidder's reference lottery is determined by the submitted bid, both on and off the equilibrium path. Therefore, even a bidder who in equilibrium abstains from bidding is exposed to (potential) losses if she deviates. Hence, secret reserve prices are not necessary to expose every bidder to potential losses. In the remainder of this section, we characterize the optimal reserve price in both the FPA and the SPA, and show that it differs between the two formats.

### 4.1 First-Price Auction

We focus on equilibria in symmetric and increasing bidding strategies,  $\beta_I : [\underline{t}, \bar{t}] \rightarrow \mathbb{R}_+$ . Under a (binding) public reserve price,  $r$ , there is a threshold type  $t_r$ , such that  $q(t) = 0$  for all  $t \in [\underline{t}, t_r)$  and  $q(t) = F_1(t)$  for  $t \in [t_r, \bar{t}]$ . That is, all types below  $t_r$  prefer not to participate in the auction,

whereas the opposite holds for all larger types. Recall that  $\Lambda^k \equiv \eta^k(\lambda^k - 1)$  for  $k \in \{g, m\}$ , with  $\Lambda^g \leq 1$ . Consider a type- $t$  bidder who mimics a larger type  $\tilde{t} > t$ . Her expected payoff is

$$EU(\tilde{t}, t) = q(\tilde{t})(t - \beta_I(\tilde{t})) - \Lambda^g(1 - q(\tilde{t}))q(\tilde{t})t - \Lambda^m(1 - q(\tilde{t}))q(\tilde{t})\beta_I(\tilde{t}),$$

where  $q(t) = F_1(t)$  if  $t \geq t_r$  and 0 otherwise. Differently from (2), under CPE a type- $t$  bidder's reference point immediately adjusts to the deviation  $\beta_I(\tilde{t})$ . Thus, the probability terms in the expected gain-loss utility depends on  $\tilde{t}$  but not on the bidder's actual type  $t$ .

Next, consider a bidder with a type  $t \in [\underline{t}, t_r)$ . In equilibrium, such a bidder does not want to mimic the threshold type,  $t_r$ , implying that

$$\begin{aligned} EU(t_r, t) &\leq EU(t, t) \\ \Leftrightarrow [F_1(t_r) - \Lambda^g F_1(t_r)(1 - F_1(t_r))]t &\leq F_1(t_r)[1 + \Lambda^m(1 - F_1(t_r))]r. \end{aligned} \quad (8)$$

The left-hand side of (8) represents a type- $t$  bidder's benefit when mimicking type  $t_r$ : with probability  $F_1(t_r)$  she wins the auction and enjoys an item she values  $t$ . However, mimicking type  $t_r$  also exposes the bidder to risk as she now expects to win with probability  $F_1(t_r)$  and to lose with probability  $(1 - F_1(t_r))$ . The term  $\Lambda^g F_1(t_r)(1 - F_1(t_r))$  is the (expected) gain-loss dis-utility generated by this risk, which reduces the benefit from mimicking a higher type. The right-hand side of (8) states the costs of mimicking: with probability  $F_1(t_r)$  the bidder wins the auction and pays price  $r$ . The second part, again, is the (expected) psychological dis-utility the bidder experiences because of the uncertainty over whether or not she will have to pay; i.e.,  $\Lambda^m F_1(t_r)(1 - F_1(t_r))r$ . In equilibrium, no type has an incentive to deviate; hence, (8) holds with equality for the threshold type  $t_r$ . Thus, the reserve price satisfies

$$r = \left\{ \frac{1 - \Lambda^g[1 - F_1(t_r)]}{1 + \Lambda^m[1 - F_1(t_r)]} \right\} t_r. \quad (9)$$

Notice the difference with the optimal reserve price when bidders play according to PPE. There, a bidder keeps his reference point fixed when considering a deviation, implying that bidders with types below  $t_r$  do not experience losses in the good dimension when deviating and bidding the reserve price. Under CPE, however, even a bidder who abstains from bidding in equilibrium is exposed to (potential) losses when placing a non-trivial bid off the equilibrium path. Next, we characterize the symmetric equilibrium bidding strategy under a public reserve price  $r$ .

**Lemma 2.** *In the equilibrium of the FPA with reserve price  $r$ , a bidder's expected payment satisfies*

$$F_1(t)\beta_I^*(t) = \frac{(1 - \Lambda^g)[F_1(t)t - \int_{t_r}^t F_1(x)dx] + \Lambda^g[F_1(t)^2t - \int_{t_r}^t F_1(x)^2dx]}{1 + \Lambda^m[1 - F_1(t)]}, \quad (10)$$

for  $t \geq t_r$ , and 0 for  $t < t_r$ .

Note that for  $\Lambda^g = \Lambda^m = 0$  both 9 and 10 reduce to their risk-neutral analogues. In light of this, the first result in the next proposition is rather intuitive.

**Proposition 7.** *Under loss aversion, the optimal probability of no trade is weakly larger than under risk neutrality. It is strictly larger if and only if  $\Lambda^m > 0$  or  $\Lambda^g t^S > 0$ . Moreover, whenever  $\Lambda^m > 0$  or  $\Lambda^g > 0$ , the optimal reserve price is strictly lower than the risk-neutral one.*

Suppose first that  $\Lambda^m = 0$ . As argued before, under CPE both the bid of type  $t_r$ , i.e., the reserve price, as well as the bids of higher types are impacted by potential losses. Thus, differently from UPE, increasing the reserve price does not reduce the probability that a bidder is exposed to the attachment effect. Hence, the seller faces the standard trade-off between raising the likelihood of no trade and amplifying competitive pressure. Yet, because of loss aversion in the item dimension, the expected payoff of the threshold type is lower than in the risk-neutral benchmark; this, in turn, implies that for the *same* threshold type the risk-neutral reserve price is larger than the one under loss aversion. Thus, if  $t^S > 0$  it is more attractive for the seller to keep the good when facing loss-averse bidders than when facing risk-neutral ones, implying a larger threshold type under loss aversion than under risk neutrality.

Next, consider the case where  $\Lambda^m > 0$ . According to Proposition 7, the probability of no trade is larger than under risk neutrality. The intuition is as follows. Compared to the risk-neutral benchmark, raising the threshold type has the same bid-enhancing effect on those types above the threshold. Yet, because loss aversion over money induces less aggressive bidding overall, the forgone revenue from the marginally excluded bidders is lower; hence, raising the threshold type is less costly for the seller. In turn, the optimal threshold type under loss aversion is larger than its risk-neutral counterpart.

Therefore, under CPE, the seller excludes more types than under risk neutrality.<sup>21</sup> However, notice the even if it excludes more types, the optimal reserve under CPE is lower than the risk-neutral one since loss aversion reduces a given threshold type's expected utility from participating in the auction. Indeed, as can be seen from equation (9), the maximal price the threshold type is willing to pay decreases in both  $\Lambda^m$  and  $\Lambda^g$ .

Our final result concerns how the set of excluded types changes with the number of bidders.

**Proposition 8.** *As  $N \rightarrow \infty$ , the probability of no trade converges to that under risk neutrality if and only if  $t^S \Lambda^m = 0$  and  $t^S \Lambda^g = 0$ . Else, the probability of no trade is larger than under risk neutrality. Moreover, the optimal reserve price converges to a value lower than  $r_{RN}$ .*

Proposition 8 shows that if and only if the bidders are loss averse (i.e.,  $\Lambda^g > 0$  or  $\Lambda^m > 0$ ) and the seller has a strictly positive value for the good (i.e.,  $t^S > 0$ ), the limit probability of no

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<sup>21</sup>As shown by Hu et al. (2010), the optimal reserve price with risk-averse bidders is lower than in the risk-neutral benchmark; hence, an immediate additional implication of Proposition 7 is that the seller excludes more types if bidders are loss averse than if they are risk averse.



trade is larger than that under risk neutrality. In particular, as  $N \rightarrow \infty$ , the probability of having to pay her bid,  $F_1(t)$ , converges to zero for each bidder except the one with the highest type in the support. Thus, raising the reserve price increases the expected payment of low and high types by the same amount, just like under risk neutrality. However, for the *same* threshold type, the dis-utility effect implies that the reserve price under loss aversion is lower than that under risk neutrality. Thus, if  $t^S > 0$ , similar to UPE, the no-trade event becomes relatively more attractive for a (risk-neutral) seller when facing loss-averse bidders than when facing risk-neutral ones. Hence, the seller continues to exclude more types compared to the risk-neutral and risk-averse benchmarks even as the number of bidders grows arbitrarily large.

## 4.2 Second-Price Auction

In the FPA, bidders do not experience any risk in their payment conditional on winning. In contrast, in the SPA bidders are uncertain about what they have to pay even conditional on winning. We will now show how this difference affects equilibrium bidding behavior and, in turn, the seller's optimal reserve price.

We start by characterizing bidders' equilibrium behavior. For a fixed reserve price  $r$ , there is a corresponding threshold type  $t_r$ , who is indifferent between participating in the auction or abstaining from it. Consider a type- $t$  bidder who plans to bid as  $\tilde{t} \geq t_r$ . Her expected utility is:

$$\begin{aligned}
EU(\tilde{t}, t) &= F_1(\tilde{t})t - F_1(t_r)r - \int_{t_r}^{\tilde{t}} \beta_{II}^*(x) f_1(x) dx \\
&\quad - \Lambda^g (1 - F_1(\tilde{t})) F_1(\tilde{t}) t - \Lambda^m (1 - F_1(\tilde{t})) \left[ \int_{t_r}^{\tilde{t}} \beta_{II}^*(x) f_1(x) dx + F_1(t_r)r \right] \\
&\quad - \Lambda^m \int_{t_r}^{\tilde{t}} \int_{t_r}^x [\beta_{II}^*(x) - \beta_{II}^*(y)] f_1(x) f_1(y) dy dx - \Lambda^m F_1(t_r) \int_{t_r}^{\tilde{t}} (\beta_{II}^*(x) - r) f_1(x) dx
\end{aligned} \tag{11}$$

The first line in (11) represents a type- $t$  bidder's material payoff when mimicking a bidder with type  $\tilde{t} \geq t_r$ . The second line represents the expected gain-loss utility from the “extensive risk”—the dis-utility stemming from the risk between winning and losing the auction—while the last line depicts the bidder's expected gain-loss utility from the risk in the payment dimension, or the “intensive risk”; see also Balzer and Rosato (2021). Notice that the latter risk is not present in the FPA, where bidders know their payment conditional on winning.

Consider now a bidder with type  $t < t_r$ . If such a bidder mimics the threshold type  $t_r$ , she does not face any risk in her payment conditional on winning. Thus, the relationship between  $r$  and the threshold type satisfies (4), as in the FPA. The next proposition describes the equilibrium bidding strategies.

**Lemma 3.** Fix a reserve price  $r$  with corresponding type  $t_r$  such that  $\left\{ \frac{1-\Lambda^g[1-F_1(t_r)]}{1+\Lambda^m[1-F_1(t_r)]} \right\} t_r = r$ . Furthermore, let  $\gamma(t) \equiv (1-\Lambda^g)F_1(t) + \Lambda^g F_1(t)^2$ . In equilibrium, a bidder with type  $t_r$  submits the lowest bid  $\beta(t_r)$ , with  $\beta(t_r) > r$ . For any  $t \in [t_r, \bar{t}]$  the equilibrium bidding function satisfies

$$\beta_{II}^*(t) = \frac{\gamma'(t)t}{f_1(t)(1+\Lambda^m)} + 2\Lambda^m \int_{t_r}^t \frac{\gamma'(x)x}{(1+\Lambda^m)^2} e^{\frac{2\Lambda^m[F_1(t)-F_1(x)]}{1+\Lambda^m}} dx + \frac{2\Lambda^m e^{\frac{2\Lambda^m[F_1(t)-F_1(t_r)]}{1+\Lambda^m}}}{1+\Lambda^m} F_1(t_r)r.$$

It is worth to highlight that, with loss aversion,  $\beta_{II}(t_r) > r$ ; that is, the threshold type submits a bid strictly larger than the reserve price. To see the intuition, assume for a moment that  $\Lambda^g = 0$  and  $\Lambda^m > 0$ , and consider a bidder with a type slightly below  $t_r$ . If such a bidder deviates and bids the reserve price, she faces no risk in her payment conditional on winning.<sup>22</sup> Now consider a bidder with a type slightly above  $t_r$ . If she bids above the reserve price (as she does in equilibrium), the bidder faces an uncertain payment conditional on winning; and this additional risk lowers her expected payoff. Hence, while the threshold type is not exposed to uncertainty in the payment conditional on winning, types above the threshold internalize the additional uncertainty in their payment. This implies that their expected payments decrease more rapidly with  $\Lambda^m$  (compared to the threshold type's payment), forcing the reserve price to be lower than the lowest bid; that is,  $\beta_{II}^*(t_r) > r$ .

The next proposition shows that a seller optimally excludes more types under loss aversion than under risk neutrality or risk aversion.

**Proposition 9.** Under loss aversion, the optimal probability of no trade is weakly larger than under risk neutrality or risk aversion. It is strictly larger if and only if  $\Lambda^m > 0$  or  $\Lambda^g t^S > 0$ .

Proposition 9 mirrors the respective result for the FPA. Indeed, despite the difference in how the reserve price affects equilibrium bids, for  $\Lambda^m = 0$  the FPA and SPA generate the same expected revenue. Moreover, as argued above, the reserve price is determined by the exact same condition as in the FPA. Next, we compare the no-trade probability in the SPA with that in the FPA.

**Proposition 10.** For  $0 < \Lambda^m \leq 1$ , more types are excluded in the SPA than in the FPA. As  $\Lambda^m \rightarrow \infty$ , more types are excluded in the FPA than in the SPA.

Proposition 10 implies that the marginal benefit from raising the reserve price is larger in the SPA than in the FPA if  $\Lambda^m$  is relatively low. In contrast, the opposite is true for large values of loss aversion over money. To see the intuition, suppose the seller raises the reserve price; this, in turn, leads every type to bid more aggressively in both auction formats. Such larger bids expose bidders to more risk in the money dimension. Besides increasing the “extensive risk” by the same

<sup>22</sup>In the FPA, conditional on winning, a bidder pays her bid. Thus, a bidder who bids the reserve price faces the same kind of risk in her monetary payment as a bidder who places a bid above the reserve price: with some probability she pays her bid, and with the remaining probability she pays nothing. Hence, the bid of the threshold type and the reserve price coincide in the FPA.

amount in the FPA and SPA, however, a higher reserve price also reduces the “intensive risk” a bidder faces in the SPA. Indeed, since fewer types participate in the auction, the price becomes less risky conditional on winning. This effect explains why raising the reserve price can be more beneficial to the seller’s revenue in the SPA than in the FPA.

Yet, for  $\Lambda^m$  large enough, there is a second channel at play. Indeed, in both formats, equilibrium bids are decreasing in  $\Lambda^m$ . In the FPA, however, an increase in  $\Lambda^m$  reduces high types’ bids by less than low types’ ones. The reason is that high types face less risk about their payment as they expect to win, and hence to pay, with a larger probability. Indeed,  $\Lambda^m$  does not affect the bid of the highest type. Hence, as  $\Lambda^m$  increases, it becomes more attractive to exclude low types, as they bid relatively little. In contrast, in the SPA an increase in  $\Lambda^m$  affects all bidder types as even high types are exposed to the “intensive risk” in the payment. Thus, excluding more types becomes less attractive, compared to the FPA.

Finally, the last result of this section mirrors the corresponding one for the FPA.

**Proposition 11.** *As  $N \rightarrow \infty$ , the probability of no trade converges to that under risk neutrality if and only if  $t^S = 0$ . Else, the probability of no trade is larger than under risk neutrality. Moreover, the optimal reserve price in the SPA converges to a value lower than that under risk neutrality.*

Thus, as the number of bidders grows large the difference in the optimal reserve prices between the SPA and FPA vanishes if  $t^S = 0$ . In this case, the optimal no-trade probability in both formats approaches that under risk neutrality. If the no-trade probability, and thus the threshold type, in the FPA is the same as in the SPA, the same holds for the resulting reserve price.

## 5 Conclusion

This paper belongs to a recent and growing literature on the market implications of expectations-based loss aversion. Indeed, over the last decade, the model of expectations-based loss aversion developed by Kőszegi and Rabin (2006, 2007, 2009) has found many fruitful applications in several areas of economics, including firms’ pricing and advertising strategies (Heidhues and Kőszegi, 2008, 2014; Rosato, 2016; Karle and Peitz, 2014, 2017; Karle and Schumacher, 2017), incentive provision (Herweg et al., 2010; Daido and Murooka, 2016; Macera, 2018), bargaining (Rosato, 2017; Herweg et al., 2018; Benkert, 2022), labor supply (Crawford and Meng, 2011), school choice (Dreyfus et al., 2021; Meisner and von Wangenheim, 2022), asset pricing (Pagel, 2016, 2018; Meng and Weng, 2018), and life-cycle consumption (Pagel, 2017). In particular, there have been several studies on the implications of expectations-based loss aversion in auctions; see Lange and Ratan (2010), Eisenhuth (2019) and Balzer and Rosato (2021) on sealed-bid auctions, and von Wangenheim (2021), Balzer et al. (2022) and Rosato (2022) on dynamic ones.

While the prior literature has mostly abstracted from considering reserve prices, the focus of our paper is on how expectations-based loss aversion affects the optimal reserve price in the FPA

and SPA.<sup>23</sup> Our analysis reveals that loss aversion delivers new implications for the design of optimal auctions that are likely to be of interest for both theorists as well as practitioners. In particular, under UPE, we find that secret and random reserve prices dominate public ones in both the FPA and SPA. Indeed, by using secret and random reserve prices, the seller introduces a small risk that exposes all bidders to the attachment effect, which in turn leads them to bid more aggressively. Under CPE, instead, we show that the optimal reserve price is public and deterministic, but it differs between the FPA and SPA. Finally, under both UPE and CPE, the optimal reserve price depends the number of bidders in the auction and is typically lower than the optimal reserve price under risk neutrality. Hence, expectations-based loss aversion can rationalize several features of reserve prices observed in real-world auctions which are hard to reconcile with the classical risk-neutral and risk-averse frameworks.

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<sup>23</sup>Three notable exceptions are Rosenkranz and Schmitz (2007), Eisenhuth (2019) and Muramoto and Togo (2022). With CPE, Eisenhuth (2019) shows that the optimal reserve price in the all-pay auction implies the same no-trade probability as under risk neutrality, while Muramoto and Togo (2022) focus on asymmetric auction design, allowing for bidder-specific reserve prices. In Rosenkranz and Schmitz (2007), differently from our setting, bidders are loss averse only with respect to their monetary payment and use the (public) reserve price as a reference point.

# A Appendix A

## Proof of Lemma 1

*Proof.* We know from Balzer and Rosato (2021) that in the FPA, a bidder's first-order condition satisfies

$$\frac{(1 + \eta^g \lambda^g F_1(t) + \eta^g [1 - F_1(t)]) f_1(t) t}{1 + \eta^m \lambda^m} = P'(t) - \frac{\Lambda^m}{1 + \eta^m \lambda^m} f_1(t) P(t). \quad (12)$$

The solution to differential equation (12) is

$$P(t) = e^{\frac{\Lambda^m}{1 + \eta^m \lambda^m} F_1(t)} \left( \int_{t_r}^t \frac{(1 + \eta^g \lambda^g F_1(x) + \eta^g [1 - F_1(x)]) f_1(x) x}{1 + \eta^m \lambda^m} e^{-\frac{\Lambda^m}{1 + \eta^m \lambda^m} F_1(x)} dx + C \right),$$

where  $C$  is a constant. Since  $P(t) = F_1(t) \beta_I^*(t)$ , the constant satisfies  $P(t_r) = F_1(t_r) \beta_I^*(t_r) = F_1(t_r) r$ . Thus,  $C = P(t_r) e^{-\frac{\Lambda^m}{1 + \eta^m \lambda^m} F_1(t_r)}$ . ■

## Proof of Proposition 1

*Proof.* Fix  $t_r$  with corresponding  $r$  (see equation (3)). The seller's objective, normalized by  $N$ , is:

$$\int_{t_r}^{\bar{t}} \left( \frac{\int_{t_r}^t \{f_1(x) x (1 + F_1(x) \lambda^g \eta^g + [1 - F_1(x)] \eta^g)\} e^{\frac{\Lambda^m [F_1(t) - F_1(x)]}{1 + \eta^m \lambda^m}}}{1 + \eta^m \lambda^m} dx \right. \\ \left. + \frac{F_1(t_r) t_r (1 + \eta^g)}{1 + \eta^m \lambda^m} e^{\frac{\Lambda^m [F_1(t) - F_1(t_r)]}{1 + \eta^m \lambda^m}} \right) f(t) dt + \frac{F(t_r)^N}{N} t^S \quad (13)$$

Notice first that the  $t_r$  which maximizes (13) is either  $t_r = \underline{t}$ ,  $t_r = \bar{t}$  or an interior solution. In the first case, the derivative of (13) is negative at  $t_r = \underline{t}$ , in the second-case it is positive at  $t_r = \bar{t}$ , while in the last case the derivative is zero at an interior solution and negative for a slightly larger  $t_r$ . Moreover, define

$$\tilde{V}(t_r) \equiv \frac{1}{f(t_r)} \int_{t_r}^{\bar{t}} \left( 1 - \left\{ \frac{\Lambda^g}{1 + \eta^g} - \frac{\Lambda^m}{1 + \eta^m \lambda^m} \right\} f_1(t_r) t_r \right) e^{\frac{\Lambda^m [F_1(t) - F_1(t_r)]}{1 + \eta^m \lambda^m}} f(t) dt. \quad (14)$$

It is straightforward to show that the derivative of (13) with respect to  $t_r$  is increasing in  $t_r$  if and only if

$$\tilde{V}(t_r) + t^S \frac{1 + \eta^m \lambda^m}{1 + \eta^g} - t_r \geq 0. \quad (15)$$

If  $\eta^m = 0$ , then  $\frac{\Lambda^m}{1 + \eta^m \lambda^m} = 0$  and  $1 + \eta^m \lambda^m = 1$ . Thus, equation (15) rules out the potential solution  $t_r = \bar{t}$  as  $\tilde{V}(\bar{t}) = 0$  and  $\bar{t} > t^S$ . Using expression (15), we see that the seller's revenue increases in  $t_r$  as long as

$$V(t_r) - t^S \leq -t^S \frac{\eta^g}{1 + \eta^g} - \frac{\eta^g (\lambda^g - 1)}{1 + \eta^g} \frac{[1 - F(t_r)]}{f(t_r)} f_1(t_r) t_r, \quad (16)$$

where  $V(t_r) = t_r - [1 - F(t_r)]/f(t_r)$  is the 'virtual valuation'. If  $t_r = \underline{t} = 0$ , the left-hand side is lower than the right-hand side.

The right-hand side of equation (16) is negative. This implies that at the optimal threshold type,  $t_r^*$ , (no matter whether it is at the lower bound or in the interior) we have  $V(t_r^*) - t^S < 0$ . Since  $V$  is an increasing function, less types than under risk neutrality are excluded, where the optimal threshold is interior, i.e.,  $V(t^{RN}) - t^S = 0$ , because  $\underline{t} = 0$ .

Moreover, note that if  $\lambda^g$  is sufficiently high, we have that  $t_r^* \rightarrow \underline{t} = 0$  as, for any  $t > 0$ , the right-hand side of (16) becomes arbitrarily negative with  $\lambda^g$  becoming arbitrarily large. Obviously, for such sufficiently large  $\lambda^g$ , the optimal reserve price,  $r^*$ , is  $r^* = (1 + \eta^g)t_r^* < t^S \leq r_{RN}$ .

Next, fix arbitrary  $t^S \geq 0$  and  $\eta^g > 0$ . Whenever (16) has an interior solution, the optimal reserve price is

$$r^* = (1 + \eta^g)t_r^* = t^S + \frac{[1 - F(t_r^*)]}{f(t_r^*)}(1 + \eta^g - \eta^g(\lambda^g - 1)f_1(t_r^*)t_r^*). \quad (17)$$

If (16) has no interior solution,  $t_r^* = \underline{t} = 0 = r^*$ .

If  $\lambda^g = 1$ , then (16) has an interior solution (as the left-hand side of (16) is larger than the right-hand side at  $t_r = \bar{t}$ , and vice versa at  $t_r = \underline{t}$ ). Using (17) we see that for  $\lambda^g = 1$ , we have that  $r^* = (1 + \eta^g)t_r^* = t^S + \frac{[1 - F(t_r^*)]}{f(t_r^*)}(1 + \eta^g) > r_{RN}$ , as  $t_r^* \leq t^{RN}$ . In contrast, when  $\lambda^g \rightarrow \infty$ , (16) has no interior solution and  $r^* = 0$  no matter how large  $t^S$  is. Finally, notice that applying the implicit function theorem to equation (16) and using that, at an interior solution, the left-hand side minus the right-hand side increases in  $t_r$ , one can establish that  $t_r^*$ , and thus  $r^* = (1 + \eta^g)t_r^*$ , are (continuously) decreasing in  $\lambda^g$ . Hence, there exists  $\bar{\lambda}^g$  with the properties stated in Proposition 1. ■

## Proof of Proposition 2

*Proof.* If  $\eta^m = 0$ , then equation (16) from the proof of Proposition 1 applies. Thus, at the optimal threshold type,  $t_r^*$ , we have  $V(t_r^*) - t^S < 0$ . Since  $V$  is an increasing function, less types than under risk neutrality are excluded. Moreover, note that the right-hand side of (16) depends on  $f_1(t_r)$  which itself depends on  $N$ . Note that, for any  $t_r < \bar{t}$ ,  $f_1(t_r) \rightarrow 0$  if  $N \rightarrow \infty$ . Thus, using (16), we have that  $t_r^* \rightarrow t^{RN}$  with  $N \rightarrow \infty$  if and only if  $t^S = 0$ .

In addition, using (16) it is straightforward to see that if  $N \rightarrow \infty$ , and thus  $f_1(t_r) \rightarrow 0$ , the optimal  $t_r^*$  is in the interior. Using (17) we see that, if  $N \rightarrow \infty$ , the optimal reserve price,  $r^*$ , satisfies  $r^* \rightarrow t^S + (1 + \eta^g)\frac{1 - F(t_r^*)}{f(t_r^*)} \geq t^S + (1 + \eta^g)\frac{1 - F(t^{RN})}{f(t^{RN})} > r_{RN}$ , as  $t_r^* < t^{RN}$  and  $[1 - F(t)]/f(t)$  is decreasing in  $t$ . ■

## Proof of Proposition 3

*Proof.* Recall equation (15) from the proof of Proposition 1. It states that the sign-determining equation of the first-order derivative with respect to  $t_r$  is  $\tilde{V}(t_r) + t^S \frac{1 + \eta^m \lambda^m}{1 + \eta^g} - t_r$ . If and only if this term is positive, the expected revenue increases in  $t_r$ . Thus, at an interior optimum,  $t_r^*$ , this term is zero, and,  $\tilde{V}'(t_r^*) - 1 < 0$ .

Note first that, if  $N \rightarrow \infty$ ,  $\tilde{V}(t_r) \rightarrow [1 - F(t_r)]/f(t_r)$ . Thus,  $t_r^* \rightarrow [1 - F(t_r^*)]/f(t_r^*) + t^S \frac{1+\eta^m \lambda^m}{1+\eta^g}$ . In addition, using the form of the optimal reserve price  $r^*$ ,  $r^* = \frac{1+\eta^g}{1+\eta^m \lambda^m} t_r^*$ , as  $N \rightarrow \infty$ ,  $r^* \rightarrow t^S + \frac{1+\eta^g}{1+\eta^m \lambda^m} \frac{1-F(t_r^*)}{f(t_r^*)}$ . If  $\eta^g > \eta^m \lambda^m$ , then  $\frac{1+\eta^g}{1+\eta^m \lambda^m} > 1$  and  $t_r^* < t^{RN}$ . Using the decreasing hazard-rate property, it follows that  $r^* > t^S + \frac{1-F(t^{RN})}{f(t^{RN})} = r^{RN}$ . The exact opposite argument applies for the case  $\eta^g < \eta^m \lambda^m$ .

Moreover, denote by  $\tilde{V}_{\eta^k}$  the partial derivative of  $\tilde{V}$  with respect to  $\eta^k$ , with  $k \in \{g, m\}$ . Applying the implicit function theorem, we observe that

$$dt_r^*(\tilde{V}'(t_r^*) - 1) + d\eta^k \left( \tilde{V}_{\eta^k}(t_r^*) + t^S \frac{d}{d\eta^k} \left( \frac{1 + \eta^m \lambda^m}{1 + \eta^g} \right) \right) = 0 \quad \Rightarrow \quad \frac{dt_r^*}{d\eta^k} = - \frac{\tilde{V}_{\eta^k}(t_r^*) + t^S \frac{d}{d\eta^k} \left( \frac{1 + \eta^m \lambda^m}{1 + \eta^g} \right)}{\tilde{V}'(t_r^*) - 1}.$$

Since  $\tilde{V}'(t_r^*) - 1 < 0$ ,  $\frac{dt_r^*}{d\eta^k} < 0$  if and only if  $\tilde{V}_{\eta^k}(t_r^*) + t^S \frac{d}{d\eta^k} \left( \frac{1 + \eta^m \lambda^m}{1 + \eta^g} \right) < 0$  at  $t_r^*$ .

First, for any  $\eta^m$  and  $t_S \geq 0$ , it is straightforward to observe that  $\tilde{V}_{\eta^g}(t_r^*) + t^S \frac{d}{d\eta^g} \left( \frac{1 + \eta^m \lambda^m}{1 + \eta^g} \right) < 0$ .

Second, fix any  $\eta^g$ , and note that

$$\begin{aligned} & \tilde{V}_{\eta^m}(t_r^*) + t^S \frac{d}{d\eta^m} \left( \frac{1 + \eta^m \lambda^m}{1 + \eta^g} \right) = - \left( \frac{\lambda^m - 1}{(1 + \eta^m \lambda^m)^2} f_1(t_r^*) t_r^* \right) \frac{1}{f(t_r^*)} \int_{t_r^*}^{\bar{t}} e^{\frac{\Lambda^m [F_1(t) - F_1(t_r^*)]}{1 + \eta^m \lambda^m}} f(t) dt \\ & + \frac{\lambda^m - 1}{(1 + \eta^m \lambda^m)^2} \frac{1}{f(t_r^*)} \int_{t_r^*}^{\bar{t}} [F_1(t) - F_1(t_r^*)] \left( 1 - \left\{ \frac{\Lambda^g}{1 + \eta^g} - \frac{\Lambda^m}{1 + \eta^m \lambda^m} \right\} f_1(t_r^*) t_r^* \right) e^{\frac{\Lambda^m [F_1(t) - F_1(t_r^*)]}{1 + \eta^m \lambda^m}} f(t) dt + t^S \frac{\lambda^m}{1 + \eta^g}. \end{aligned}$$

Considering  $\tilde{V}_{\eta^m}(t_r^*) + t^S \frac{d}{d\eta^m} \left( \frac{1 + \eta^m \lambda^m}{1 + \eta^g} \right)$ , we have that this term is positive if and only if

$$\begin{aligned} & \left( 1 - \left\{ \frac{\Lambda^g}{1 + \eta^g} - \frac{\Lambda^m}{1 + \eta^m \lambda^m} \right\} f_1(t_r^*) t_r^* \right) \frac{\int_{t_r^*}^{\bar{t}} [F_1(t) - F_1(t_r^*)] e^{\frac{\Lambda^m [F_1(t)]}{1 + \eta^m \lambda^m}} f(t) dt}{(N - 1) F^{N-2}(t_r^*) f(t_r^*) t_r^* \int_{t_r^*}^{\bar{t}} e^{\frac{\Lambda^m [F_1(t)]}{1 + \eta^m \lambda^m}} f(t) dt} \\ & > 1 - \frac{t^S \frac{\lambda^m}{1 + \eta^g} e^{-\frac{\Lambda^m F_1(t_r^*)}{1 + \eta^m \lambda^m}}}{\frac{\lambda^m - 1}{(1 + \eta^m \lambda^m)^2} (N - 1) F^{N-2}(t_r^*) t_r^* \int_{t_r^*}^{\bar{t}} e^{\frac{\Lambda^m [F_1(t)]}{1 + \eta^m \lambda^m}} f(t) dt}. \end{aligned} \quad (18)$$

Since both  $e^{\frac{\Lambda^m [F_1(t)]}{1 + \eta^m \lambda^m}}$  and  $F_1(t) - F_1(t_r^*)$  are increasing in  $t$ , a lower bound for the left-hand side is given by

$$\begin{aligned} & \left( 1 - \left\{ \frac{\Lambda^g}{1 + \eta^g} - \frac{\Lambda^m}{1 + \eta^m \lambda^m} \right\} f_1(t_r^*) t_r^* \right) \frac{\int_{t_r^*}^{\bar{t}} [F_1(t) - F_1(t_r^*)] dt \int_{t_r^*}^{\bar{t}} e^{\frac{\Lambda^m [F_1(t)]}{1 + \eta^m \lambda^m}} f(t) dt}{(N - 1) F^{N-2}(t_r^*) f(t_r^*) t_r^* \int_{t_r^*}^{\bar{t}} e^{\frac{\Lambda^m [F_1(t)]}{1 + \eta^m \lambda^m}} f(t) dt} \\ & = \left( 1 - \left\{ \frac{\Lambda^g}{1 + \eta^g} - \frac{\Lambda^m}{1 + \eta^m \lambda^m} \right\} f_1(t_r^*) t_r^* \right) \frac{\int_{t_r^*}^{\bar{t}} [F_1(t) - F_1(t_r^*)] dt}{(N - 1) F^{N-2}(t_r^*) f(t_r^*) t_r^*} \\ & = \frac{\left( 1 - \left\{ \frac{\Lambda^g}{1 + \eta^g} - \frac{\Lambda^m}{1 + \eta^m \lambda^m} \right\} f_1(t_r^*) t_r^* \right)}{f(t_r^*) t_r^*} \int_{t_r^*}^{\bar{t}} \left( F(t) \frac{\left( \frac{F(t)}{F(t_r^*)} \right)^{N-2}}{N - 1} - \frac{F(t_r^*)}{N - 1} \right) dt. \end{aligned}$$

Note that for  $N \rightarrow \infty$  the first term of the above expression converges to  $1/(f(t_r^*)t_r^*)$ . Moreover, the integral diverges to plus infinity as  $F(t)/F(t_r^*) > 1$  and  $t_r^* = [1 - F(t_r^*)]/f(t_r^*) + t^S(1 - \eta^m \lambda^m)/(1 + \eta^g) < \bar{t}$  as argued above. Thus, the the left-hand side of (18) is larger than 1. By continuity in  $N$ , the left-hand side of (18) is strictly larger than 1 if  $N$  is sufficiently large, implying  $\frac{dt_r^*}{d\mu^m} > 0$ .

Next, consider the case of  $N = 2$ . It is easy to see that the left-hand side of equation (18) becomes

$$\left(1 - \left\{\frac{\Lambda^g}{1 + \eta^g} - \frac{\Lambda^m}{1 + \eta^m \lambda^m}\right\} f_1(t_r^*)t_r^*\right) \frac{\int_{t_r^*}^{\bar{t}} [F(t) - F(t_r^*)] e^{\frac{\Lambda^m [F(t)]}{1 + \eta^m \lambda^m}} f(t) dt}{f(t_r^*)t_r^* \int_{t_r^*}^{\bar{t}} e^{\frac{\Lambda^m [F(t)]}{1 + \eta^m \lambda^m}} f(t) dt}. \quad (19)$$

In the following we show that (19) is strictly below 1 whenever  $t_r^* < t^{RN}$  for some  $\eta^m$ . Thus, by continuity of (18), for sufficiently low  $t^S$  (if  $t^S = 0$  the right-hand side of (18) is 1) and  $N$ , we have  $\frac{dt_r^*}{d\mu^m} < 0$ . Start with  $\eta^m = 0$ . There we have that  $f(t_r^*)t_r^* = 1 - F(t_r^*)$ . Thus, (19) becomes

$$\frac{\int_{t_r^*}^{\bar{t}} [F(t) - F(t_r^*)] e^{\frac{\Lambda^m [F(t)]}{1 + \eta^m \lambda^m}} f(t) dt}{[1 - F(t_r^*)] \int_{t_r^*}^{\bar{t}} e^{\frac{\Lambda^m [F(t)]}{1 + \eta^m \lambda^m}} f(t) dt} < 1,$$

where the inequality follows from  $F(t) - F(t_r^*) < 1 - F(t_r^*)$ . Since the expression is continuous in  $\eta^m$ , the conclusion holds for  $\eta^m > 0$  but sufficiently small. ■

#### Proof of Proposition 4

*Proof.* We prove the proposition in three steps. First, we present a function that bounds the seller's expected-revenue function (with the reserve price being the argument) from above. Second, we find the public reserve price that maximizes that upper bound. Third, we show that there exist secret and random reserve prices that lead to a revenue arbitrarily close to the maximized upper bound.

**Step 1: Bounding the Seller's Revenue.** Recall that the seller's revenue under a deterministic reserve price is given by

$$\int_{t_r}^{\bar{t}} \frac{\int_{t_r}^t \{F_1'(s)s(1 + F_1(s)\lambda^g\eta^g + [1 - F_1(s)]\eta^g)\} e^{\frac{\Lambda^m [F_1(t) - F_1(s)]}{1 + \eta^m \lambda^m}} ds}{1 + \eta^m \lambda^m} + \frac{F_1(t_r)t_r(1 + \eta^g)}{1 + \lambda^m \eta^m} e^{\frac{\Lambda^m [F_1(t) - F_1(t_r)]}{1 + \eta^m \lambda^m}} f(t) dt.$$

We bound the seller's revenue from above and then show that secret and random reserve prices yield a revenue that is arbitrary close to that bound,  $\bar{P}(t_r)$ . If  $\eta^m > 0$ , we construct the bound by replacing  $F_1(t_r)t_r(1 + \eta^g)/(1 + \lambda^m \eta^m)$  with  $\bar{P}(t_r)$ , where

$$\bar{P}(t_r) \equiv (1 + \eta^g)t_r \frac{e^{\frac{\Lambda^m}{c} F_1(t_r)} - 1}{\Lambda^m} + \frac{c\Lambda^g}{\Lambda^m} t_r \frac{e^{\frac{\Lambda^m}{c} F_1(t_r)} - 1 - \frac{\Lambda^m}{c} F_1(t_r)}{\Lambda^m}, \quad (20)$$



and  $c \equiv 1 + \eta^m \lambda^m$ .<sup>24</sup> If  $\eta^m = 0$ , we construct the bound by using the limit of (20) when  $\eta^m \rightarrow 0$ , i.e.,  $\bar{P}(t_r) = (1 + \eta^g)t_r F_1(t_r) + \Lambda^g t_r F_1(t_r)^2/2$ . Further define  $h(s) \equiv F_1'(s)sv(s)$  with  $v(s) \equiv 1 + \eta^g + \Lambda^g F_1(s)$ , then the expected revenue's upper bound,  $\bar{R}(t_r)$  (times  $c$ ) is

$$c\bar{R}(t_r) = \int_{t_r}^{\bar{t}} \left( e^{\frac{\Lambda^m}{c} F_1(t)} \int_{t_r}^t h(s) e^{-\frac{\Lambda^m}{c} F_1(s)} ds + e^{\frac{\Lambda^m [F_1(t) - F_1(t_r)]}{c}} c\bar{P}(t_r) \right) f(t) dt. \quad (21)$$

### Step 2: Maximizing the upper Bound.

The derivative of  $t^S F(t_r)^N / N + c\bar{R}(t_r)$  with respect to  $t_r$  is

$$\begin{aligned} & t^S f(t_r) F_1(t_r) - f(t_r) c\bar{P}(t_r) \\ + \int_{t_r}^{\bar{t}} & \left( -h(t_r) e^{\frac{\Lambda^m [F_1(t) - F_1(t_r)]}{c}} - \frac{\Lambda^m}{c} F_1'(t_r) e^{\frac{\Lambda^m [F_1(t) - F_1(t_r)]}{c}} c\bar{P}(t_r) + c\bar{P}'(t_r) e^{\frac{\Lambda^m [F_1(t) - F_1(t_r)]}{c}} \right) f(t) dt \\ & = t^S f(t_r) F_1(t_r) - f(t_r) c\bar{P}(t_r) + \int_{t_r}^{\bar{t}} \left( e^{\frac{\Lambda^m [F_1(t) - F_1(t_r)]}{c}} \frac{c\bar{P}(t_r)}{t_r} \right) f(t) dt \\ & = t^S f(t_r) F_1(t_r) - c \frac{\bar{P}(t_r)}{t_r} \left( f(t_r) t_r - [1 - F(t_r)] \right) + \frac{c\bar{P}(t_r)}{t_r} \int_{t_r}^{\bar{t}} \left( e^{\frac{\Lambda^m [F_1(t) - F_1(t_r)]}{c}} - 1 \right) f(t) dt, \end{aligned} \quad (22)$$

where we used that

$$\begin{aligned} \bar{P}'(t_r) &= \frac{\bar{P}(t_r)}{t_r} + (1 + \eta^g) \frac{\Lambda^m}{c} F_1'(t_r) t_r \frac{e^{\frac{\Lambda^m}{c} F_1(t_r)}}{\Lambda^m} + \frac{c\Lambda^g}{\Lambda^m} t_r F_1'(t_r) \frac{\Lambda^m}{c} \frac{e^{\frac{\Lambda^m}{c} F_1(t_r)} - 1}{\Lambda^m} \\ &= \frac{\bar{P}(t_r)}{t_r} + (1 + \eta^g) F_1'(t_r) t_r \frac{e^{\frac{\Lambda^m}{c} F_1(t_r)}}{c} + \frac{\Lambda^g}{\Lambda^m} t_r F_1'(t_r) (e^{\frac{\Lambda^m}{c} F_1(t_r)} - 1). \end{aligned}$$

Recall that  $V(t) = t - [1 - F(t)]/f(t)$ . (22) reveals that the optimal threshold type,  $\tilde{t}_r^*$ , satisfies

$$V(\tilde{t}_r^*) = \frac{1}{f(\tilde{t}_r^*)} \int_{\tilde{t}_r^*}^{\bar{t}} \left( e^{\frac{\Lambda^m [F_1(t) - F_1(\tilde{t}_r^*)]}{c}} - 1 \right) f(s) ds + \frac{F_1(\tilde{t}_r^*) \tilde{t}_r^*}{(1 + \eta^m \lambda^m) \bar{P}(\tilde{t}_r^*)} t^S. \quad (23)$$

### Step 3: Upper bound is the Supremum when using Secret Reserve Prices.

Next, we show that the seller can achieve the maximized upper bound with the following secret and random reserve-price scheme. For given  $K \in \mathbb{R}^+$ , introduce a random variable,  $\tilde{T}_r$ , with realization  $\tilde{t}_r \in [t, \tilde{t}_r^*]$ , drawn according to CDF  $F_0(\tilde{t}_r) = (\frac{\tilde{t}_r}{\tilde{t}_r^*})^K$ . Moreover, for  $q(t) = F_0(t)F_1(t)$  consider

$$\beta_I^*(\tilde{t}_r) = \left( \int_t^{\tilde{t}_r} \frac{1 + \eta^g \lambda^g q(s) + \eta^g [1 - q(s)]}{1 + \eta^m \lambda^m} e^{\frac{\Lambda^m [q(\tilde{t}_r) - q(s)]}{1 + \eta^m \lambda^m}} q'(s) s ds + \right) / q(\tilde{t}_r). \quad (24)$$

If the secret reserve price follows the random function  $\beta_I^* \circ \tilde{T}_r : [t, \tilde{t}_r^*] \mapsto \Delta([\beta_I^*(t), \beta_I^*(\tilde{t}_r^*)])$ , then, in a symmetric equilibrium in increasing strategies, type- $t$  bidder expects to win the auction with probability  $q(t) = F_0(t)F_1(t)$ , which by Lemma 1 reinforces (24) as the equilibrium bidding strategy (when replacing

<sup>24</sup>Noting that, for fixed  $t_r$ ,  $e^{\frac{\Lambda^m}{c} F_1(t_r)}$  is as a convex function of  $\Lambda^m$  and applying a first-order Taylor approximation around 0 proves that we bounded the revenue from above.

$F_1(t)$  with  $q(t)$  and replacing the support of (24) with  $[\underline{t}, \bar{t}]$ . Define  $h_q(s) \equiv q'(s)sv(s)$  with  $v_q(s) \equiv 1 + \eta^g + \Lambda^g q(s)$ . Then, replacing  $F_1$  with  $q$  is straightforward to observe that the expected payment,  $P(t)$ , satisfies

$$cP(t) = e^{\frac{\Lambda^m}{c}q(t)} \int_{\underline{t}}^t h_q(s) e^{-\frac{\Lambda^m}{c}q(s)} ds + cP(\underline{t}), \quad (25)$$

for some constant  $P(\underline{t})$  (see Lemma (1)). Obviously,  $P(\underline{t}) = \beta_I^*(\underline{t})q(\underline{t}) = 0$ , as  $q(\underline{t}) = 0$ .

Finally, we want to show that for  $K \rightarrow \infty$  the seller's payoff converges to the upper bound (21). Indeed, first note that

$$\begin{aligned} cP(t) &= e^{\frac{\Lambda^m}{c}q(t)} \left( \int_{\tilde{t}_r^*}^t h_q(s) e^{-\frac{\Lambda^m}{c}q(s)} ds + \int_{\underline{t}}^{\tilde{t}_r^*} h_q(s) e^{-\frac{\Lambda^m}{c}q(s)} ds \right) \\ &= e^{\frac{\Lambda^m}{c}q(t)} \int_{\tilde{t}_r^*}^t h_q(s) e^{-\frac{\Lambda^m}{c}q(s)} ds + \underbrace{e^{\frac{\Lambda^m[q(t)-q(\tilde{t}_r^*)]}{c}} e^{\frac{\Lambda^m}{c}q(\tilde{t}_r^*)} \int_{\underline{t}}^{\tilde{t}_r} h_q(s) e^{-\frac{\Lambda^m}{c}q(s)} ds}_{\equiv c\hat{P}(\tilde{t}_r^*)}. \end{aligned} \quad (26)$$

Observe first that, for any  $t \geq \tilde{t}_r^*$ , we have  $q(t) = F_1(t)$ , implying  $h_q(t) = h(t)$ . Thus, the expectation of the first sum of  $cP(t)$  stated in (26) is equal to  $c\bar{R}(t_r) - \int_{t_r}^t e^{\frac{\Lambda^m[F_1(t)-F_1(t_r)]}{c}} c\bar{P}(t_r) f(t) dt$  (see (21)).

It thus remains to show that  $\hat{P}(\tilde{t}_r^*)$  (defined in the under-bracket of (26)) converges to  $\bar{P}(\tilde{t}_r^*)$  with  $K \rightarrow \infty$ . Applying partial integration reveals that

$$\begin{aligned} \hat{P}(\tilde{t}_r^*) &= -\frac{e^{\frac{\Lambda^m}{c}q(\tilde{t}_r^*)}}{\Lambda^m} \left( e^{-\frac{\Lambda^m}{c}q(\tilde{t}_r^*)} \tilde{t}_r^* v_q(\tilde{t}_r^*) - e^{-\frac{\Lambda^m}{c}q(\underline{t})} \underline{t} v_q(\underline{t}) - \int_{\underline{t}}^{\tilde{t}_r^*} e^{-\frac{\Lambda^m}{c}q(s)} v_q(s) ds + \right. \\ &\quad \left. \frac{c}{\Lambda^m} \Lambda^g (e^{-\frac{\Lambda^m}{c}q(\tilde{t}_r^*)} \tilde{t}_r^* - e^{-\frac{\Lambda^m}{c}q(\underline{t})} \underline{t}) - \frac{c}{\Lambda^m} \Lambda^g \int_{\underline{t}}^{\tilde{t}_r^*} e^{-\frac{\Lambda^m}{c}q(s)} ds \right) \\ &= \frac{\int_{\underline{t}}^{\tilde{t}_r^*} e^{\frac{\Lambda^m[q(\tilde{t}_r^*)-q(s)]}{c}} v_q(s) ds}{\Lambda^m} + \frac{\frac{c}{\Lambda^m} \Lambda^g \int_{\underline{t}}^{\tilde{t}_r^*} e^{\frac{\Lambda^m[q(\tilde{t}_r^*)-q(s)]}{c}} ds}{\Lambda^m} \\ &\quad - \frac{\tilde{t}_r^* v_q(\tilde{t}_r^*) - e^{-\frac{\Lambda^m[q(\tilde{t}_r^*)-q(\underline{t})]}{c}} \underline{t} v_q(\underline{t})}{\Lambda^m} - \frac{\tilde{t}_r^* - e^{-\frac{\Lambda^m[q(\tilde{t}_r^*)-q(\underline{t})]}{c}} \underline{t}}{\Lambda^m} \frac{c}{\Lambda^m} \Lambda^g. \end{aligned}$$

We now use that  $q(t) \rightarrow 0$  if  $t < \tilde{t}_r^*$ . For such a  $t$ , we have that  $v_q(t) \rightarrow 1 + \eta^g$  and  $\hat{P}(\tilde{t}_r^*)$  becomes

$$\begin{aligned} \hat{P}(\tilde{t}_r^*) &\rightarrow \frac{\int_{\underline{t}}^{\tilde{t}_r^*} e^{\frac{\Lambda^m}{c}q(\tilde{t}_r^*)} (1 + \eta^g) ds}{\Lambda^m} + \frac{\frac{c}{\Lambda^m} \Lambda^g \int_{\underline{t}}^{\tilde{t}_r^*} e^{\frac{\Lambda^m}{c}q(\tilde{t}_r^*)} ds}{\Lambda^m} - \frac{(\tilde{t}_r^* - \underline{t})(1 + \eta^g)}{\Lambda^m} \left( 1 + \frac{c}{\Lambda^m} \Lambda^g \right) - \frac{\tilde{t}_r^*}{\Lambda^m} \Lambda^g q(\tilde{t}_r^*) \\ &= (1 + \eta^g) \tilde{t}_r^* \frac{e^{\frac{\Lambda^m}{c}q(\tilde{t}_r^*)} - 1}{\Lambda^m} + \frac{c \Lambda^g \tilde{t}_r^* (e^{\frac{\Lambda^m}{c}q(\tilde{t}_r^*)} - 1) - \tilde{t}_r^* \Lambda^m q(\tilde{t}_r^*)}{\Lambda^m}. \end{aligned}$$

Thus  $\hat{P}(\tilde{t}_r^*) \rightarrow \bar{P}(\tilde{t}_r)$  and the claim follows. ■

### Proof of Proposition 5

*Proof.* Consider equation (23) from Step 2 of Proposition 4's proof. Assume that  $t^S = 0$ . For  $\eta^m = 0$ , the

largest optimal threshold  $\tilde{t}_r^*$ , satisfies  $V(\tilde{t}_r^*) = 0$ , and thus the threshold coincides with the risk-neutral one. Since  $V$  is increasing in  $t$ , and the right-hand side of equation (23) is increasing in  $\eta^m$ ,  $\tilde{t}_r^*$  is strictly larger than the risk-neutral threshold type if  $\eta^m > 0$ . Moreover, it is easy to observe that  $\tilde{t}_r^*$  converges to the risk-neutral optimal threshold if  $N \rightarrow \infty$ . Now assume that  $t^S > 0$  and  $\eta^m = 0$ . The optimal threshold then satisfies  $V(\tilde{t}_r^*) = t^S / \left(1 + \eta^g + F_1(\tilde{t}_r^*)\Lambda^g/2\right) < t^S$ . Since  $V$  is increasing in  $t$ ,  $\tilde{t}_r^*$  is strictly smaller than the risk-neutral threshold type. Moreover, it is easy to observe that  $V(\tilde{t}_r^*)$  converges to  $t^S/(1 + \eta^g)$  if  $N \rightarrow \infty$  and thus is lower than the risk-neutral optimal threshold. ■

### Proof of Proposition 6

*Proof.* Let  $ER_k(q)$  be the expected revenue of the FPA ( $k = I$ ) and the SPA ( $k = II$ ). The argument is  $q : [\underline{t}, \bar{t}] \mapsto [0, 1]$  where  $q(t)$  is the probability with which type- $t$  bidder expects to win the auction before submitting her bid. Suppose the seller uses random (and secret) reserve prices. The reserve price distribution shapes the bidding function, and, in equilibrium, each realization of the reserve price, say  $r$ , has a corresponding threshold type,  $\tilde{t}_r$ , which is the largest type bidding below  $r$ . Let  $\alpha(\tilde{t}_r)$  be the CDF of the stochastic threshold types, generated by the above randomness of the reserve price. For fixed realization  $\tilde{t}_r$ , define  $q_{\tilde{t}_r}$  as follows:  $q_{\tilde{t}_r}(t) = F_1(\tilde{t}_r)$  if  $t \geq \tilde{t}_r$ , and else zero. For a given CDF over  $\tilde{t}_r$ ,  $\alpha(\tilde{t}_r)$ , we have that  $q = \int_{\underline{t}}^t q_{\tilde{t}_r} d\alpha(\tilde{t}_r)$ , i.e.,  $q(t) = \int_{\underline{t}}^t q_{\tilde{t}_r}(t) d\alpha(\tilde{t}_r) \forall t \in [\underline{t}, \bar{t}]$ . The following result will prove useful in proving the desired result:

**Lemma 4.** *There exists a convex functional  $\bar{E}R_k \geq ER_k$  for  $k \in \{I, II\}$ . Moreover, for any  $q = q_{\tilde{t}_r}$ , i.e., an allocation policy generated by a deterministic reserve price,  $\bar{E}R_k = ER_k$  for  $k \in \{I, II\}$ .*

*Proof.* See Appendix B. ■

Then, it follows from Lemma 4 that it is without loss of generality to focus on deterministic reserve prices. Indeed,  $\bar{E}R_k(q) = \bar{E}R_k(\int_{\underline{t}}^t q_{\tilde{t}_r} d\alpha(\tilde{t}_r)) \leq \int_{\underline{t}}^t \bar{E}R_k(q_{\tilde{t}_r}) d\alpha(\tilde{t}_r) \leq \bar{E}R_k(q_{\tilde{t}_r}^*) = ER_k(q_{\tilde{t}_r}^*)$ , where  $q_{\tilde{t}_r}^*$  is the allocation policy induced by the deterministic reserve price generating the largest expected revenue. ■

### Proof of Lemma 2

*Proof.* We know from Balzer and Rosato (2021) that in the FPA, for a monotonic allocation policy  $q$ , a bidder's first-order condition satisfies

$$\frac{(1 - \Lambda^g)q'(t) + 2\Lambda^g q'(t)q(t)}{1 + \Lambda^m[1 - q(t)]} t = P'(t) - \frac{\Lambda^m q'(t)}{1 + \Lambda^m[1 - q(t)]} P(t), \quad (27)$$

The solution to differential equation (27) is

$$P(t) = \frac{\int_{\tilde{t}_r}^t [(1 - \Lambda^g)q'(s) + 2\Lambda^g q'(s)q(s)] dt}{1 + \Lambda^m[1 - q(t)]} + C,$$

where  $C$  is pinned down by the boundary condition  $C = P(t_r) = q(t_r) \frac{1 + \Lambda^g + \Lambda^g q(t_r)}{1 + \Lambda^m[1 - q(t_r)]}$ . Using that  $q(t) = F_1(t)$  and applying partial integration, the result follows. ■

## Proof of Proposition 7

*Proof.* Using bidder's expected payment, that is,  $P(t)$  defined in (10), the seller's objective (normalized by  $N$ ) is:

$$\underbrace{\int_{t_r}^{\bar{t}} \left( \frac{\Lambda^g [F_1(t)t - \int_{t_r}^t F_1(s)ds] + (1 - \Lambda^g) [F_1(t)^2 t - \int_{t_r}^t F_1(s)^2 ds]}{1 + \Lambda^m [1 - F_1(t)]} \right) f(t) dt}_{= \int_{t_r}^{\bar{t}} P(t) f(t) dt} + \frac{F(t_r)^N t^S}{N}, \quad (28)$$

where all types smaller than  $t_r$  receive the good with probability 0.

Define  $\tilde{H}(t) \equiv \int_{t_r}^t \frac{f(s)}{1 + \Lambda^m [1 - F_1(s)]} ds$  and  $\tilde{h}(t) \equiv \tilde{H}'(t) = \frac{f(t)}{1 + \Lambda^m [1 - F_1(t)]}$ . Applying partial integration to the seller's (normalized) expected revenue, we observe that

$$\int_{t_r}^{\bar{t}} P(t) f(t) dt = \int_{t_r}^{\bar{t}} [(1 - \Lambda^g) F_1(t) + \Lambda^g F_1(t)^2] \left( t - \frac{\tilde{H}(\bar{t}) - \tilde{H}(t)}{\tilde{h}(t)} \right) \tilde{h}(t) dt. \quad (29)$$

Note that the last bracket simplifies to

$$t - \frac{\int_{t_r}^{\bar{t}} \frac{f(s)}{1 + \Lambda^m [1 - F_1(s)]} ds}{\frac{f(t)}{1 + \Lambda^m [1 - F_1(t)]}} = t - \int_t^{\bar{t}} \frac{f(t)}{f(s)} \frac{1 + \Lambda^m [1 - F_1(t)]}{1 + \Lambda^m [1 - F_1(s)]} dt.$$

Substituting into the seller's objective, (28), and differentiating with respect to  $t_r$ , we see that the optimal threshold type,  $t_r^I$ , satisfies

$$t_r^I - t^S = \int_{t_r^I}^{\bar{t}} \frac{f(t)}{f(t_r^I)} \frac{1 + \Lambda^m [1 - F_1(t_r^I)]}{1 + \Lambda^m [1 - F_1(t)]} dt + \frac{(\Lambda^m + \Lambda^g) [1 - F_1(t_r^I)]}{1 - \Lambda^g [1 - F_1(t_r^I)]} t^S.$$

Note that, by the intermediate value theorem (and because  $\bar{t} > t^S$ ), the first-order condition has a solution. Moreover, it is straightforward to see that, if  $\Lambda^m = 0$ , the first-order condition implies  $V(t_r^I) \geq t^S$ , which holds with equality if and only if  $t^S = 0$ . Thus, if  $t^S = 0$ , as  $r_{RN} = t^{RN}$  and  $t_r^I = t^{RN}$ , we have that the optimal reserve price under loss aversion,  $r_I$ , satisfies  $r_I = r_{RN} (1 - \Lambda^g [1 - F_1(t^{RN})]) < r_{RN}$ . Now suppose that  $\Lambda^m \geq 0$  and  $t^S \geq 0$ . First note that  $\int_{t_r^I}^{\bar{t}} \frac{f(t)}{f(t_r^I)} \frac{1 + \Lambda^m [1 - F_1(t_r^I)]}{1 + \Lambda^m [1 - F_1(t)]} dt > \int_{t_r^I}^{\bar{t}} \frac{f(t)}{f(t_r^I)} dt = \frac{1 - F(t_r^I)}{f(t_r^I)}$ , since  $F_1(t_r^I) < F_1(t)$ . Thus,

$$\begin{aligned} t_r^I - t^S &= \int_{t_r^I}^{\bar{t}} \frac{f(t)}{f(t_r^I)} \frac{1 + \Lambda^m [1 - F_1(t_r^I)]}{1 + \Lambda^m [1 - F_1(t)]} dt + \frac{(\Lambda^m + \Lambda^g) [1 - F_1(t_r^I)]}{1 - \Lambda^g [1 - F_1(t_r^I)]} t^S \\ &\geq \int_{t_r^I}^{\bar{t}} \frac{f(t)}{f(t_r^I)} dt + \frac{(\Lambda^m + \Lambda^g) [1 - F_1(t_r^I)]}{1 - \Lambda^g [1 - F_1(t_r^I)]} t^S = \frac{1 - F(t_r^I)}{f(t_r^I)} + \frac{(\Lambda^m + \Lambda^g) [1 - F_1(t_r^I)]}{1 - \Lambda^g [1 - F_1(t_r^I)]} t^S \geq \frac{1 - F(t_r^I)}{f(t_r^I)}, \end{aligned}$$

where the first inequality is strict if  $\Lambda^m > 0$  and, for any  $\Lambda^m \geq 0$ , the last inequality is strict if  $\Lambda^g t^S > 0$ . Thus, under these conditions we have that  $V(t_r^I) > t^S$ . Since  $V(t)$  is increasing, and  $V(t^{RN}) = t^S$ , it follows that  $t_r^I$  is larger than the risk-neutral threshold  $t^{RN}$ .

Further, observe that the optimal loss-averse reserve price satisfies

$$\begin{aligned}
r_I &= \frac{1 - \Lambda^g[1 - F_1(t_r^I)]}{1 + \Lambda^m[1 - F_1(t_r^I)]} \left( \int_{t_r^I}^{\bar{t}} \frac{f(t)}{f(t_r^I)} \frac{1 + \Lambda^m[1 - F_1(t_r^I)]}{1 + \Lambda^m[1 - F_1(t)]} dt + \left[ \frac{(\Lambda^m + \Lambda^g)[1 - F_1(t_r^I)]}{1 - \Lambda^g[1 - F_1(t_r^I)]} + 1 \right] t^S \right) \\
&= \int_{t_r^I}^{\bar{t}} \frac{f(t)}{f(t_r^I)} \frac{1 - \Lambda^g[1 - F_1(t_r^I)]}{1 + \Lambda^m[1 - F_1(t)]} dt + \frac{1 + \Lambda^m[1 - F_1(t_r^I)]}{1 + \Lambda^m[1 - F_1(t_r^I)]} t^S \\
&< \int_{t_r^I}^{\bar{t}} \frac{f(t)}{f(t_r^I)} dt + t^S = \frac{1 - F(t_r^I)}{f(t_r^I)} + t^S < \frac{1 - F(t^{RN})}{f(t^{RN})} + t^S = r_{RN},
\end{aligned}$$

where the last inequality follows from  $t_r^I > t^{RN}$  and the fact that the hazard rate is decreasing. ■

### Proof of Proposition 8

*Proof.* Recall from the proof of Proposition 7 that the optimal threshold type,  $t_r^I$ , satisfies  $t_r^I - t^S = \int_{t_r^I}^{\bar{t}} \frac{f(t)}{f(t_r^I)} \frac{1 + \Lambda^m[1 - F_1(t_r^I)]}{1 + \Lambda^m[1 - F_1(t)]} dt + \frac{(\Lambda^m + \Lambda^g)[1 - F_1(t)]}{1 - \Lambda^g[1 - F_1(t)]} t^S$ . If  $N \rightarrow \infty$ , then  $F_1(t) \rightarrow 0$  for any  $t < \bar{t}$ . Thus,

$$t_r^I - t^S \rightarrow \frac{1 - F(t_r^I)}{f(t_r^I)} + \frac{\Lambda^m + \Lambda^g}{1 - \Lambda^g} t^S,$$

and therefore  $V(t_r^I) - t^S \rightarrow \frac{\Lambda^m + \Lambda^g}{1 - \Lambda^g} t^S \geq 0$ , where the inequality holds strict if  $t^S \Lambda^m > 0$  or  $t^S \Lambda^g > 0$ . In contrast, the optimal reserve price,  $r_I$ , converges to

$$r_I \rightarrow \frac{1 - \Lambda^g}{1 + \Lambda^m} \frac{1 - F(t_r^I)}{f(t_r^I)} + t^S \leq \frac{1 - \Lambda^g}{1 + \Lambda^m} \frac{1 - F(t^{RN})}{f(t^{RN})} + t^S \leq \frac{1 - F(t^{RN})}{f(t^{RN})} + t^S = r_{RN},$$

where at least one inequality holds strict if  $\Lambda^g > 0$  or  $\Lambda^m > 0$ . ■

### Proof of Proposition 9

*Proof.* Recall that  $\gamma(t) \equiv (1 - \Lambda^g)F_1(t) + \Lambda^g F_1(t)^2$ . The seller's objective, normalized by  $N$ , is:

$$\int_{t_r}^{\bar{t}} \left( \int_{t_r}^t \frac{\gamma'(x)x}{1 + \Lambda^m} e^{\frac{2\Lambda^m[F_1(t) - F_1(x)]}{1 + \Lambda^m}} dx + P(t_r) e^{\frac{2\Lambda^m[F_1(t) - F_1(t_r)]}{1 + \Lambda^m}} \right) f(t) dt + \frac{F(t_r)^N t^S}{N}.$$

The first-order condition with respect to the threshold type reveals that the maximizer,  $t_r^{II}$ , satisfies

$$\left( P'(t_r^{II}) - \frac{2\Lambda^m}{1 + \Lambda^m} f_1(t_r^{II}) P(t_r^{II}) - \frac{\gamma'(t_r^{II}) t_r^{II}}{1 + \Lambda^m} \right) \int_{t_r^{II}}^{\bar{t}} e^{\frac{2\Lambda^m[F_1(t) - F_1(t_r^{II})]}{1 + \Lambda^m}} f(t) dt = P(t_r^{II}) f(t_r^{II}) - f(t_r^{II}) F_1(t_r^{II}) t^S = 0. \quad (30)$$

Moreover note that

$$P'(t_r) = \frac{\gamma'(t_r) t_r}{1 + \Lambda^m[1 - F_1(t_r)]} + \frac{\gamma(t_r)}{1 + \Lambda^m[1 - F_1(t_r)]} + \frac{\Lambda^m f_1(t_r)}{1 + \Lambda^m[1 - F_1(t_r)]} P(t_r).$$

Using the above relationship, the bracket in (30) is

$$\begin{aligned}
& \frac{\Lambda^m F_1(t_r^{II}) \gamma'(t_r^{II}) t_r^{II}}{(1 + \Lambda^m [1 - F_1(t_r^{II})]) (1 + \Lambda^m)} + \frac{\gamma(t_r^{II})}{(1 + \Lambda^m [1 - F_1(t_r^{II})])} \\
& - f_1(t_r^{II}) P(t_r^{II}) \Lambda^m \frac{[1 + \Lambda^m - 2\Lambda^m F_1(t_r^{II})]}{(1 + \Lambda^m [1 - F_1(t_r^{II})]) (1 + \Lambda^m)} \\
= & \frac{\Lambda^m f_1(t_r^{II}) P(t_r^{II})}{1 + \Lambda^m} + \frac{\Lambda^m \Lambda^g F_1(t_r^{II})^2 f_1(t_r^{II}) t_r^{II}}{(1 + \Lambda^m [1 - F_1(t_r^{II})]) (1 + \Lambda^m)} + \frac{\gamma(t_r^{II})}{(1 + \Lambda^m [1 - F_1(t_r^{II})])} \\
& - \frac{f_1(t_r^{II}) P(t_r^{II}) \Lambda^m [1 + \Lambda^m - 2\Lambda^m F_1(t_r^{II})]}{(1 + \Lambda^m [1 - F_1(t_r^{II})]) (1 + \Lambda^m)} \\
= & \frac{f_1(t_r^{II}) P(t_r^{II}) \Lambda^m \Lambda^m F_1(t_r^{II})}{(1 + \Lambda^m [1 - F_1(t_r^{II})]) (1 + \Lambda^m)} + \frac{\gamma(t_r^{II})}{(1 + \Lambda^m [1 - F_1(t_r^{II})])} + \frac{\Lambda^m \Lambda^g F_1(t_r^{II})^2 f_1(t_r^{II}) t_r^{II}}{(1 + \Lambda^m [1 - F_1(t_r^{II})]) (1 + \Lambda^m)}.
\end{aligned}$$

Thus, dividing (30) by  $f(t_r^{II}) \gamma(t_r^{II}) / [1 + \Lambda^m [1 - F_1(t_r^{II})]] > 0$  and re-arranging yields

$$\begin{aligned}
\left( \frac{f_1(t_r^{II}) t_r^{II} \Lambda^m \Lambda^m F_1(t_r^{II})}{(1 + \Lambda^m [1 - F_1(t_r^{II})]) (1 + \Lambda^m)} + \frac{1}{f(t_r^{II})} + \frac{\Lambda^m \Lambda^g F_1(t_r^{II})^2 f_1(t_r^{II}) t_r^{II}}{(1 + \Lambda^m) f(t_r^{II}) \gamma(t_r^{II})} \right) \int_{t_r^{II}}^{\bar{t}} e^{\frac{2\Lambda^m [F_1(t) - F_1(t_r^{II})]}{1 + \Lambda^m}} f(t) dt \quad (31) \\
+ \frac{F_1(t_r^{II}) (1 + \Lambda^m [1 - F_1(t_r^{II})])}{\gamma(t_r^{II})} t^S - t_r^{II} = 0.
\end{aligned}$$

Note first that  $\frac{F_1(t_r^{II}) (1 + \Lambda^m [1 - F_1(t_r^{II})])}{\gamma(t_r^{II})} t^S = \frac{1 + \Lambda^m [1 - F_1(t_r^{II})]}{1 - \Lambda^g [1 - F_1(t_r^{II})]} t^S > t^S$  if and only if  $\Lambda^g t^S > 0$  or  $\Lambda^m t^S > 0$ . Moreover, it is easy to see that the first line of (31) is larger than  $[1 - F_1(t_r^{II})] / f(t_r^{II})$  if and only if  $\Lambda^m > 0$ . Whenever one of the two conditions is satisfied, we have that the left-hand side of (31) is larger than  $-V(t_r^{II}) + t^S$ . Thus,  $-V(t_r^{II}) + t^S < 0 \Leftrightarrow V(t_r^{II}) > t^S$ . Since,  $V$  is increasing,  $t_r^{II} > t^{RN}$ , where  $t^{RN}$  satisfies  $V(t^{RN}) = t^S$ .

Thus, if  $\Lambda^m = 0$  the no-trade probability is larger than under risk neutrality if and only if  $\Lambda^g t^S > 0$ . If  $\Lambda^g > 0$ , the probability of no trade is larger than under risk neutrality if and only if  $\Lambda^m > 0$  or  $t^S > 0$ .

Finally note that under risk aversion, the probability of no trade is either lower than that under risk neutrality (FPA) or the same (SPA). Thus, the claim follows. ■

### Proof of Proposition 10

*Proof.* In the FPA, the virtual valuation  $V$  evaluated at the optimal threshold,  $t_r^I$ , satisfies:

$$V(t_r^I) = \int_{t_r^I}^{\bar{t}} \frac{f(t)}{f(t_r^I)} \frac{\Lambda^m [F_1(t) - F_1(t_r^I)]}{1 + \Lambda^m (1 - F_1(t))} dt + \frac{F_1(t_r^I) (1 + \Lambda^m [1 - F_1(t_r^I)])}{\gamma(t_r^I)} t^S. \quad (32)$$

In contrast, in the SPA the optimal threshold  $t_r^{II}$  satisfies (by (31)):

$$V(t_r^{II}) \geq \int_{t_r^{II}}^{\bar{t}} (e^{\frac{2\Lambda^m [F_1(t) - F_1(t_r^{II})]}{1 + \Lambda^m}} - 1) \frac{f(t)}{f(t_r^{II})} dt + \frac{F_1(t_r^{II}) (1 + \Lambda^m [1 - F_1(t_r^{II})])}{\gamma(t_r^{II})} t^S. \quad (33)$$

Assume first that  $\Lambda^m \leq 1$ . To show that the right-hand side of (33) is larger than that of (32), it is

sufficient to show that

$$e^{\frac{2\Lambda^m[F_1(t)-F_1(t_r^{II})]}{1+\Lambda^m}} - 1 \geq \Lambda^m[F_1(t) - F_1(t_r^{II})]. \quad (34)$$

We now show that the left-hand side of (34) is larger than the right-hand side, by linearly approximating the convex function  $e^{\frac{2\Lambda^m x}{1+\Lambda^m}}$  from below:

$$\begin{aligned} e^{\frac{2\Lambda^m[F_1(t)-F_1(t_r^{II})]}{1+\Lambda^m}} - 1 &\geq 1 + \frac{2\Lambda^m}{1+\Lambda^m} e^{\frac{2\Lambda^m[0]}{1+\Lambda^m}} [F_1(t) - F_1(t_r^{II})] - 1 \\ &= \frac{2\Lambda^m}{1+\Lambda^m} [F_1(t) - F_1(t_r^{II})] \geq \Lambda^m [F_1(t) - F_1(t_r^{II})] \\ &\Leftrightarrow \Lambda^m \leq 1. \end{aligned}$$

Thus, the threshold type in the SPA is larger than that in the FPA if  $\Lambda^m \leq 1$ .

Next, consider an arbitrary  $\Lambda^m > 1$ . First, it is easy to see that, if there is weakly more exclusion in the FPA than in the SPA for  $t^S = 0$ , then this result continuous to hold for any  $t^S \geq 0$ . Thus, assume  $t^S = 0$  for the moment. In the SPA we have that, by (31),

$$V(t_r^{II}) = \left( \frac{f_1(t_r^{II})t_r^{II}\Lambda^m\Lambda^m F_1(t_r^{II})}{(1+\Lambda^m[1-F_1(t_r^{II})])(1+\Lambda^m)} + \frac{1}{f(t_r^{II})} + \frac{\Lambda^m\Lambda^g F_1(t_r^{II})^2 f_1(t_r^{II})t_r^{II}}{(1+\Lambda^m)f(t_r^{II})\gamma(t_r^{II})} \right) \int_{t_r^{II}}^{\bar{t}} e^{\frac{2\Lambda^m[F_1(t)-F_1(t_r^{II})]}{1+\Lambda^m}} f(t) dt - \frac{1-F(t_r^{II})}{f(t_r^{II})}. \quad (35)$$

Observe that, for  $\Lambda^m \rightarrow \infty$ , the solution to (32) converges to  $\bar{t}$  as the first term of the right-hand side of (32) converges to  $\int_{t_r^{II}}^{\bar{t}} \frac{f(t)}{f(t_r^{II})} \frac{F_1(t)-F_1(t_r^{II})}{1-F_1(t)} dt = \infty$ . In contrast, for  $\Lambda^m \rightarrow \infty$ , the right-hand side of (35) is bounded. Thus, for sufficiently large  $\Lambda^m$  the optimal threshold of the FPA is (weakly) larger than that of the SPA. ■

### Proof of Proposition 11

*Proof.* It is straightforward to observe that, if  $N \rightarrow \infty$ , the maximizer  $t_r^{II}$  satisfying (31), converges to the solution of

$$\frac{1-F(t_r^{II})}{f(t_r^{II})} + \frac{1+\Lambda^m}{1-\Lambda^g} t^S - t_r^{II} = 0.$$

Thus,  $V(t_r^{II}) = \frac{1+\Lambda^m}{1+\Lambda^g} t^S > t^S$  if and only if  $t^S > 0$ . Since  $V$  is an increasing function and  $V(t^{RN}) = t^S$ , the claim follows. The proof of the statement regarding the reserve price follows exactly that of the FPA, given in the proof of Proposition (8). ■

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## B Supplementary Appendix B

This appendix contains the computationally more intensive proofs of Lemma 3 and Lemma 4; the latter one is introduced in the proof of Proposition 6 in Appendix A.

### Proof of Lemma 3

*Proof.* Consider bidder type  $t$  who behaves as type  $\tilde{t}$ . The bidder's dis-utility caused by the monetary payment,  $\mathcal{T}$ , takes the following form:

$$\begin{aligned} \mathcal{T}(\tilde{t}) &\equiv F_1(t_r)r + \int_{t_r}^{\tilde{t}} \beta_{II}^*(x)f_1(x)dx + \Lambda^m[1 - F_1(\tilde{t})] \left( \int_{t_r}^{\tilde{t}} \beta_{II}^*(x)f_1(x)dx + F_1(t_r)r \right) \\ &+ \Lambda^m \int_{t_r}^{\tilde{t}} \left( \int_{t_r}^x [\beta_{II}^*(x) - \beta_{II}^*(v)]f_1(v)dv \right) f_1(x)dx + \Lambda^m F_1(t_r) \int_{t_r}^{\tilde{t}} (\beta_{II}^*(x) - r)f_1(x)dx. \end{aligned}$$

The expected payment from bidder type  $\tilde{t}$  is  $P(\tilde{t}) = F_1(t_r)r + \int_{t_r}^{\tilde{t}} \beta_{II}^*(x)f_1(x)ds$ , and thus  $P(t_r) = F_1(t_r)r$ . Then, the dis-utility reads

$$\mathcal{T}(\tilde{t}) = P(\tilde{t}) + \Lambda^m[1 - F_1(\tilde{t})]P(\tilde{t}) \quad (36)$$

$$+ \Lambda^m \int_{t_r}^{\tilde{t}} \left( \int_{t_r}^x [\beta_{II}^*(x) - \beta_{II}^*(v)]f_1(v)dv \right) f_1(x)dx + \Lambda^m F_1(t_r)[P(\tilde{t}) - P(t_r)] - \Lambda^m P(t_r)[F_1(\tilde{t}) - F_1(t_r)].$$

Next note that

$$\begin{aligned} &\int_{t_r}^{\tilde{t}} \left( \int_{t_r}^x [\beta_{II}^*(x) - \beta_{II}^*(v)]f_1(v)dv \right) f_1(x)dx \quad (37) \\ &= \int_{t_r}^{\tilde{t}} \beta_{II}^*(x)f_1(x)F_1(x)dx - [P(\tilde{t}) - P(t_r)]F_1(t_r) - F_1(\tilde{t}) \int_{t_r}^{\tilde{t}} \beta_{II}^*(x)f_1(x)dx + \int_{t_r}^{\tilde{t}} \beta_{II}^*(x)f_1(x)F_1(x)dx. \end{aligned}$$

Observe that  $[P(\tilde{t}) - P(t_r)]' = \beta_{II}^*(t)f_1(t)$ . Thus,

$$\begin{aligned} \int_{t_r}^{\tilde{t}} \beta_{II}^*(x)f_1(x)F_1(x)dx &= [P(\tilde{t}) - P(t_r)]F_1(\tilde{t}) - \int_{t_r}^{\tilde{t}} [P(x) - P(t_r)]f_1(x)dx \\ &= F_1(\tilde{t})P(\tilde{t}) - F_1(t_r)P(t_r) - \int_{t_r}^{\tilde{t}} P(x)f_1(x)dx. \end{aligned}$$

Thus, (37) is

$$(F_1(\tilde{t}) - F_1(t_r))[P(\tilde{t}) + P(t_r)] - 2 \int_{t_r}^{\tilde{t}} P(x)f_1(x)dx. \quad (38)$$

Substituting (38) into (36) we have

$$\mathcal{T}(\tilde{t}) = P(\tilde{t})(1 + \Lambda^m) - \Lambda^m F_1(t_r)P(t_r) - \Lambda^m 2 \int_{t_r}^{\tilde{t}} P(x)f_1(x)dx.$$

Recall that  $\gamma(t) \equiv (1 - \Lambda^g)F_1(t) + \Lambda^g F_1(t)^2$ . Define  $\hat{\gamma}(t) \equiv \gamma(t)t - \int_{t_r}^t \gamma(s)ds$ ,  $C \equiv \Lambda^m F_1(t_r)P(t_r)$ , and  $y(t) \equiv \int_{t_r}^t P(x)f_1(x)dx$ . Then, applying the envelope theorem, we have that in equilibrium

$$\begin{aligned} \hat{\gamma}(t) = \mathcal{T}(t) &\Leftrightarrow \hat{\gamma}(t) + C = y'(t) \frac{1 + \Lambda^m}{f_1(t)} - 2\Lambda^m y(t) \\ &\Leftrightarrow \frac{f_1(t)(\hat{\gamma}(t) + C)}{1 + \Lambda^m} = y'(t) - \frac{2\Lambda^m f_1(t)}{1 + \Lambda^m} y(t). \end{aligned}$$

The solution to this differential equation, given the boundary condition  $y(t_r) = 0$ , is

$$y(t) = e^{\frac{2\Lambda^m F_1(t)}{1 + \Lambda^m}} \int_{t_r}^t \frac{f_1(x)(\hat{\gamma}(x) + C)}{1 + \Lambda^m} e^{-\frac{2\Lambda^m F_1(x)}{1 + \Lambda^m}} dx.$$

Differentiating both sides and dividing by  $f_1(t)$  gives us  $P(t)$ . It is

$$\begin{aligned} P(t) &= 2\Lambda^m e^{\frac{2\Lambda^m F_1(t)}{1 + \Lambda^m}} \int_{t_r}^t \frac{f_1(x)(\hat{\gamma}(x) + C)}{(1 + \Lambda^m)^2} e^{-\frac{2\Lambda^m F_1(x)}{1 + \Lambda^m}} dx + \frac{\hat{\gamma}(t) + C}{1 + \Lambda^m} \\ &= \int_{t_r}^t \frac{\hat{\gamma}'(x)}{1 + \Lambda^m} e^{\frac{2\Lambda^m [F_1(t) - F_1(x)]}{1 + \Lambda^m}} dx + P(t_r) e^{\frac{2\Lambda^m [F_1(t) - F_1(t_r)]}{1 + \Lambda^m}}, \end{aligned}$$

where the last equality follows from partial integration and from  $P(t_r) = (\hat{\gamma}(t_r) + C)/(1 + \Lambda^m)$ .

Taking the derivative with respect to  $t$  and dividing by  $f_1(t)$  reveals the bidding function:

$$\beta_{II}^*(t) = 2\Lambda^m \int_{t_r}^t \frac{\gamma'(x)x}{(1 + \Lambda^m)^2} e^{\frac{2\Lambda^m [F_1(t) - F_1(x)]}{1 + \Lambda^m}} dx + \frac{\gamma'(t)t}{f_1(t)(1 + \Lambda^m)} + 2\Lambda^m \frac{P(t_r) e^{\frac{2\Lambda^m [F_1(t) - F_1(t_r)]}{1 + \Lambda^m}}}{1 + \Lambda^m},$$

where  $\gamma'(t)t = \hat{\gamma}'(t) = f_1(t)t \left( (1 - \Lambda^g) + 2\Lambda^g F_1(t) \right)$ . Finally, we need to show that the bid of the threshold type is above the reserve price; that is,

$$\begin{aligned} \beta_{II}^*(t_r) &= \frac{\gamma'(t_r)t_r}{f_1(t_r)(1 + \Lambda^m)} + \frac{2\Lambda^m}{1 + \Lambda^m} F_1(t_r)r \geq r \\ &\Leftrightarrow [1 - \Lambda^g + 2\Lambda^g F_1(t_r)]t_r > [1 - \Lambda^g + \Lambda^g F_1(t_r)]t_r \frac{1 + \Lambda^m [1 - 2F_1(t_r)]}{1 + \Lambda^m [1 - F_1(t_r)]}, \end{aligned}$$

which is true. ■

### Proof of Lemma 4

*Proof.* We first bound  $ER_I$  from above and show that this bound is convex in  $q$ . Using (10) (and replacing the allocation policy  $F_1$  with  $q$ ) it is straightforward to observe that  $ER_I$  is bounded from above by

$$\bar{ER}_I(q) = \int_{\underline{t}}^{\bar{t}} \frac{(1 - \Lambda^g)q(t)t - \int_{\underline{t}}^t q(s)ds + \Lambda^g[q(t)^2t - \int_{\underline{t}}^t q(s)^2ds]}{1 + \Lambda^m[1 - F_1(t)]},$$

since  $[1 + \Lambda^m(1 - q(t))] \geq [1 + \Lambda^m(1 - F_1(t))]$  as  $F_1(t) \geq q(t)$  for all  $t$ . Moreover, if  $q = q_{t_r}$ , then  $q_{t_r}(t) = F_1(t)$  for  $t \geq t_r$  and  $q_{t_r}(t) = 0$  otherwise. Thus,  $\bar{ER}_I(q_{t_r}) = ER_I(q_{t_r})$ .

Recall the definition  $\tilde{H}(t) \equiv \int_{\underline{t}}^t \frac{f(s)}{1 + \Lambda^m[1 - F_1(s)]} ds$  and  $\tilde{h}(t) \equiv \tilde{H}'(t) = \frac{f(t)}{1 + \Lambda^m[1 - F_1(t)]}$ . Applying partial integration to the maximized upper bound, we observe that

$$\bar{ER}_I(q) = \int_{\underline{t}}^{\bar{t}} [(1 - \Lambda^g)q(t) + \Lambda^g q(t)^2] \left( t - \frac{\tilde{H}(\bar{t}) - \tilde{H}(t)}{\tilde{h}(t)} \right) \tilde{h}(t) dt, \quad (39)$$

which is obviously a convex functional.

Regarding the SPA, using the steps from the proof of Lemma 3 (and replacing the allocation policy  $F_1$  with  $q$ ), the seller's expected payment collected from type- $t$  bidder given  $q$ ,  $P(t; q)$ , is

$$P(t; q) \equiv \int_{t_r}^t \frac{q'(x)x\{(1 - \Lambda^g) + 2\Lambda^g q(x)\}}{1 + \Lambda^m} e^{\frac{2\Lambda^m[q(t) - q(x)]}{1 + \Lambda^m}} dx + P(t_r; q) e^{\frac{2\Lambda^m[q(t) - q(t_r)]}{1 + \Lambda^m}}, \quad (40)$$

where  $t_r$  is the smallest type such that  $q(t) > 0$  and  $P(t_r; q) = q(t_r) \frac{1 - \Lambda^g[1 - q(t_r)]}{1 + \Lambda^m[1 - q(t_r)]}$ . Observe that

$$\begin{aligned} & e^{\frac{2\Lambda^m[q(t)]}{1 + \Lambda^m}} \int_{t_r}^t 2\Lambda^g \frac{q'(x)q(x)x}{1 + \Lambda^m} e^{\frac{2\Lambda^m[-q(x)]}{1 + \Lambda^m}} dx = -e^{\frac{2\Lambda^m[q(t)]}{1 + \Lambda^m}} \frac{2\Lambda^g}{2\Lambda^m} \int_{t_r}^t q(x)x \left( e^{-\frac{2\Lambda^m q(x)}{1 + \Lambda^m}} \right)' dx \\ &= -e^{\frac{2\Lambda^m[q(t)]}{1 + \Lambda^m}} \frac{2\Lambda^g}{2\Lambda^m} \left( q(t)t e^{-\frac{2\Lambda^m q(t)}{1 + \Lambda^m}} - q(t_r)t_r e^{-\frac{2\Lambda^m q(t_r)}{1 + \Lambda^m}} - \int_{t_r}^t q(x) e^{-\frac{2\Lambda^m q(x)}{1 + \Lambda^m}} dx - \int_{t_r}^t q'(x)x e^{-\frac{2\Lambda^m q(x)}{1 + \Lambda^m}} dx \right) \\ &= \frac{\Lambda^g}{\Lambda^m} \left( e^{\frac{2\Lambda^m[q(t)]}{1 + \Lambda^m}} \left\{ \int_{t_r}^t q'(x)x e^{-\frac{2\Lambda^m q(x)}{1 + \Lambda^m}} dx + \int_{t_r}^t q(x) e^{-\frac{2\Lambda^m q(x)}{1 + \Lambda^m}} dx \right\} + q(t_r)t_r e^{\frac{2\Lambda^m[q(t) - q(t_r)]}{1 + \Lambda^m}} - q(t)t \right). \end{aligned}$$

Moreover,

$$\int_{t_r}^t \frac{q'(x)x}{1 + \Lambda^m} e^{\frac{2\Lambda^m[q(t) - q(x)]}{1 + \Lambda^m}} dx = \frac{1}{2\Lambda^m} \left( \int_{t_r}^t e^{\frac{2\Lambda^m[q(t) - q(x)]}{1 + \Lambda^m}} dx - t + t_r e^{\frac{2\Lambda^m[q(t) - q(t_r)]}{1 + \Lambda^m}} \right).$$

Merging the relevant terms, (40) becomes

$$\begin{aligned} P(t; q) &= \frac{\Lambda^g + \Lambda^m}{\Lambda^m} \frac{1}{2\Lambda^m} \left( \int_{t_r}^t e^{\frac{2\Lambda^m[q(t) - q(x)]}{1 + \Lambda^m}} dx - t + t_r e^{\frac{2\Lambda^m[q(t) - q(t_r)]}{1 + \Lambda^m}} \right) \\ &+ \frac{\Lambda^g}{\Lambda^m} \left( e^{\frac{2\Lambda^m[q(t)]}{1 + \Lambda^m}} \int_{t_r}^t q(x) e^{-\frac{2\Lambda^m q(x)}{1 + \Lambda^m}} dx + q(t_r)t_r e^{\frac{2\Lambda^m[q(t) - q(t_r)]}{1 + \Lambda^m}} - q(t)t \right) + P(t_r; q) e^{\frac{2\Lambda^m[q(t) - q(t_r)]}{1 + \Lambda^m}}, \end{aligned}$$

and, in turn,

$$ER_{II}(q) = \int_{t_r}^{\bar{t}} \left( \frac{\Lambda^g + \Lambda^m}{\Lambda^m} \frac{1}{2\Lambda^m} \left( \int_{t_r}^t e^{\frac{2\Lambda^m[q(t)-q(x)]}{1+\Lambda^m}} dx - t + t_r e^{\frac{2\Lambda^m[q(t)-q(t_r)]}{1+\Lambda^m}} \right) \right. \\ \left. + \frac{\Lambda^g}{\Lambda^m} \left( e^{\frac{2\Lambda^m[q(t)]}{1+\Lambda^m}} \int_{t_r}^t q(x) e^{\frac{-2\Lambda^m q(x)}{1+\Lambda^m}} dx + q(t_r) t_r e^{\frac{2\Lambda^m[q(t)-q(t_r)]}{1+\Lambda^m}} - q(t)t \right) + P(t_r; q) e^{\frac{2\Lambda^m[q(t)-q(t_r)]}{1+\Lambda^m}} \right) f(t) dt. \quad (41)$$

Note that an upper bound for (41) is

$$\bar{E}R_{II}(q) = \int_{t_r}^{\bar{t}} \left( \frac{\Lambda^g + \Lambda^m}{\Lambda^m} \frac{1}{2\Lambda^m} \left( \int_{t_r}^t e^{\frac{2\Lambda^m[q(t)-q(x)]}{1+\Lambda^m}} dx - t + t_r e^{\frac{2\Lambda^m[q(t)-q(t_r)]}{1+\Lambda^m}} \right) \right. \\ \left. + \frac{\Lambda^g}{\Lambda^m} \left( e^{\frac{2\Lambda^m[q(t)]}{1+\Lambda^m}} \int_{t_r}^t F_1(x) e^{\frac{-2\Lambda^m q(x)}{1+\Lambda^m}} dx + q(t_r) t_r e^{\frac{2\Lambda^m[q(t)-q(t_r)]}{1+\Lambda^m}} - q(t)t \right) + \bar{P}(t_r; q) e^{\frac{2\Lambda^m[q(t)-q(t_r)]}{1+\Lambda^m}} \right) f(t) dt, \quad (42)$$

as  $F_1(t) \geq q(t)$ . Note that we replaced  $q(t)$  with  $F_1(t)$  in the first integral of the second line and  $\bar{P}(t_r; q) \equiv q(t_r) \frac{1-\Lambda^g[1-q(t_r)]}{1+\Lambda^m[1-F_1(t_r)]} \geq P(t_r; q)$ . Moreover, if  $q = q_{t_r}$ , it holds that  $\bar{E}R_{II}(q_{t_r}) = ER_{II}(q_{t_r})$  as  $F_1(t) = q(t)$  for all  $t \geq t_r$  under a deterministic reserve price. As  $q(t) - q(x) \geq 0$  for  $t \geq x$ ,  $e^x$  is convex, and  $\bar{P}(t_r; q)$  is convex and increasing in  $q(t_r)$  (and thus in  $q$ ), it holds that  $\bar{E}R_{II}$  is a convex functional. ■